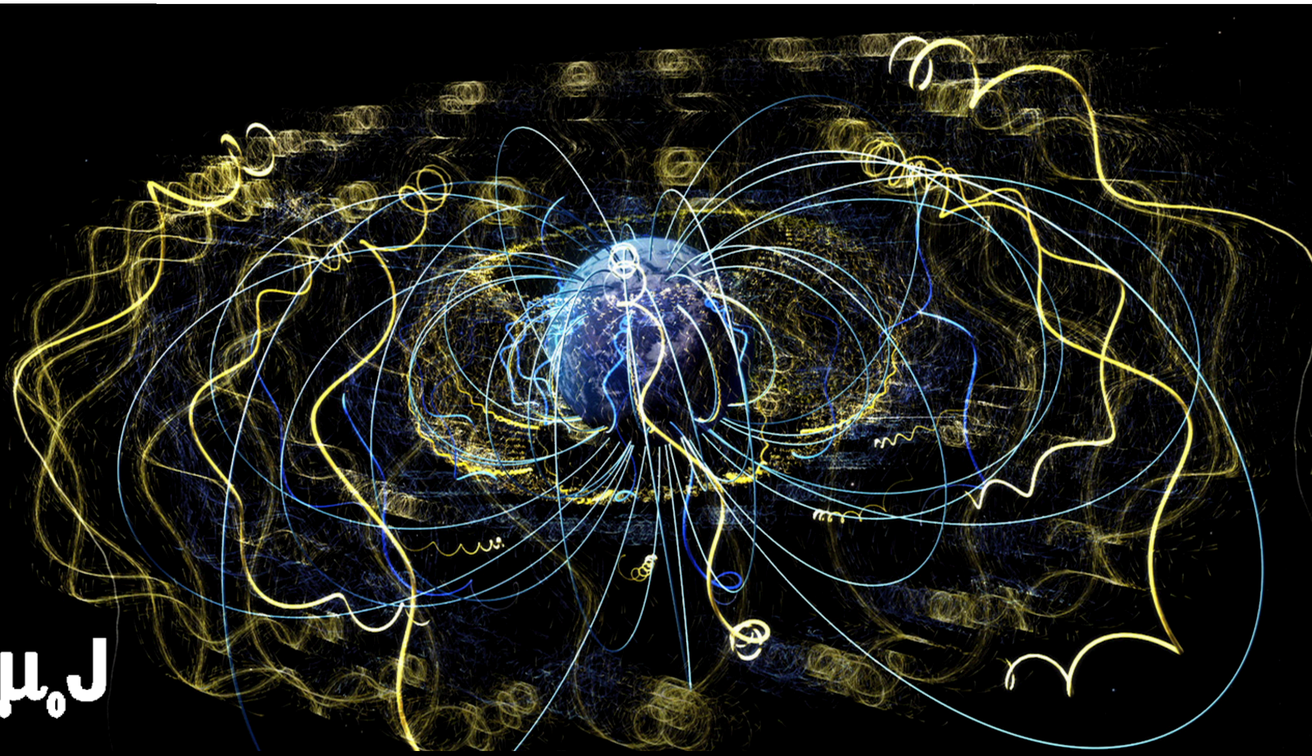
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

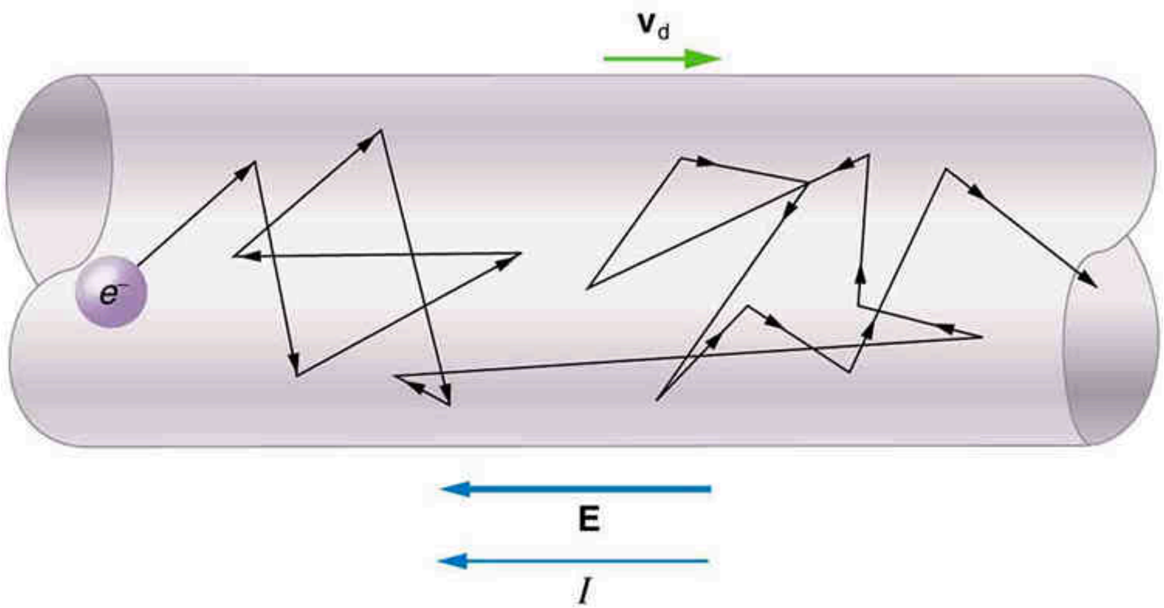
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



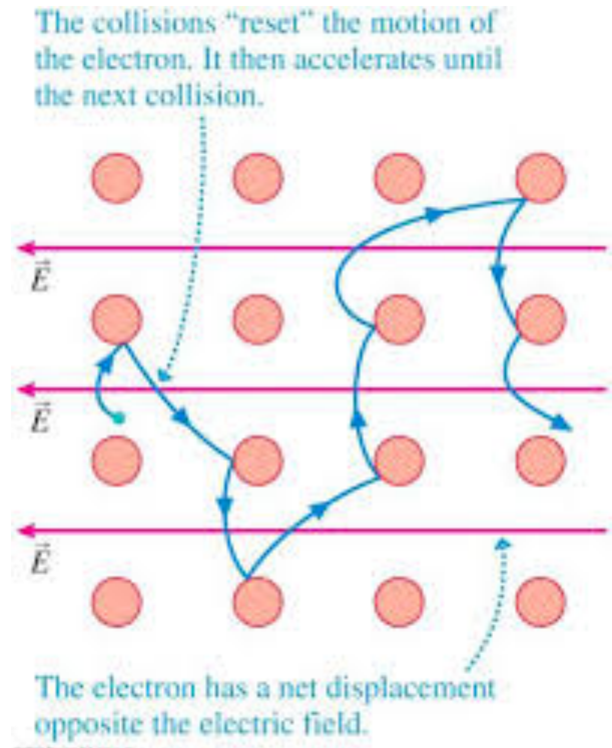
Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Ohm's Law



$\mathbf{v}_d$
e^-
E
I



The collisions "reset" the motion of the electron. It then accelerates until the next collision.
$\vec{E}$
$\vec{E}$
$\vec{E}$
The electron has a net displacement opposite the electric field.

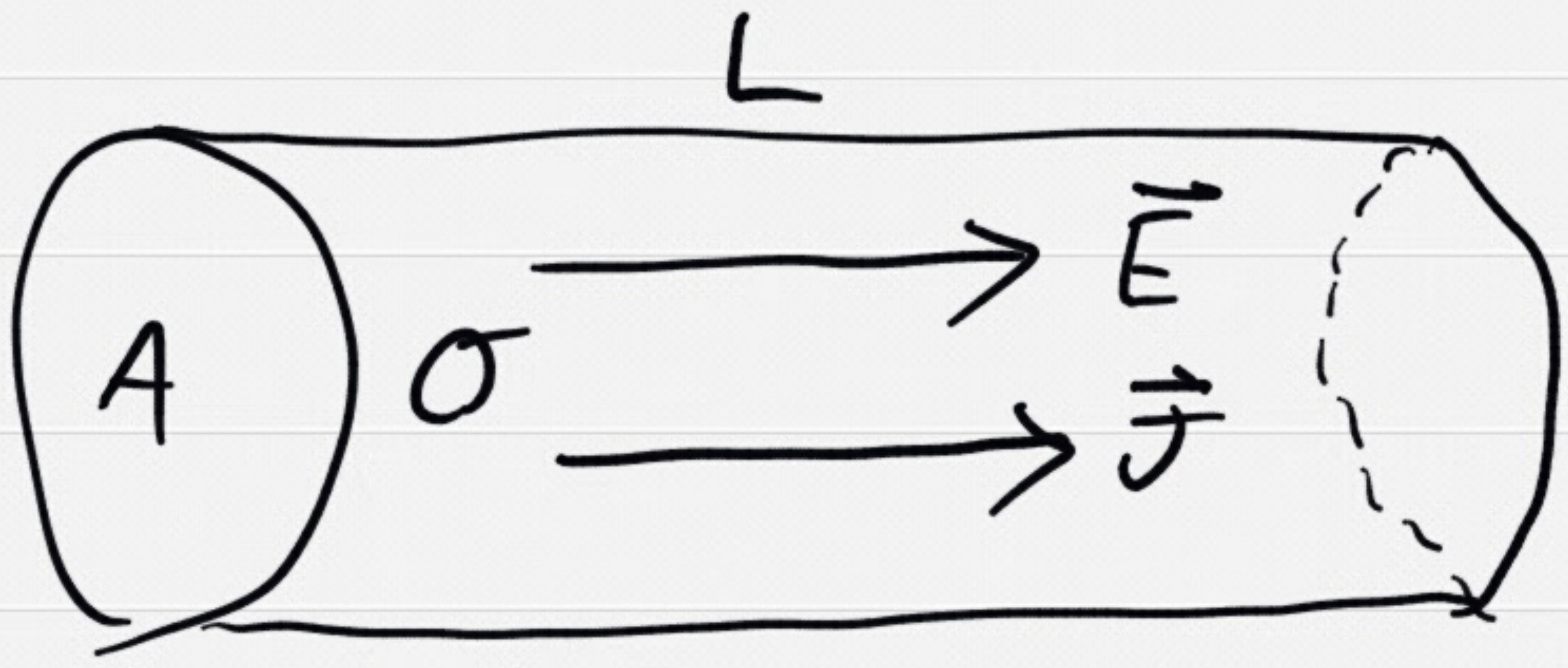
- Ohm's law implies dissipation of energy

$$\vec{J} = \sigma \vec{E}$$

$$P = dW/dt = d/dt \int \vec{F} \cdot d\vec{l}$$
$$= \vec{F} \cdot \vec{v}$$
$$= Q \cdot \vec{E} \cdot \vec{v}$$
$$= \rho \cdot \text{volume} \cdot \vec{E} \cdot \vec{v}$$

$$= (\vec{J} \cdot \vec{E}) \cdot \text{volume}$$
if $\vec{J} = \rho \vec{v}$

$\boxed{\text{Microscopic} \rightarrow \text{Macroscopic}}$



$$\vec{J} = \sigma \vec{E}$$

$I = J \cdot A$, $\Delta V = -\int \vec{E} \cdot d\vec{l}$
$= -EL$

$\Rightarrow I/A = -\sigma \cdot \Delta V/L$

$\Rightarrow \Delta V = -\frac{L}{\sigma A} \cdot I = -IR$

w/ $R = L/\sigma A$

$$P = (\vec{J} \cdot \vec{E}) \cdot \text{Volume}$$

$$= \frac{I}{A} \cdot \frac{\Delta V}{L} \cdot LA$$

$$= I \cdot \Delta V$$ usually shortened to $P = VI$

$$P = VI = I^2 R = V^2/R$$

(since $V = IR$)

- This power dissipation is called "Joule heating" and is due to collisions

- In a perfect conductor

$$\sigma \to \infty$$

$$\Rightarrow R, \vec{E}, \Delta V \to 0$$

$$\Rightarrow P \to 0$$

$\boxed{\text{EMF}}$

$$\mathcal{E} = \oint \vec{f} \cdot d\vec{l}$$

= "Electromotive Force"

- Loop integral of force per charge

- Many sources of EMF

  - Mechanical (generator)
  - Chemical (battery)
  - Electric field

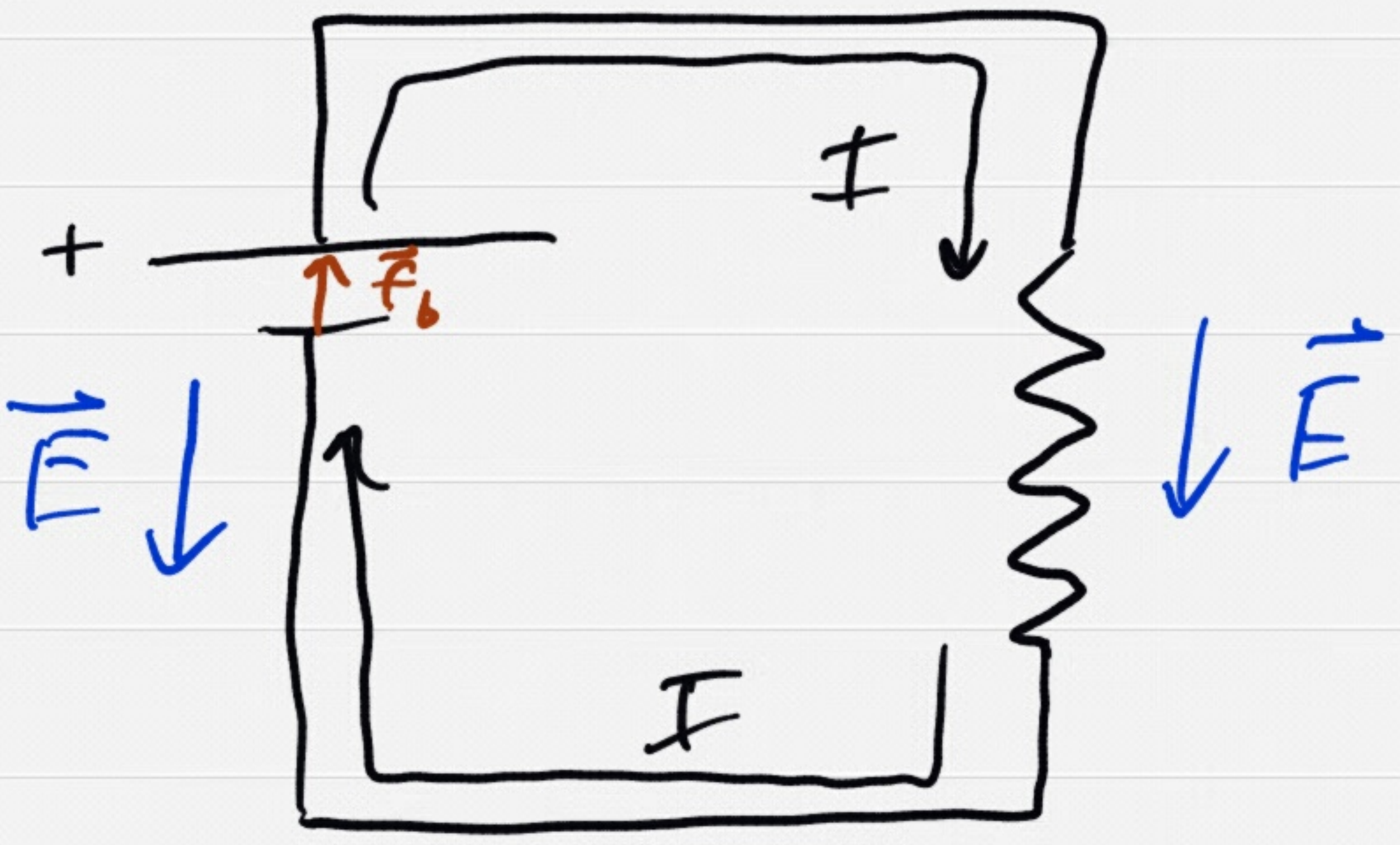
- Electrostatic EMF is zero

$$\oint \vec{E} \cdot d\vec{l} = \int (\nabla \times \vec{E}) \cdot d\vec{a}$$

zero if $\nabla \times \vec{E} = 0$

$\Rightarrow$ Kirchoff's Law:
Sum of voltage drops around circuit equal to zero

Simple Circuit



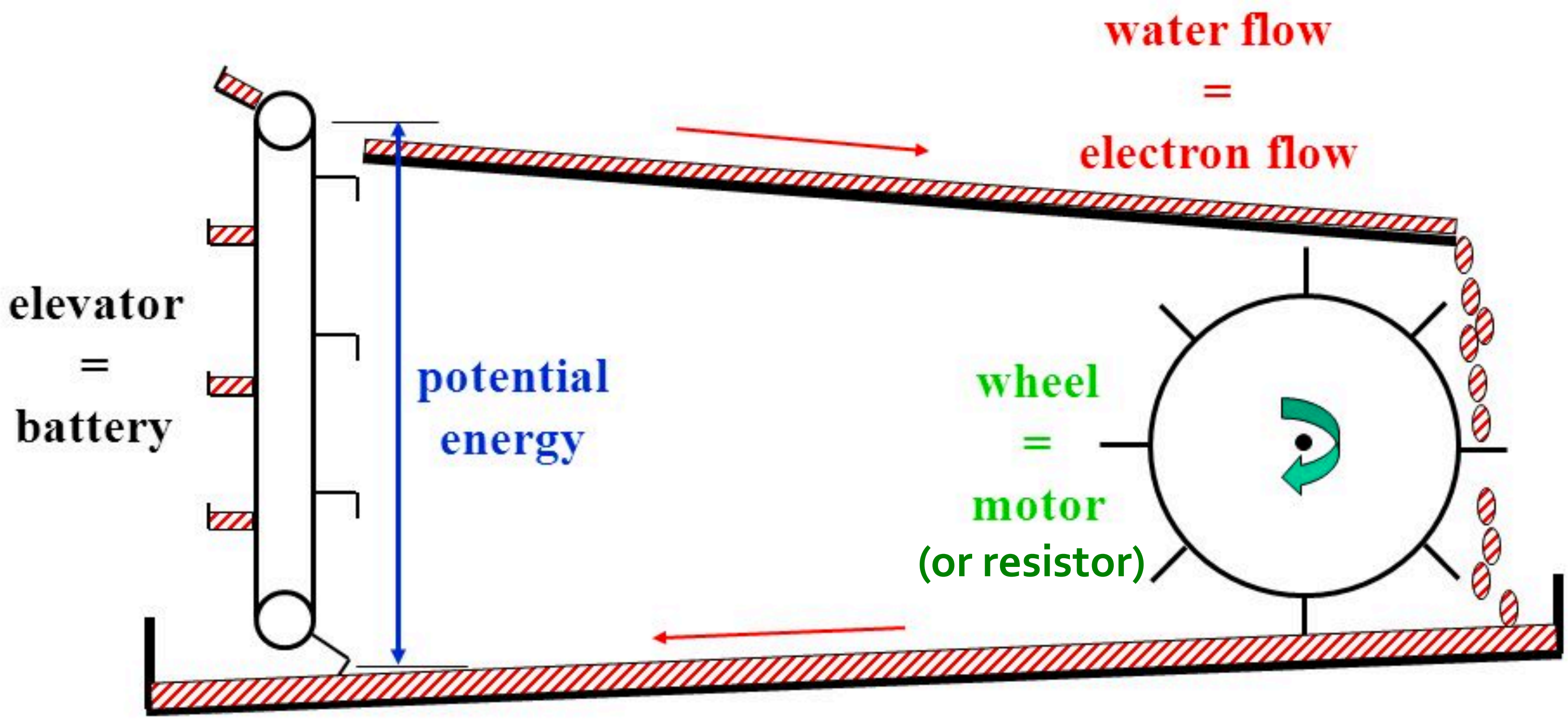
$$\Delta V_{battery} = +V_0$$
$$\Delta V_{resistor} = -V_0$$
$$\Delta V_{total} = V_0 - V_0 = 0$$
$$= -\oint \vec{E} \cdot d\vec{l}$$

$$\mathcal{E} = \oint \vec{f} \cdot d\vec{l} = \oint \vec{f}_b \cdot d\vec{l}$$

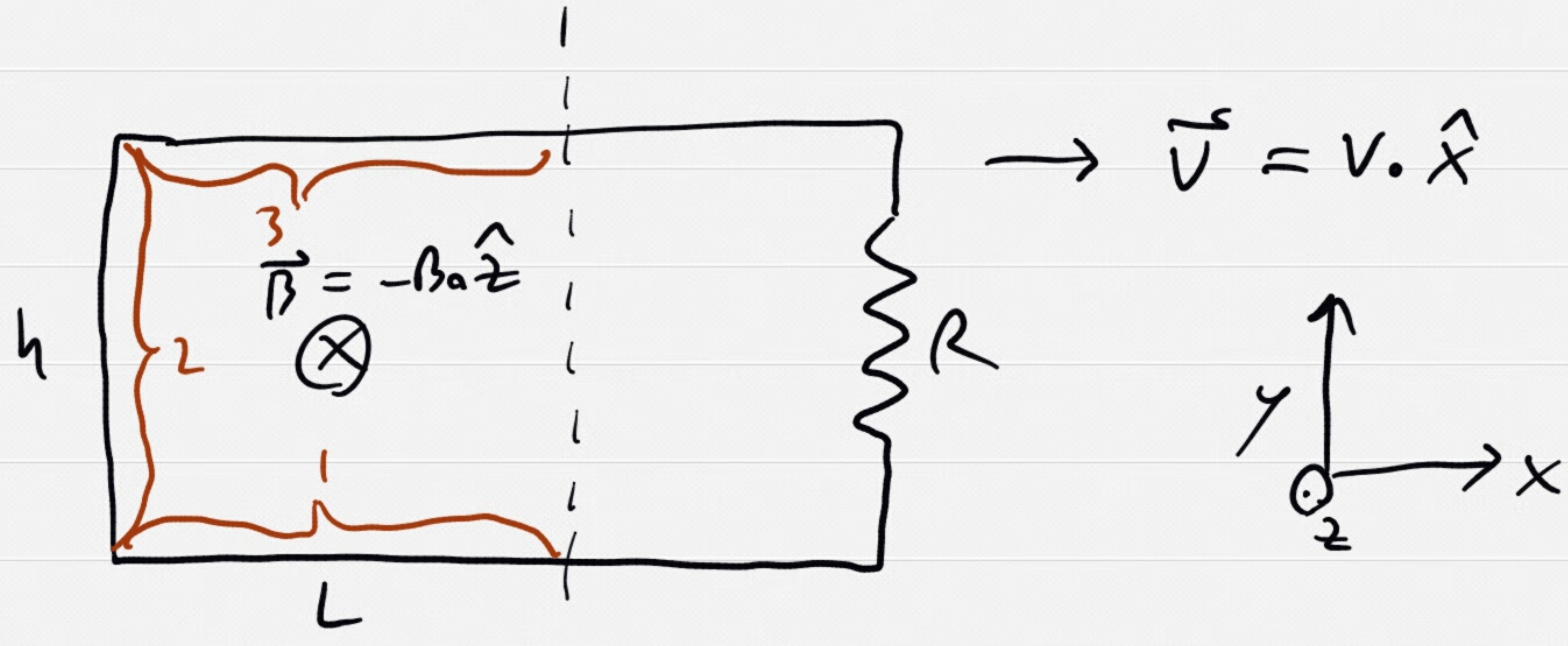
- In battery, $\vec{f}_b = -\vec{E}$
  - $\vec{f}_{total} = \vec{f}_b + \vec{E} = 0$ in battery
  - ohm's law not satisfied in battery
  - $\mathcal{E} = V_0$

- $\vec{f}_b$ allows charges to overcome $\vec{E}$ in battery
- then they flow in response to $\vec{E}$ through circuit

Electric Circuit Analogy



Motional EMF



$$\vec{F}_1 = q_1 \vec{v}_1 \times \vec{B}$$
$$= q_1 v_0 B_0 \hat{y}$$

$$\vec{F}_2 = q_2 v_0 B_0 \hat{y}$$

$$\vec{F}_3 = q_3 v_0 B_0 \hat{y}$$

$\vec{f} = \vec{F}/q = v_0 B_0 \hat{y}$ for all three

$\mathcal{E} = \oint \vec{f} \cdot d\vec{\ell}$ w/ $d\vec{\ell}$ clockwise

$= v_0 B_0 h$ (only from segment 2)

Drives current $I = \mathcal{E}/R$

$$= \frac{v_0 B_0 h}{R}$$

Who does work?

w/ current flowing

$$I = \mathcal{E}/R = qu$$

$$\vec{F}_{1u} = q_1 u \cdot -\hat{x} \times \vec{B}$$
$$= -q_1 u B_0 \hat{y}$$
$$\vec{F}_{2u} = q_2 u \hat{y} \times \vec{B}$$
$$= -q_2 u B_0 \hat{x}$$
$$\vec{F}_{3u} = q_3 u \hat{x} \times \vec{B}$$
$$= q_3 u B_0 \hat{y}$$

$$q_1 = q_3 \rightarrow \vec{F}_u = -q_2 u B_0 \hat{x}$$

$$\vec{F}_{pull} = -\vec{F}_u = q_2 u B_0 \hat{x}$$

$$\vec{f}_{pull} = \vec{F}_{pull}/q_2 = u B_0 \hat{x}$$

$$W/q = \int \vec{f}_{pull} \cdot d\vec{l} = u B_0 \cdot \Delta x$$
$$= u B_0 \cdot v_0 \Delta t$$

Δt is time for charge to traverse segment 2

$$\Rightarrow W/q = B_0 v_0 \cdot h = \mathcal{E}$$