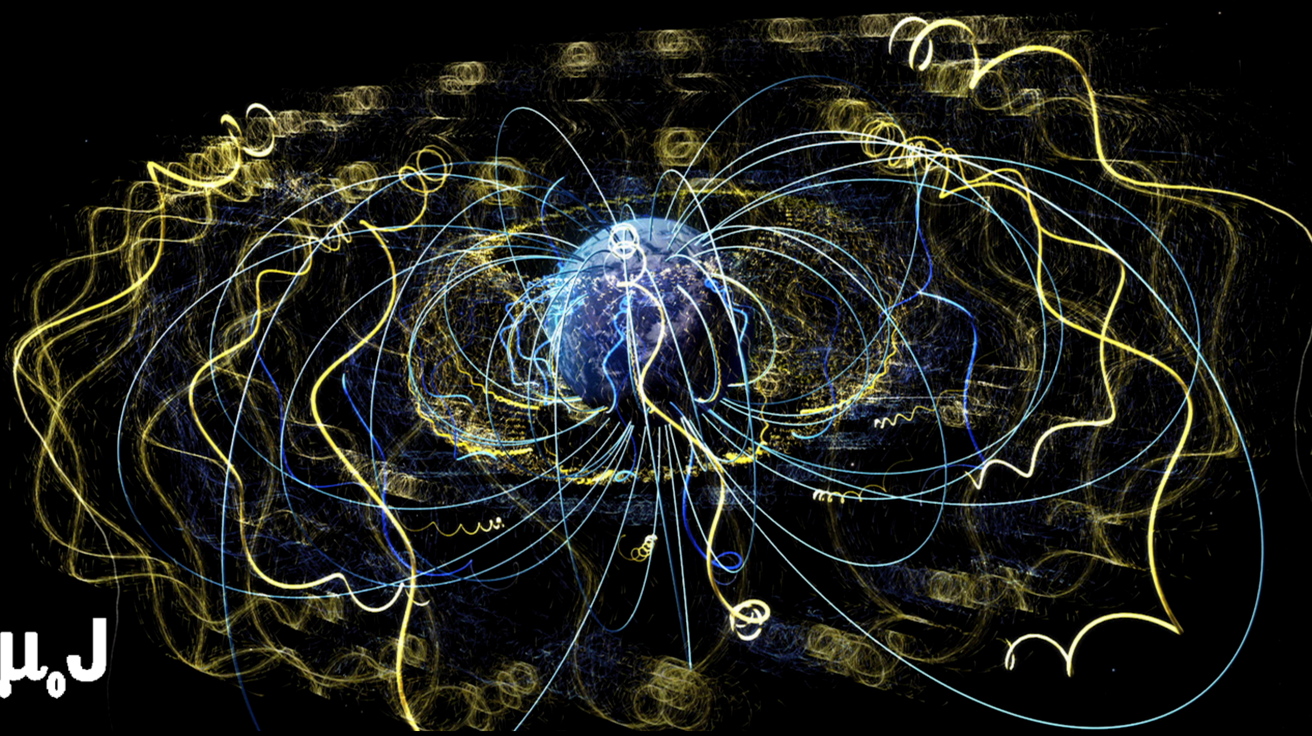
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



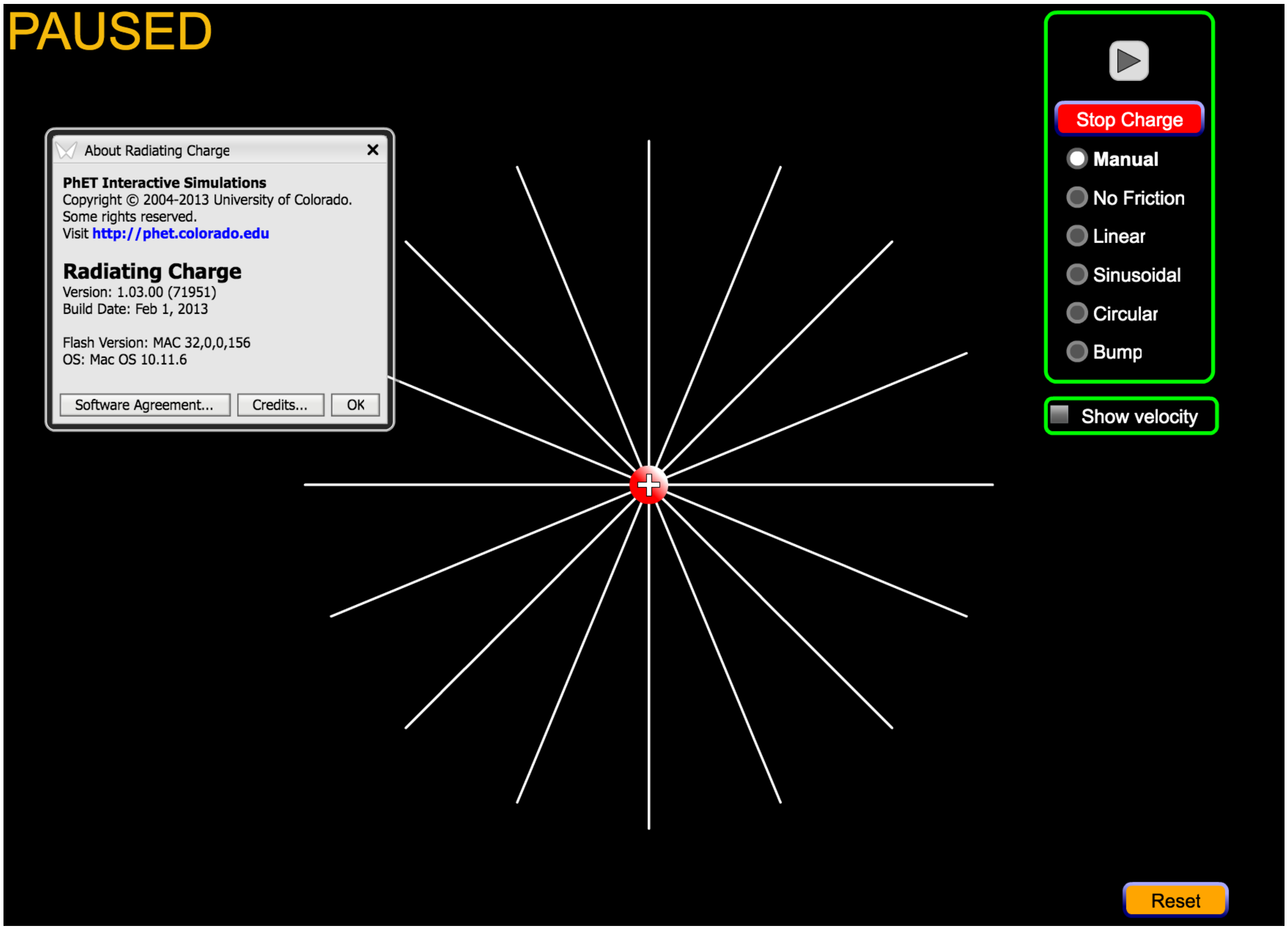
Electricity and Magnetism II: 3812

Professor Jasper Halekas

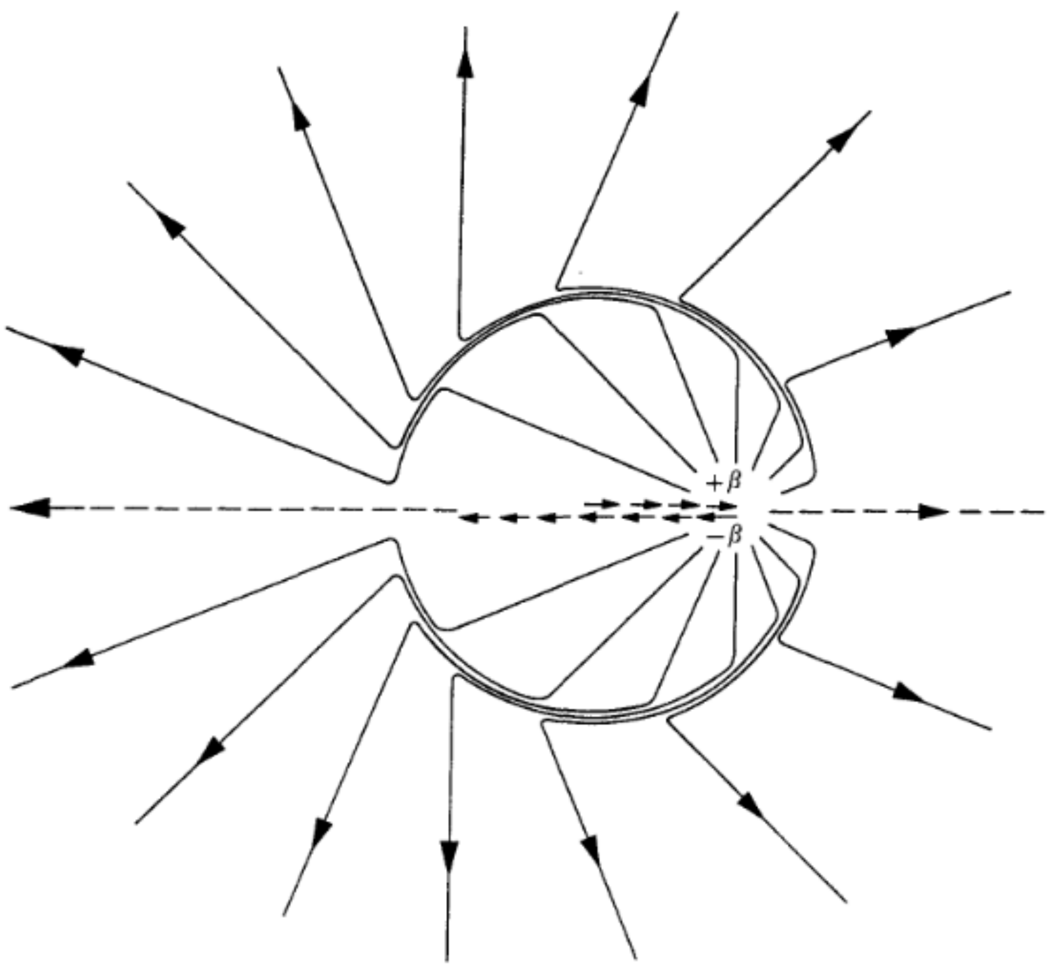
Virtual by Zoom!

MWF 9:30-10:20 Lecture

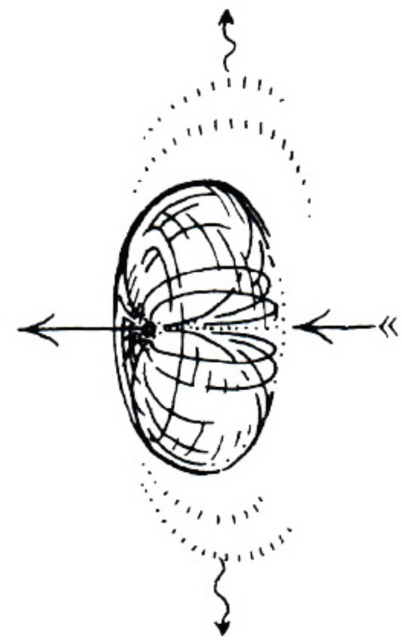
Radiation Simulation



Radiation from Accelerated Point Charges



RADIATION FROM A POINT-CHARGE



AN ACCELERATING POINT-CHARGE RELEASES A FRONT OF RADIATION IN THE SHAPE OF A TOROID.

Ch. 11.2	Radiation From Point Charges

From Ch. 10

$$\vec{E}(\vec{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{\Delta r}{(\vec{\Delta r}\cdot\vec{u})^3}\left[(c^2-v^2)\vec{u} + \vec{\Delta r}\times(\vec{u}\times\vec{a})\right]$$

w/ $\vec{u} = c\hat{\Delta r} - \vec{v}$

$\vec{\Delta r} = \vec{r} - \vec{w}(t_r)$

$\vec{v} = d\vec{w}/dt \,|_{t_r}$

$\vec{a} = d\vec{v}/dt \,|_{t_r}$

$\Delta r = c(t - t_r)$

$$\vec{B}(\vec{r},t) = \frac{1}{c}\hat{\Delta r}\times\vec{E}(\vec{r},t)$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$= \frac{1}{\mu_0 c}\left[E^2\hat{\Delta r} - (\hat{\Delta r}\cdot\vec{E})\vec{E}\right]$$

- Many terms!

- For radiation, we only care about terms that fall of as $1/r^2$ or slower

- These are the acceleration terms

$$\vec{E}_{rad} = \frac{q}{4\pi\varepsilon_0} \frac{\Delta r}{(\vec{\Delta r}\cdot\vec{u})^3} \left[\vec{\Delta r} \times (\vec{u} \times \vec{a}) \right]$$

$$\vec{S}_{rad} = \frac{1}{\mu_0 c} E_{rad}^2 \, \hat{\Delta r}$$

(other term drops since it involves $\hat{\Delta r} \cdot \vec{E}_{rad} = 0$)

Low-Velocity Limit

$$v \ll c \Rightarrow \vec{u} \sim c\hat{\Delta r}$$

$$\Rightarrow \vec{\Delta r} \cdot \vec{u} \sim c\Delta r$$

$$\Rightarrow \vec{E}_{rad} \cong \frac{q}{4\pi\varepsilon_0} \frac{1}{c^3 \Delta r^2} \left[\Delta r \times (c\hat{\Delta r} \times \vec{a}) \right]$$

$$= \frac{q}{4\pi\varepsilon_0 c^2 \Delta r} \left[\hat{\Delta r} \times (\hat{\Delta r} \times \vec{a}) \right]$$

$$= \frac{\mu_0 q}{4\pi \Delta r} \left[(\hat{\Delta r} \cdot \vec{a}) \hat{\Delta r} - \vec{a} \right]$$

$$\vec{S}_{rad} = \frac{1}{\mu_0 c} \vec{E}_{rad} \vec{E}_{rad} \hat{\Delta r}$$

$$= \frac{\mu_0 q^2}{16\pi^2 c \Delta r^2} \left[(\hat{\Delta r} \cdot \vec{a})^2 + a^2 - 2(\hat{\Delta r} \cdot \vec{a}) \hat{\Delta r} \cdot \vec{a} \right] \hat{\Delta r}$$

$$= \frac{\mu_0 q^2}{16\pi^2 c \Delta r^2} \left[a^2 - (\hat{\Delta r} \cdot \vec{a})^2 \right] \hat{\Delta r}$$

Low velocity radiation in spherical coordinates

Define θ as angle between $\hat{\Delta r}$ & $\vec{a}$

$$\Rightarrow \hat{\Delta r} \cdot \vec{a} = a \cos\theta$$

$$\Rightarrow a^2 - (\hat{\Delta r} \cdot \vec{a})^2 = a^2 \sin^2\theta$$

So
$$\boxed{\vec{S}_{rad} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^2\theta}{\Delta r^2} \hat{\Delta r}}$$

$$P_{rad} = \oint \bar{S} \cdot d\vec{a}$$

$$\boxed{P_{rad} = \frac{\mu_0 q^2 a^2}{6\pi c}}$$

same as Larmor formula we derived from multipole expansion of radiation

Compare

Electric Dipole

$$\vec{S}_{rad} = \frac{\mu_0 p_0^2 \omega^4}{16\pi^2 c} \frac{\sin^2\theta}{r^2} \cos^2(\omega(t - r/c)) \hat{r}$$

$$S_{rad} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2\theta}{r^2} \hat{r}$$

Accelerated Charge

$$\vec{S}_{rad} = \langle \vec{S}_{rad} \rangle = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{\sin^2\theta}{\Delta r^2} \Delta\hat{r}$$

- Same angular and radial dependence
- But radiation from point charge doesn't come in ripples