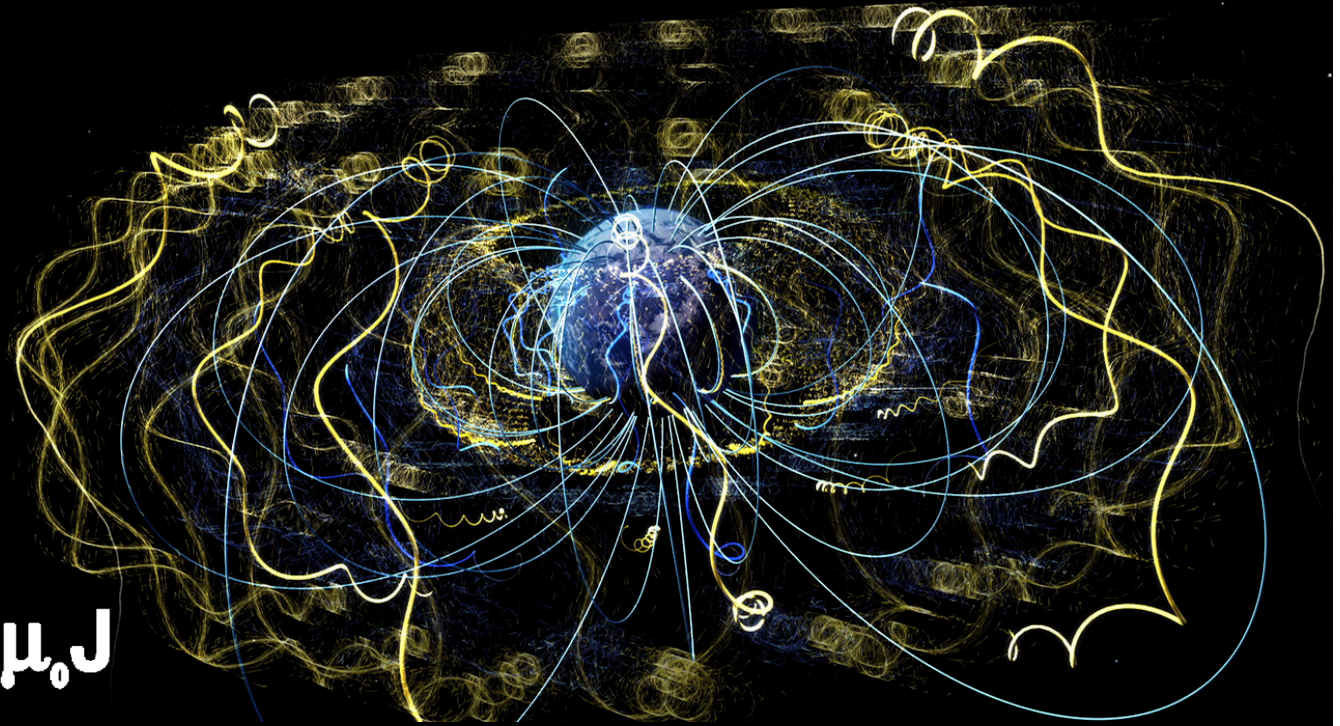


$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



# Electricity and Magnetism II: 3812

Professor Jasper Halekas  
Virtual by Zoom!  
MWF 9:30-10:20 Lecture

# Radiation Simulation

PAUSED

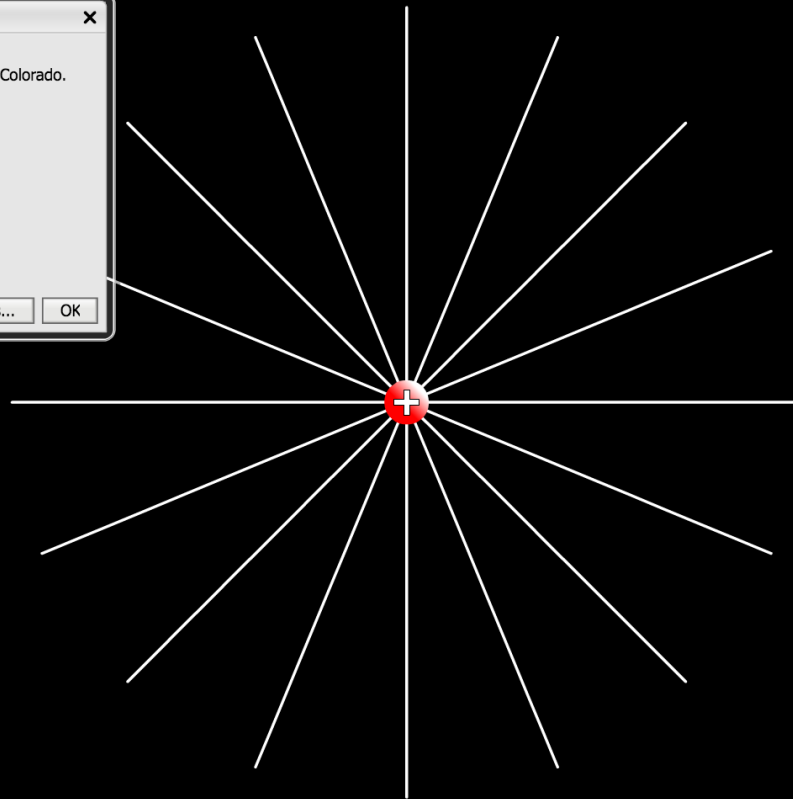
About Radiating Charge


**PhET Interactive Simulations**  
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Visit <http://phet.colorado.edu>

**Radiating Charge**  
Version: 1.03.00 (71951)  
Build Date: Feb 1, 2013

Flash Version: MAC 32,0,0,156  
OS: Mac OS 10.11.6

Software Agreement... Credits... OK





**Stop Charge**

Manual

No Friction

Linear

Sinusoidal

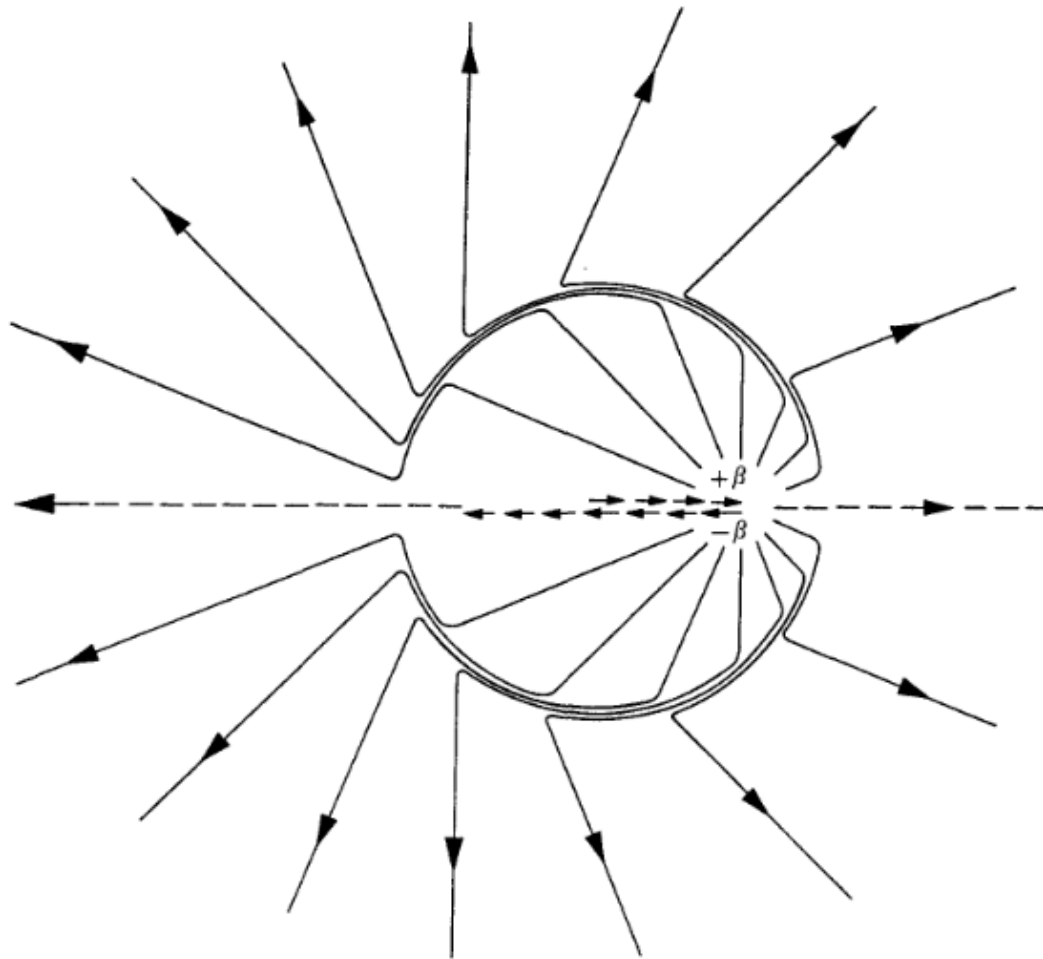
Circular

Bump

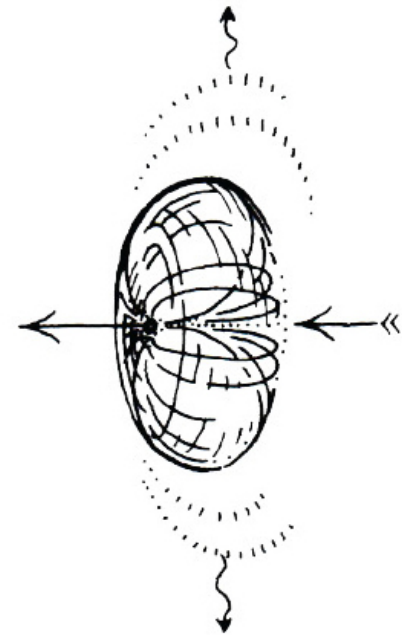
Show velocity

Reset

# Radiation from Accelerated Point Charges



RADIATION FROM A POINT-CHARGE



AN ACCELERATING POINT-CHARGE RELEASES A FRONT OF RADIATION IN THE SHAPE OF A TOROID.

Ch. 11.2

## Radiation From Point Charges

From Ch. 10

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\Delta r}{(\Delta r - \bar{u})^3} [(c^2 - v^2)\vec{u} + \Delta\vec{r} \times (\vec{u} \times \vec{a})]$$

$$w/ \quad \vec{u} = c\hat{\Delta r} - \vec{v}$$

$$\Delta\vec{r} = \vec{r} - \vec{w}(t_r)$$

$$\vec{v} = d\vec{w}/dt|_{t_r}$$

$$\vec{a} = d\vec{v}/dt|_{t_r}$$

$$\Delta r = c(t - t_r)$$

$$\vec{B}(\vec{r}, t) = \frac{1}{c} \hat{\Delta r} \times \vec{E}(\vec{r}, t)$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$= \frac{1}{\mu_0 c} [E^2 \hat{\Delta r} - (\hat{\Delta r} \cdot \vec{E}) \vec{E}]$$

— Many terms!

— For radiation, we only care about terms that fall off as  $1/r^2$  or slower

— These are the acceleration terms

$$\vec{E}_{\text{rad}} = \frac{q}{4\pi\epsilon_0} \frac{\Delta r}{(\Delta \vec{r} \cdot \vec{u})^3} [\Delta \vec{r} \times (\vec{u} \times \vec{a})]$$

$$\vec{S}_{\text{rad}} = \frac{1}{\mu_0 c} E_{\text{rad}}^2 \hat{\Delta r}$$

(other term drops since it involves  $\hat{\Delta r} \cdot \vec{E}_{\text{rad}} = 0$ )

## Low-Velocity Limit

$$v \ll c \Rightarrow \vec{u} \sim c \hat{\Delta r}$$

$$\Rightarrow \Delta \vec{r} \cdot \vec{u} \sim c \Delta r$$

$$\Rightarrow \vec{E}_{\text{rad}} \approx \frac{q}{4\pi\epsilon_0} \frac{1}{c^3 \Delta r^2} [\Delta r \times (c \hat{\Delta r} \times \vec{a})]$$

$$= \frac{q}{4\pi\epsilon_0 c^2 \Delta r} [\hat{\Delta r} \times (\hat{\Delta r} \times \vec{a})]$$

$$= \frac{\mu_0 q}{4\pi \Delta r} [(\hat{\Delta r} \cdot \vec{a}) \hat{\Delta r} - \vec{a}]$$

$$\vec{S}_{\text{rad}} = \frac{1}{\mu_0 c} \vec{E}_{\text{rad}} \vec{E}_{\text{rad}} \hat{\Delta r}$$

$$= \frac{\mu_0 q^2}{16\pi^2 c \Delta r^2} [(\hat{\Delta r} \cdot \vec{a})^2 + a^2 - 2(\hat{\Delta r} \cdot \vec{a}) \hat{\Delta r} \cdot \vec{a}] \hat{\Delta r}$$

$$= \frac{\mu_0 q^2}{16\pi^2 c \Delta r^2} [a^2 - (\hat{\Delta r} \cdot \vec{a})^2] \hat{\Delta r}$$

## Low velocity radiation in spherical coordinates

Define  $\theta$  as angle between  
 $\hat{\Delta r}$  &  $\vec{a}$

$$\Rightarrow \hat{\Delta r} \cdot \vec{a} = a \cos \theta$$

$$\Rightarrow a^2 - (\hat{\Delta r} \cdot \vec{a})^2 = a^2 \sin^2 \theta$$

$$\text{So } \boxed{\vec{S}_{\text{rad}} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^2 \theta}{\Delta r^2} \hat{\Delta r}}$$

$$P_{\text{rad}} = \oint \vec{S} \cdot d\vec{a}$$

$$\boxed{P_{\text{rad}} = \frac{\mu_0 q^2 a^2}{6\pi c}}$$

Same as Larmor formula  
we derived from multipole  
expansion of radiation

## Compare

### Electric Dipole

$$\vec{S}_{\text{rad}} = \frac{\mu_0 p_0^2 \omega^4}{16 \pi^2 c} \frac{\sin^2 \theta}{r^2} \cos^2(\omega(t - r/c)) \hat{r}$$

$$S_{\text{rad}} = \frac{\mu_0 p_0^2 \omega^4}{32 \pi^2 c} \frac{\sin^2 \theta}{r^2} \hat{r}$$

### Accelerated Charge

$$\vec{S}_{\text{rad}} = \langle \vec{S}_{\text{rad}} \rangle = \frac{\mu_0 q^2 a^2}{16 \pi^2 c} \frac{\sin^2 \theta}{\Delta r^2} \Delta \hat{r}$$

— Same angular and radial dependence

— But radiation from point charge doesn't come in ripples