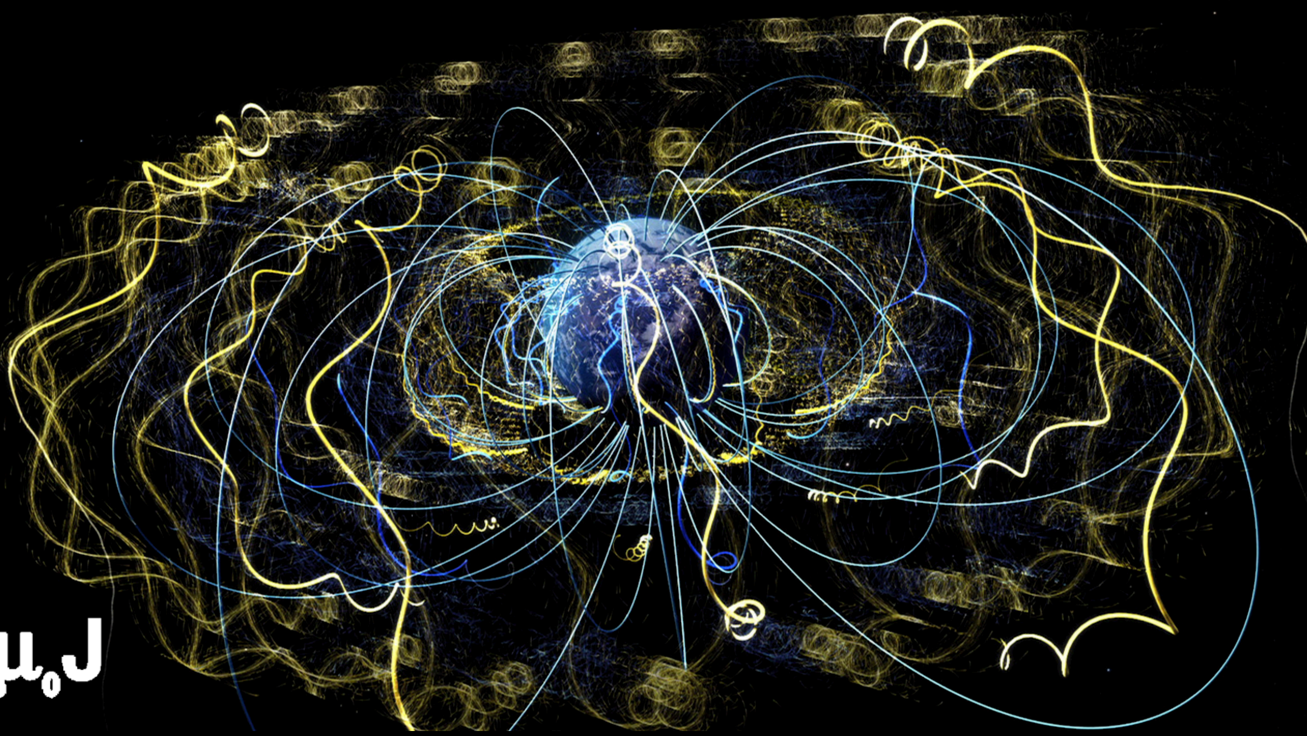
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

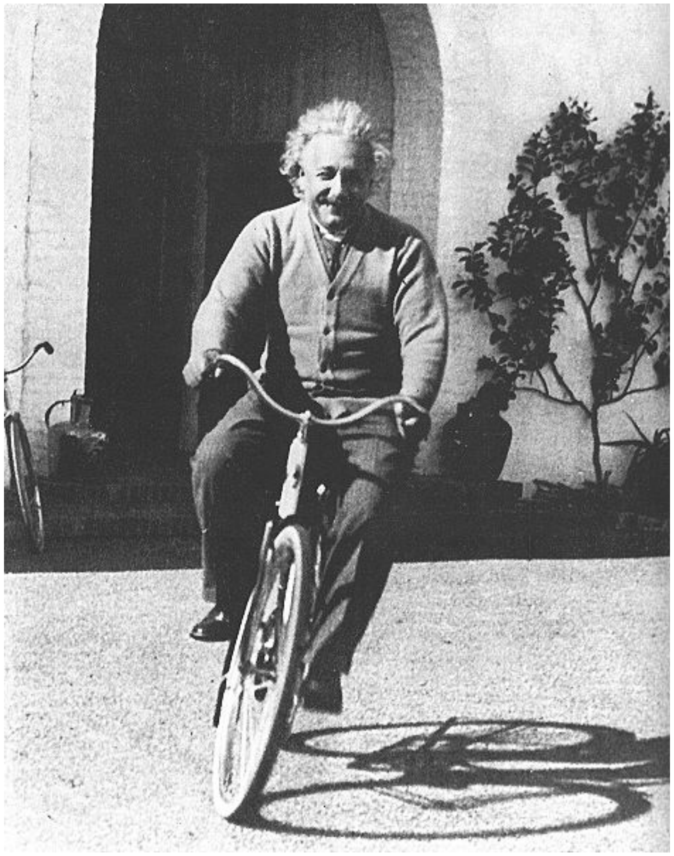
Virtual by Zoom!

MWF 9:30-10:20 Lecture

Einstein's Postulates

The laws of physics are the same in all inertial frames of reference.

The speed of light is the same in all inertial frames of reference.



Proper Time

- "Proper Time" = $\Delta\tau = \Delta t_o$
  - The time interval measured in a frame where the two events occur at the same spatial coordinate – i.e. the frame moving with your clock
- The time interval Δt measured in **any** other frame moving with respect to this frame will be longer
  - $\Delta t = \gamma \Delta t_0$

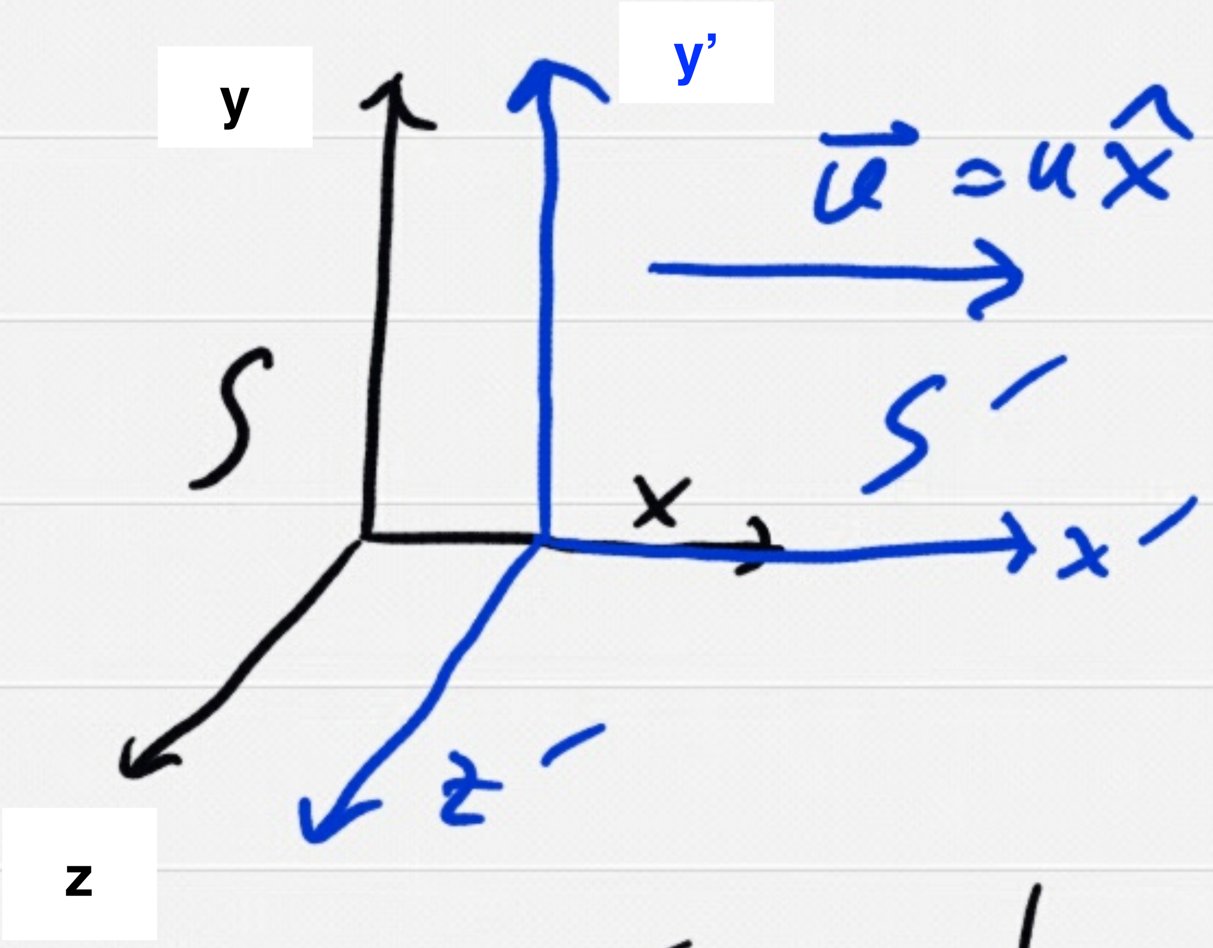
Proper Length

- "Proper Length" = L_o
  - The length of an object measured in a frame where it is at rest

- The length L measured in **any** other frame moving with a velocity with respect to this frame that has a component along the length will be shorter
  - $L = L_0/\gamma$

Lorentz Transformation

$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{ux}{c^2}\right)$$



$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Inverse:

$$x = \gamma(x' + ut')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + ux'/c^2\right)$$

$x' = $ const. $\Rightarrow \Delta t = \gamma \Delta t'$

$t = $ const. $\Rightarrow \Delta x' = \gamma \Delta x$

$\Rightarrow \Delta x = \Delta x'/\gamma$

Velocity

$$v_x' = dx'/dt' = \frac{\gamma(dx - u\,dt)}{\gamma(dt - u\,dx/c^2)}$$
$$= \frac{dx/dt - u}{1 - \frac{u}{c^2}\,dx/dt}$$
$$= (v_x - u)/(1 - uv_x/c^2)$$

But $v_y' = dy'/dt' = \dfrac{v_y}{\gamma(1 - uv_x/c^2)}$ etc.

Transformations

If S′ is moving with speed u in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

Galilean transformation
(classical)

$$x' = x - ut$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Lorentz transformation
(relativistic)

$$x' = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{u}{c^2}x\right)$$

Note: This assumes (0,0,0,0) is the same event in both frames.

Interval Transformations

If S' is moving with speed u in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

Galilean transformation (classical)

$$\Delta x' = \Delta x - u\Delta t$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

$$\Delta t' = \Delta t$$

Lorentz transformation (relativistic)

$$\Delta x' = \gamma(\Delta x - u\Delta t)$$

$$\Delta y' = \Delta y$$

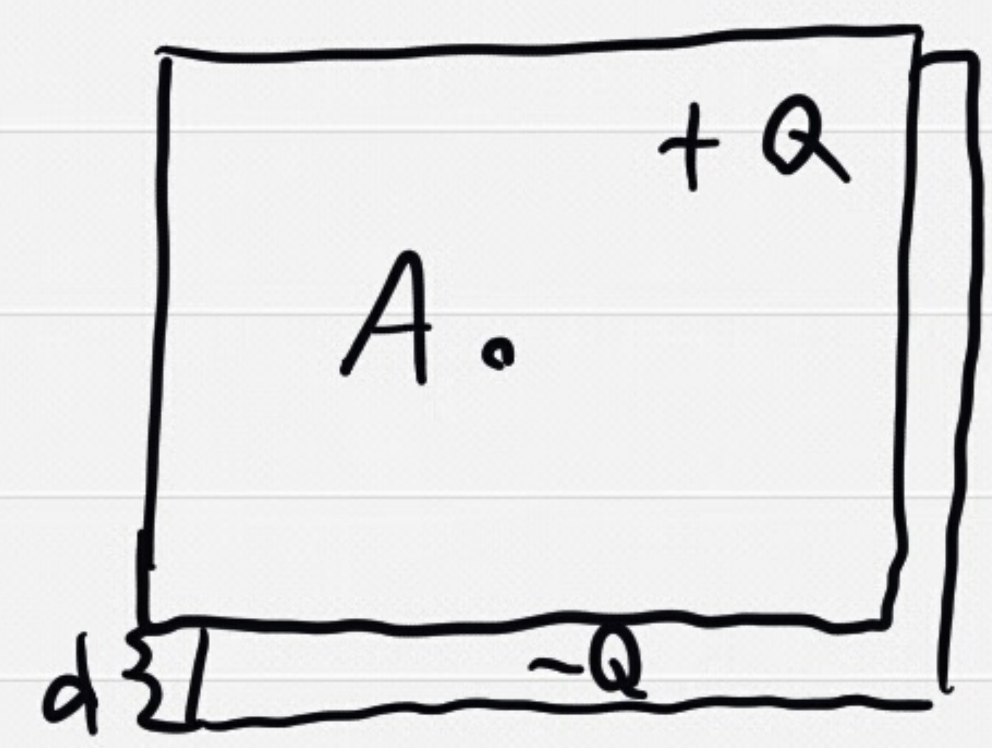
$$\Delta z' = \Delta z$$

$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x)$$

12.3 | Relativistic Electrodynamics

- E&M is already correct
- By making one assumption, we can derive transformation for $\vec{E}$ & $\vec{B}$

$$\boxed{\text{Charge is invariant}}$$

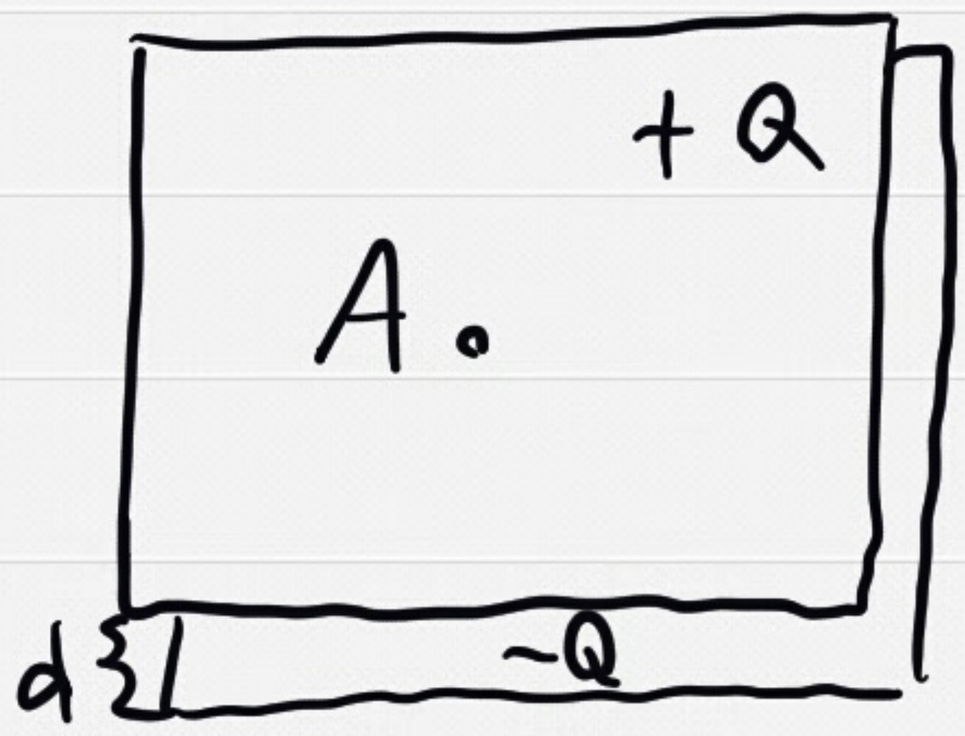


Frame S

$\sigma_0 = Q/A_0$

$E_0 = \sigma_0/\varepsilon_0$ between plates

$\xrightarrow{\bar{u}}$

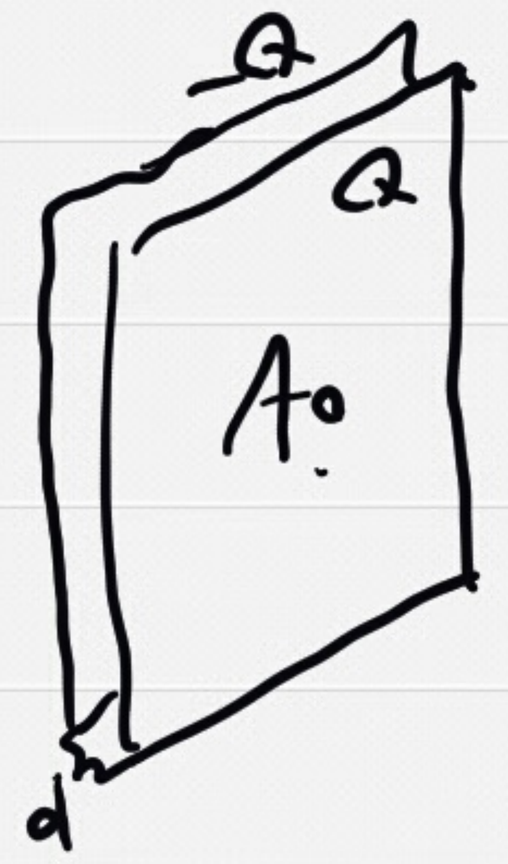


Frame S'

$A' = A_0/\gamma$

$\Rightarrow \sigma' = Q/A' = \gamma \sigma_0$

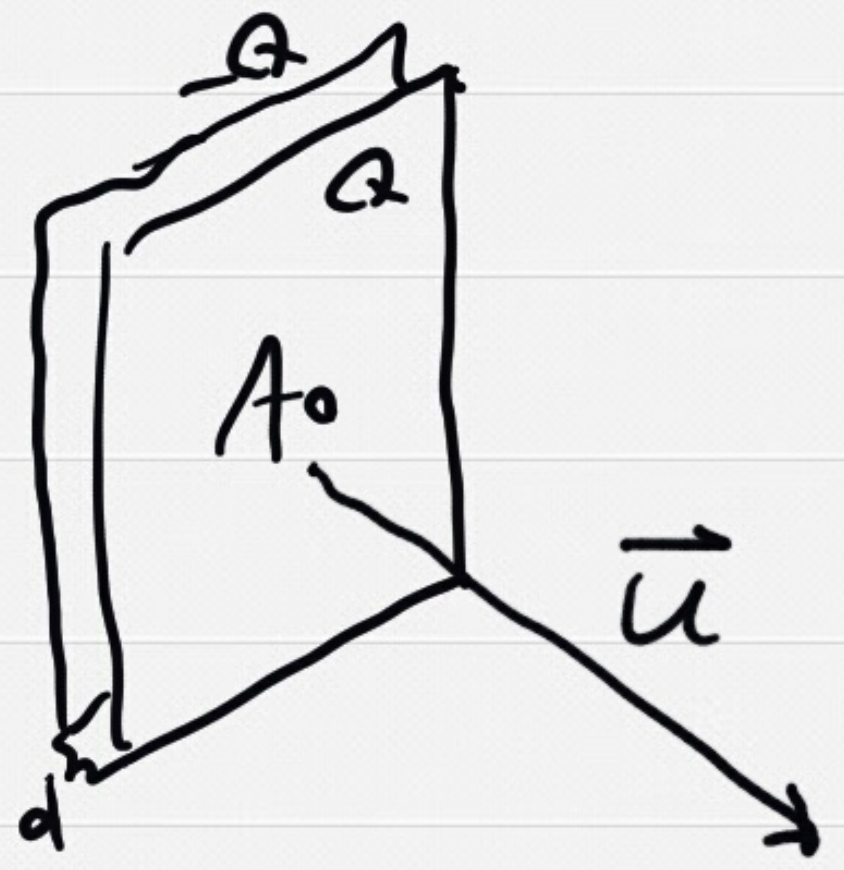
$E' = \gamma E_0$



Frame S

$A = A_0$, $\sigma_0 = Q/A_0$

$E_0 = \sigma_0/\varepsilon_0$



Frame S'

$A' = A_0$, $\sigma' = Q/A_0 = \sigma_0$

$E' = E_0$

Conclusion

$E_x' = E_x$ (w/ $\vec{u} = u\hat{x}$)

$E_y' = \gamma E_y$

$E_z' = \gamma E_z$

Moving Point Charge

$$\vec{E}_0 = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{x\hat{x} + y\hat{y} + z\hat{z}}{(x^2+y^2+z^2)^{3/2}}\right]$$

- Transform to S'
  where particle moves w/
  $\vec{v} = v\hat{x}$ (i.e. $\vec{u} = -\vec{v} = -v\hat{x}$)

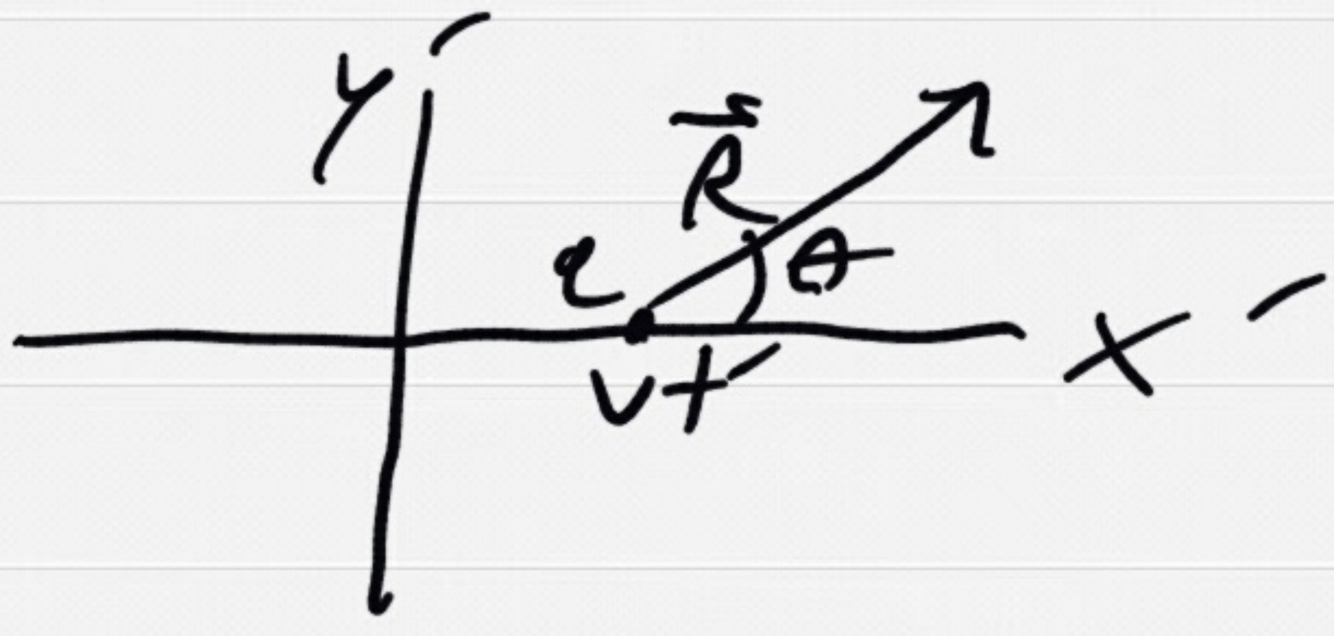
$$E_x' = E_x$$
$$E_y' = \gamma E_y$$
$$E_z' = \gamma E_z$$

so $$\vec{E}' = \frac{q}{4\pi\varepsilon_0}\left[\frac{x\hat{x} + \gamma y\hat{y} + \gamma z\hat{z}}{(x^2+y^2+z^2)^{3/2}}\right]$$

- write in terms of x', y', z'

$$x = \gamma(x' - vt') = \gamma R_x$$
$$y = y' = R_y$$
$$z = z' = R_z$$

w/ $\vec{R}$ vector from
q to $\vec{r}' = (x', y', z')$



$$\Rightarrow \vec{E}' = \frac{q}{4\pi\varepsilon_0}\left[\frac{\gamma R_x\hat{x} + \gamma R_y\hat{y} + \gamma R_z\hat{z}}{\left((\gamma R_x)^2 + R_y^2 + R_z^2\right)^{3/2}}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{\gamma\vec{R}}{\left(\gamma^2R^2\cos^2\theta + R^2\sin^2\theta\right)^{3/2}}$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{\vec{R}}{R^3}\left[\frac{1/\sqrt{1-u^2/c^2}}{\left(\cos^2\theta/(1-u^2/c^2) + \sin^2\theta\right)^{3/2}}\right]$$

$$= \frac{q\hat{R}}{4\pi\varepsilon_0 R^2}\left[\frac{1-u^2/c^2}{\left(\cos^2\theta + (1-u^2/c^2)\sin^2\theta\right)^{3/2}}\right]$$

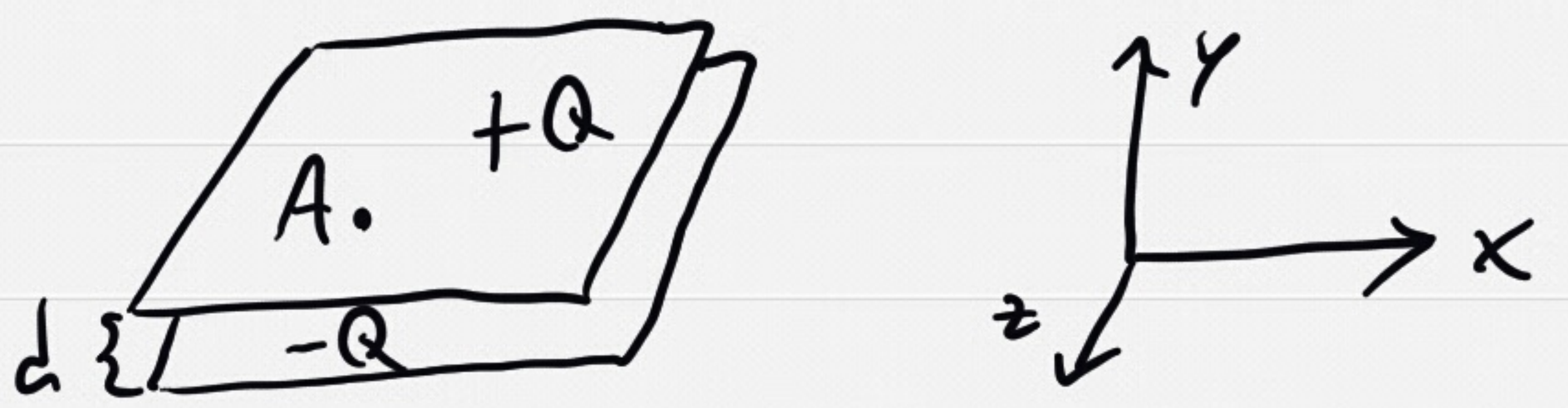
$$\boxed{\vec{E}' = \frac{q\hat{R}}{4\pi\varepsilon_0 R^2}\left[\frac{1-v^2/c^2}{\left(1-\frac{v^2}{c^2}\sin^2\theta\right)^{3/2}}\right]}$$

Note $v^2 = u^2$

- Same as from Lienard-Wiechert potentials

- The magic that makes $\vec{E}'$ point along $\hat{R}$
  - Lorentz boost of x and of E_y, E_z balance exactly

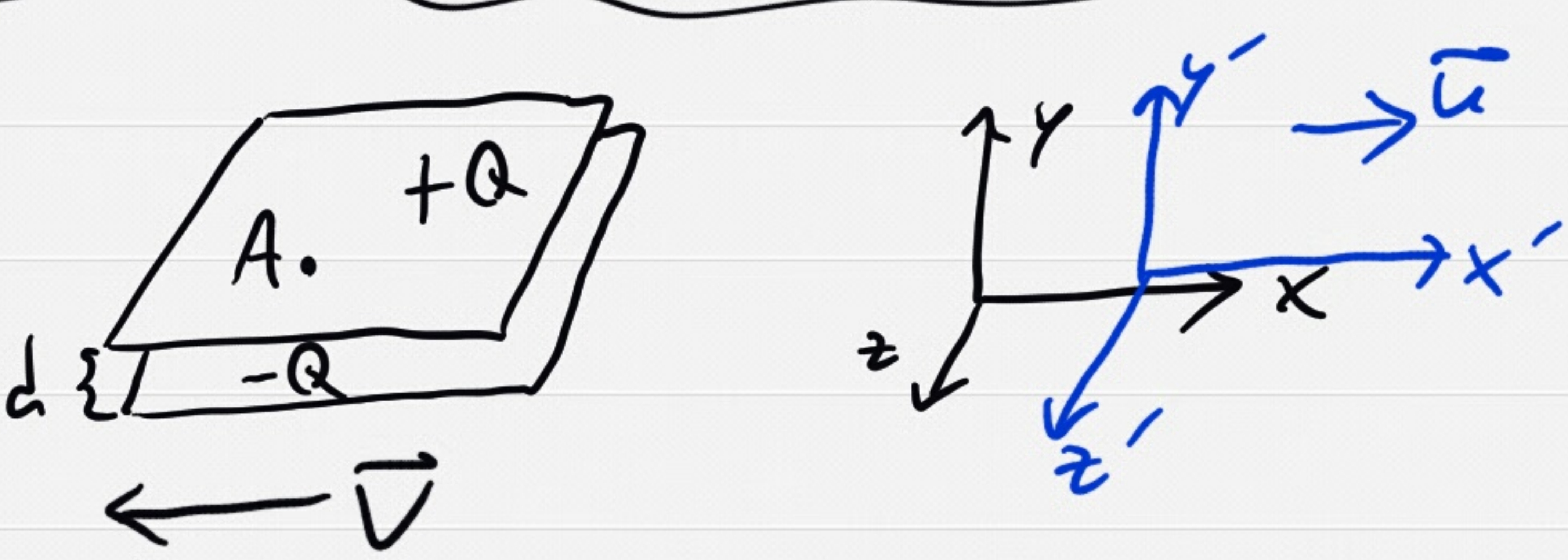
Back to Capacitor



S

$$\vec{E}_0 = -\frac{Q}{\varepsilon_0 A_0}\hat{y} = -\frac{\sigma_0}{\varepsilon_0}\hat{y}$$

$$\vec{B}_0 = 0$$



S'

In S' capacitor moves w/
$\vec{v} = -\vec{u}$

$$\vec{E}' = \gamma \vec{E}_0 = -\frac{\gamma\sigma_0}{\varepsilon_0}\hat{y} = -\frac{\sigma'}{\varepsilon_0}\hat{y}$$

σ' moving at $\vec{v}$ produces

$$\vec{K}' = \sigma'\vec{v} = -\sigma' v\hat{x} = -\sigma' u\hat{x}$$

$$\Rightarrow \vec{B}' = \mu_0 K'\hat{z} = \mu_0\sigma' u\hat{z}$$

or $$B_z' = -\mu_0\varepsilon_0 u E_y'$$
$$= -\gamma\mu_0\varepsilon_0 u E_{y0}$$

Note: $\vec{B}' = \mu_0\varepsilon_0 \vec{v}\times\vec{E}' = \frac{1}{c^2}\vec{v}\times\vec{E}'$ as expected