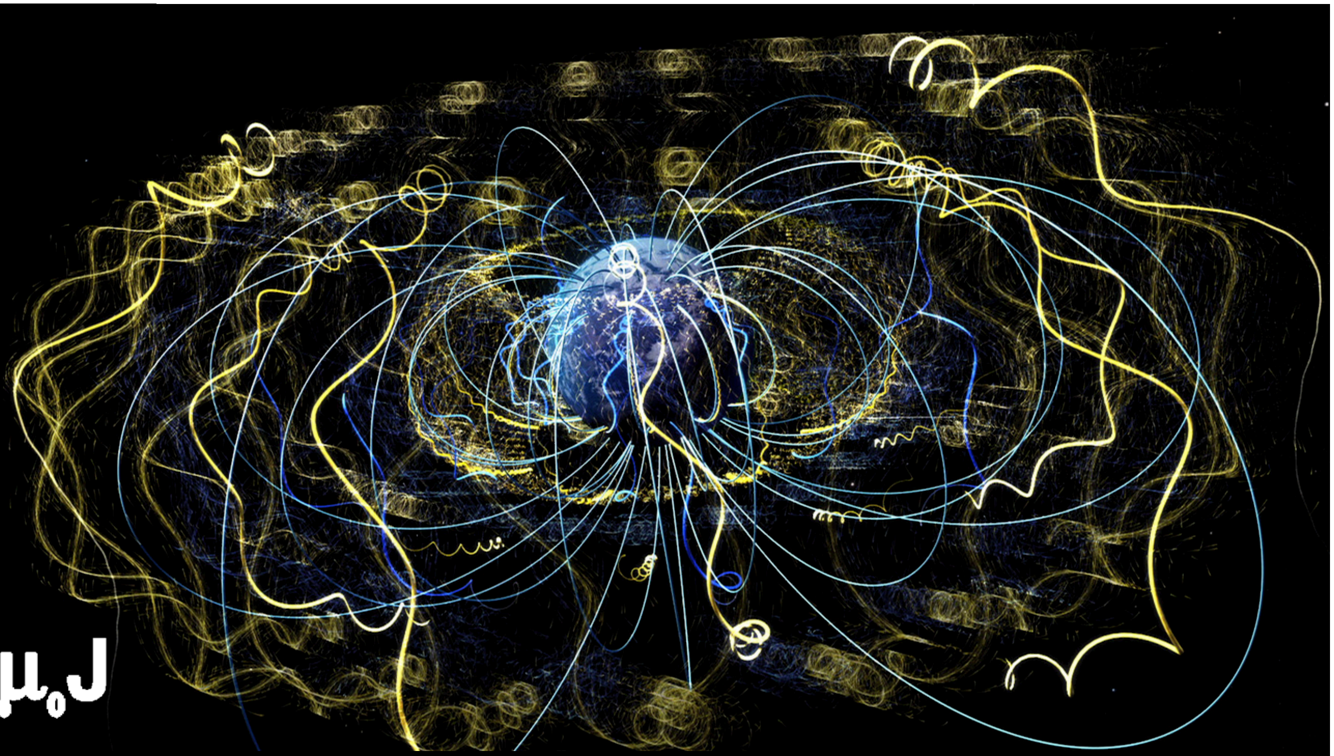
$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t} + \mu_0 J$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas

Virtual by Zoom!

MWF 9:30-10:20 Lecture

Announcements

- Final Exam
  - 3:00-5:00 pm (+1 hr grace) Monday 5/11 as scheduled
    - Take-home open-book (same format as Midterm 2)
  - Covers Chapters 7-12 in Griffiths
    - Final equation sheet posted
    - Last year's final & answers posted
- Course evaluations are open through Sunday
  - As always, feedback is much appreciated!

Announcements II

- Final Exam will cover Ch. 7-12, with the exception of the following sections:
  - 7.3.4 Magnetic charge
  - 8.2.4 Angular momentum
  - 8.3 Magnetic forces do no work
  - 9.4.3 Frequency dependence of permittivity
  - 9.5 Guided waves
  - 10.1.4 Lorentz force law in potential form
  - 10.2.2 Jefimenko's equations
  - 11.2.2-11.2.3 Radiation reaction
  - 12.2.3-12.2.4 Relativistic kinematics and dynamics
  - 12.3.3 The field tensor

Final Exam Instructions

- The exam will be posted on the course web page by 3:00pm. You must submit your answers to me by e-mail by 6:00pm. The exam is intended to take roughly two hours – the extra hour is grace period to check your work, scan it, and submit it.
- This exam is open book and open notes. However, it is not open internet, open solutions manual, or open classmate. Please do not utilize solutions or consult with anyone to solve the problems. I trust you all not to abuse this unique situation.
- Read all the questions carefully and answer every part of each question. Show your work on all problems. Partial credit may be granted for correct logic or intermediate steps, even if the final answer is incorrect. Make sure to clearly indicate (e.g. circle) your final answers.
- The solution for each of the five problems is ~½ page. If it looks like your solution is going to require substantially more work, you may be doing it the hard way!
- Unless otherwise instructed, express your answers in terms of fundamental constants like μ_o and ε_o, rather than calculating numerical values.
- Please ask if you have any questions, including clarification about the instructions, during the exam. A Zoom meeting (ID 992-770-55648) will be open during the exam.

Ch. 7: Electrodynamics

Maxwell's Equations

Differential form

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \vec{E} = \boxed{-\frac{\partial \vec{B}}{\partial t}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \boxed{\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

Maxwell's Equations

Integral form

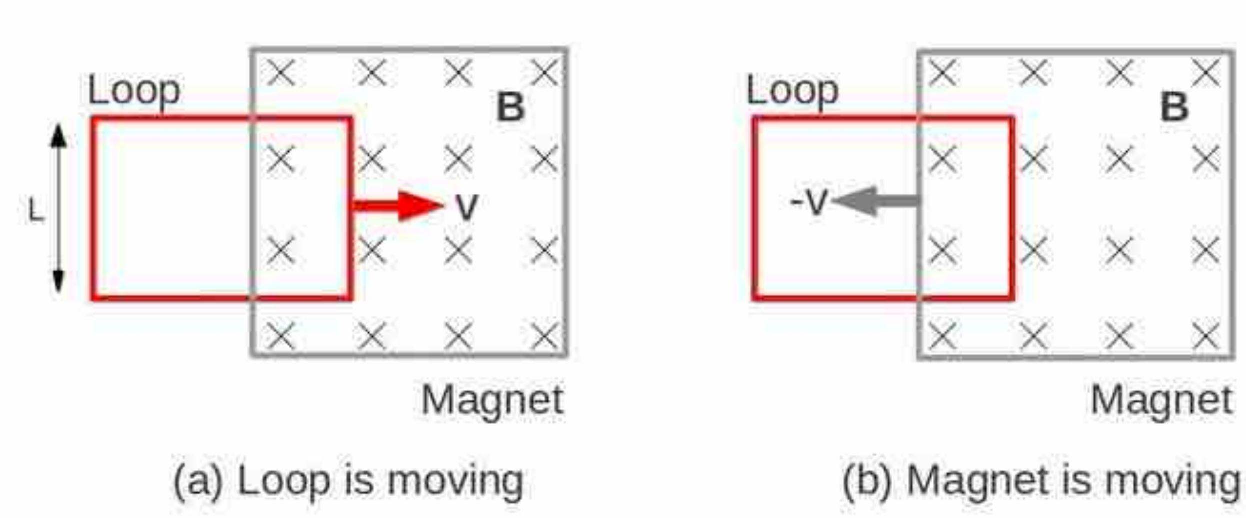
$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

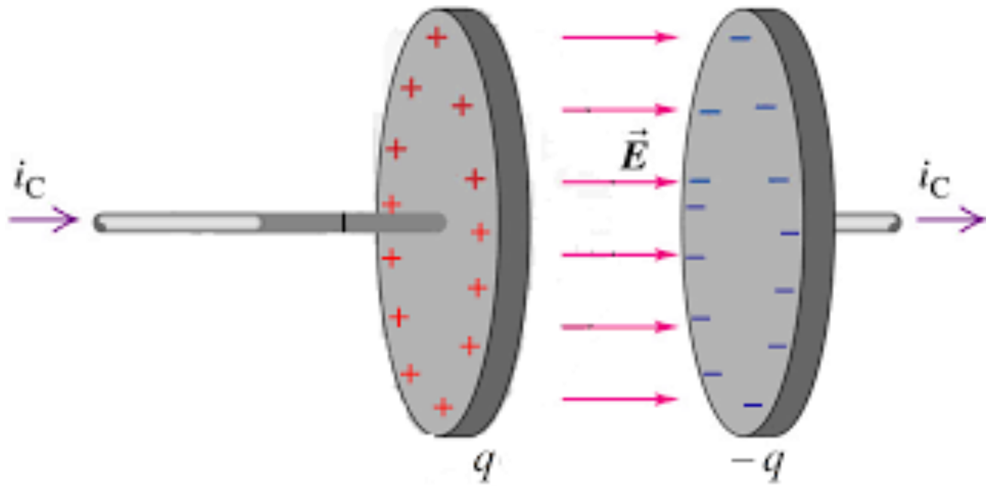
$$\oint \vec{E} \cdot d\vec{l} = \boxed{-\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}}$$

$$\oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \boxed{\mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a}}$$

Motional EMF and Displacement Current





Maxwell's Equations in Linear Dielectrics

(i) $\nabla \cdot \mathbf{D} = \rho_f,$

(iii) $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$

(ii) $\nabla \cdot \mathbf{B} = 0,$

(iv) $\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}.$

with $D = \varepsilon E$

$$H = \frac{1}{\mu} B$$

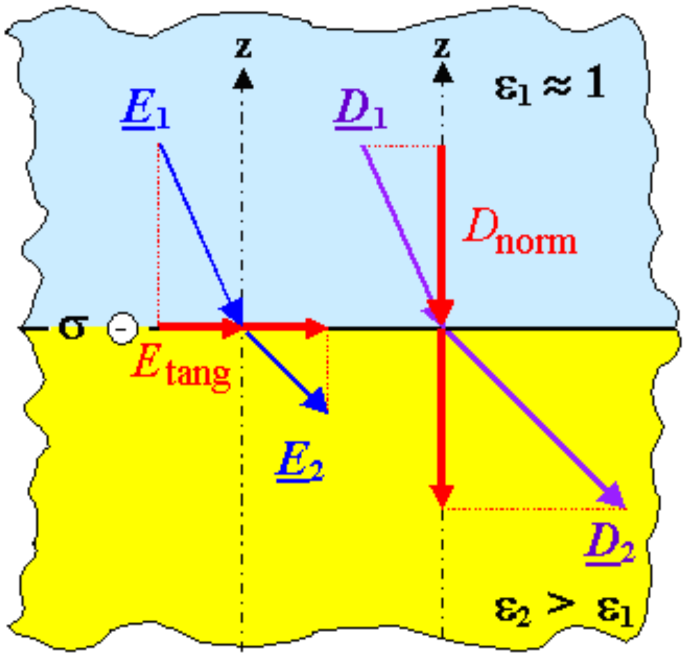
$$\varepsilon = \varepsilon_0 (1 + \chi_e) = \varepsilon_0 \varepsilon_r$$

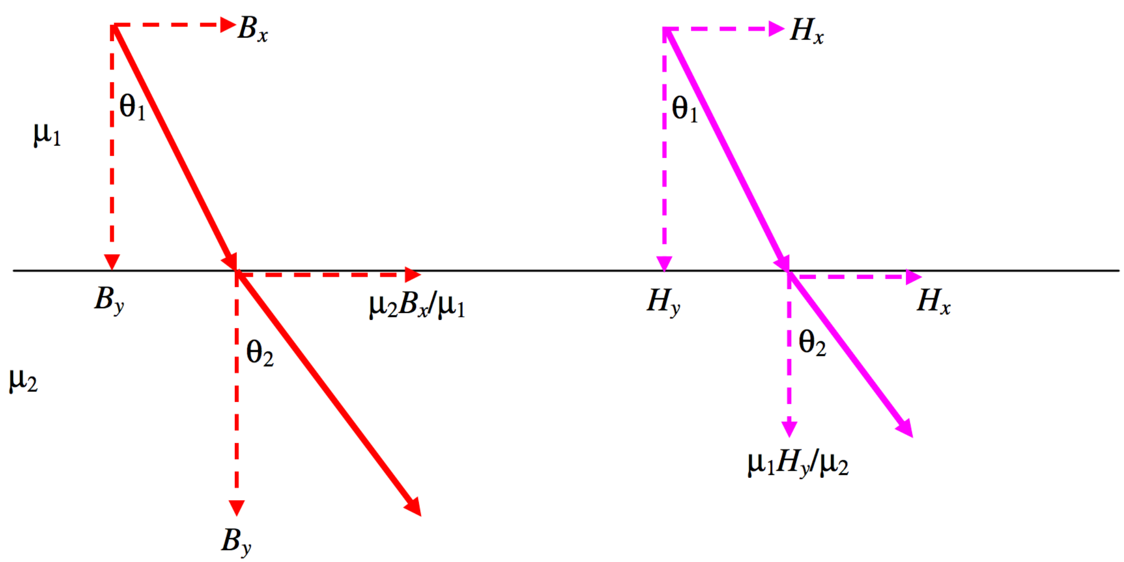
$$\mu \equiv \mu_0 (1 + \chi_m)$$

Boundary Conditions

Boundary Conditions: $\Delta D_{\perp} = \sigma_f$ $\quad \Delta \vec{E}_{||} = 0$ $\quad \Delta \vec{D}_{||} = \Delta \vec{P}_{||}$

$\Delta \vec{H}_{||} = \vec{K}_f \times \hat{n}$ $\quad \Delta B_{\perp} = 0$ $\quad \Delta H_{\perp} = -\Delta M_{\perp}$





Images show boundary conditions for case w/ no free charge or current at the boundary

Ch. 8: Conservation Laws

$$\frac{dW}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} u_{\text{em}}\, d\tau - \oint_{\mathcal{S}} \mathbf{S}\cdot d\mathbf{a} \quad \longleftrightarrow \quad \frac{d\mathbf{p}_{\text{mech}}}{dt} = -\epsilon_0\mu_0\frac{d}{dt}\int_{\mathcal{V}} \mathbf{S}\, d\tau + \oint_{\mathcal{S}} \overleftrightarrow{\mathbf{T}}\cdot d\mathbf{a}$$

$$\frac{\partial}{\partial t}\left(u_{\text{mech}} + u_{\text{em}}\right) = -\nabla\cdot\mathbf{S} \quad \longleftrightarrow \quad \frac{\partial}{\partial t}\left(\mathbf{P}_{\text{mech}} + \mathbf{P}_{\text{em}}\right) = -\nabla\cdot\left(-\overleftrightarrow{\mathbf{T}}\right)$$

$$\mathbf{P}_{\text{em}} = \int_{\mathcal{V}} \left(\varepsilon_0\mu_0\mathbf{S}\right) d\tau = \int_{\mathcal{V}} \varepsilon_0\left(\mathbf{E}\times\mathbf{B}\right) d\tau$$

$$u_{\text{em}} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) \quad \longleftrightarrow \quad \mathbf{g} = \varepsilon_0\mu_0\mathbf{S} = \varepsilon_0\left(\mathbf{E}\times\mathbf{B}\right)$$

Poynting Vector S

- S : *Energy per unit area (Energy flux density), per unit time transport by EM fields*
- $\varepsilon_0\mu_0 S$: *Momentum per unit volume (Momentum density) stored in EM fields*

Stress Tensor $\overleftrightarrow{\mathbf{T}}$

- $\overleftrightarrow{\mathbf{T}}$: *EM field stress (Force per unit area) acting on a surface*
- $-\overleftrightarrow{\mathbf{T}}$: *Flow of momentum (momentum per unit area, unit time) carried by EM fields*

Continuity Equations of EM fields in empty space

$$\frac{\partial\rho}{\partial t} = -\left(\nabla\cdot\mathbf{J}\right)$$

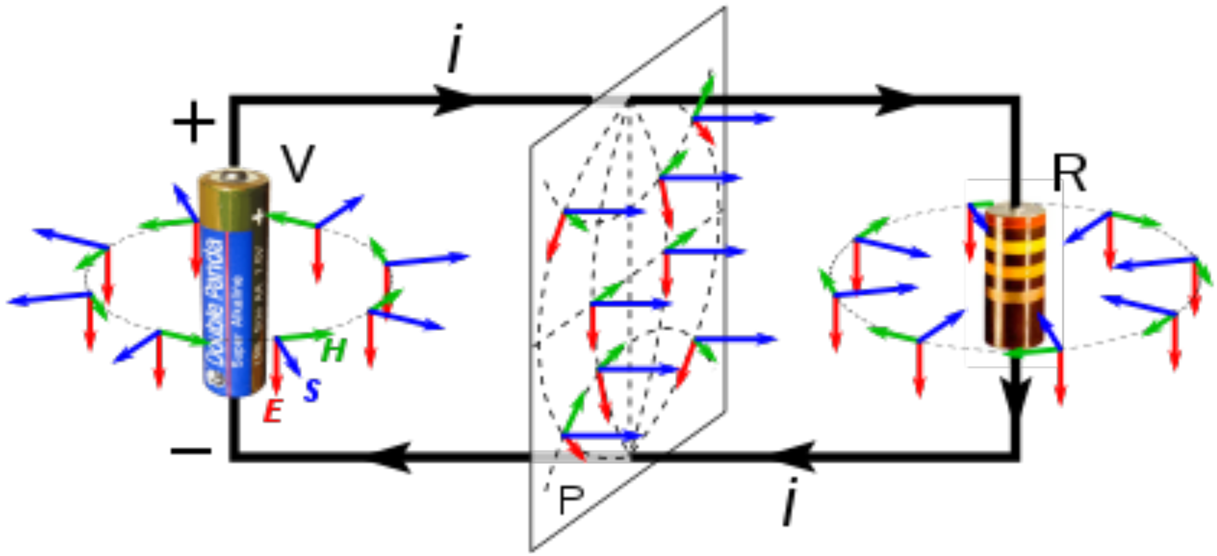
$$\frac{\partial u_{em}}{\partial t} = -\left(\nabla\cdot\mathbf{S}\right)$$

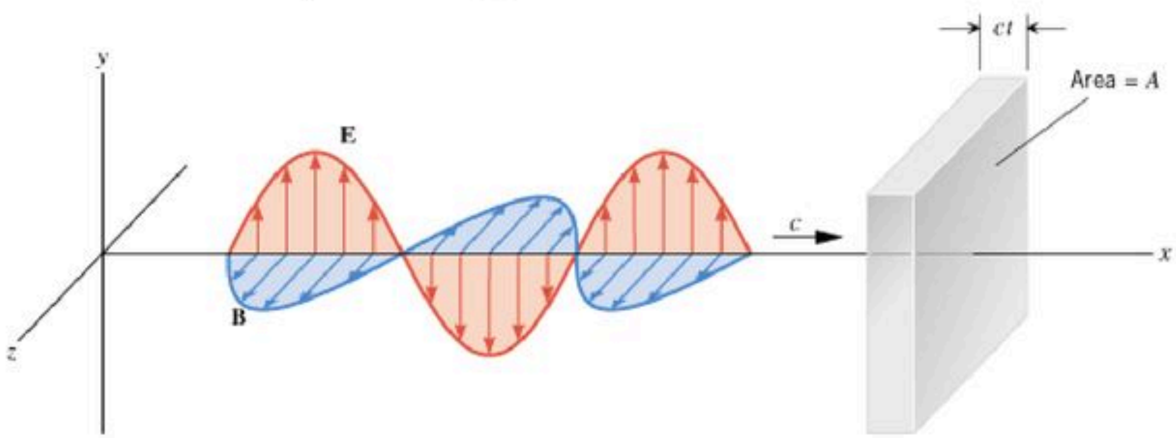
$(\mathbf{S})$ *playing the part of J → Local conservation of field energy*

$$\frac{\partial\mathbf{g}}{\partial t} = -\nabla\cdot\left(-\overleftrightarrow{\mathbf{T}}\right)$$

$\left(-\overleftrightarrow{\mathbf{T}}\right)$ *playing the part of J → Local conservation of field momentum*

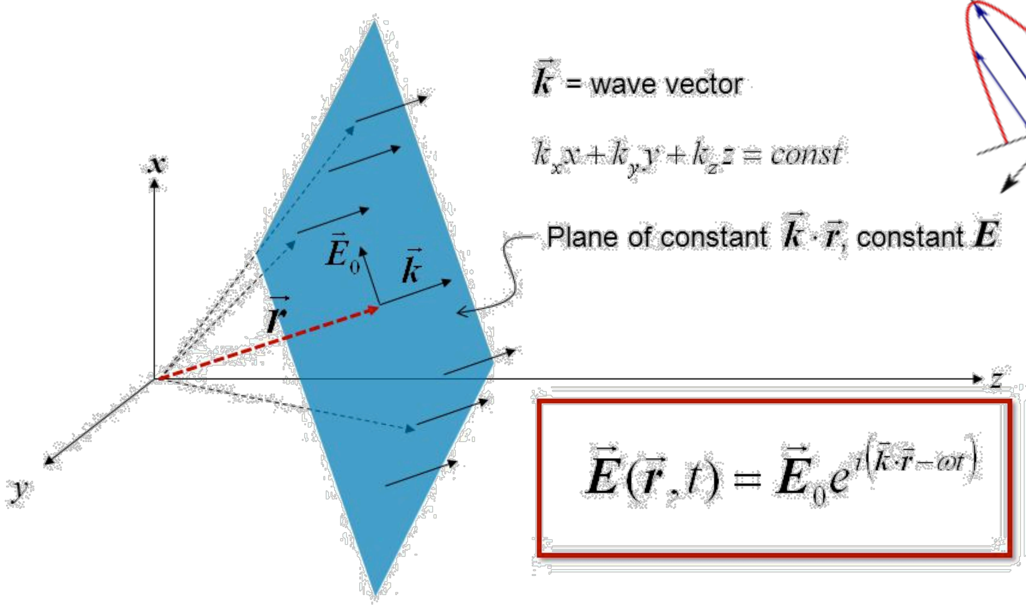
Electromagnetic Energy



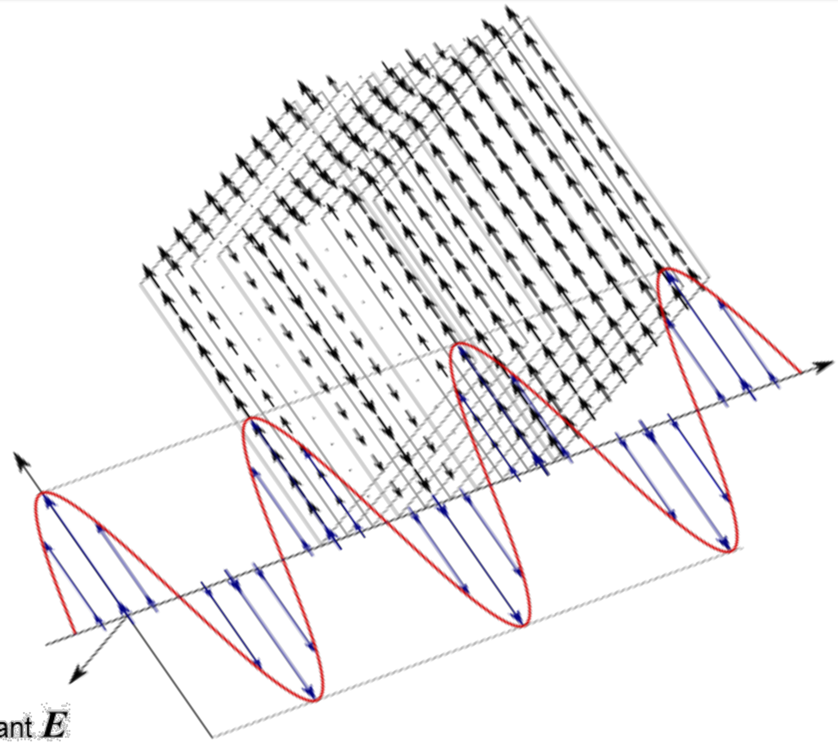


Ch. 9: Electromagnetic Waves

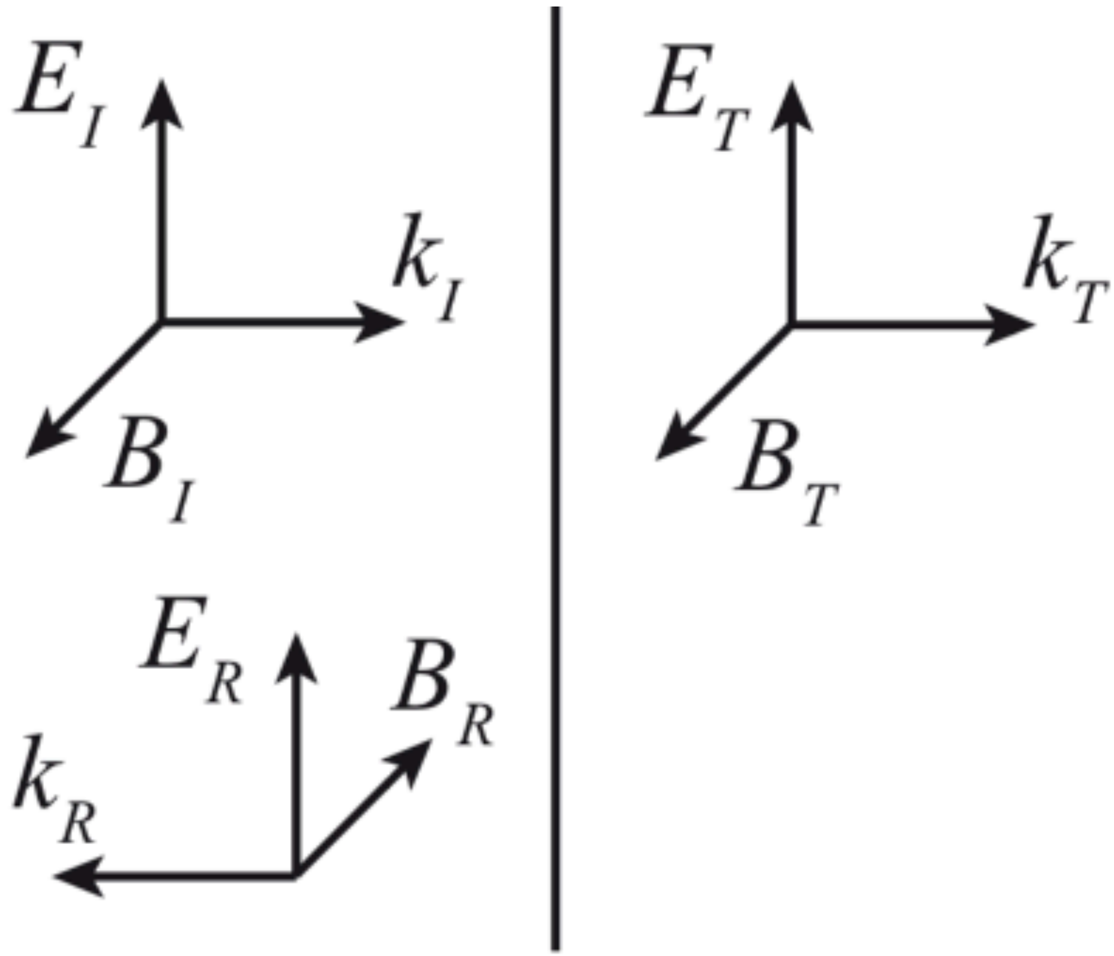
$$\vec{E}(x,y,z,t)=\vec{E}_0\sin\left(\vec{k}\cdot\vec{r}-\omega t\right)$$



$$\vec{E}(\vec{r},t)=\vec{E}_0 e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)}$$



Reflection & Transmission



Ch. 10: Potentials and Fields

Maxwell's equations in terms of the *scalar & vector potentials*

$$\nabla^2\phi + \frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A}\right) = -\frac{\rho}{\varepsilon_0}$$ ← Gauss' Law

$$\left[\nabla^2 - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\right]\vec{A} - \vec{\nabla}\left[\vec{\nabla}\cdot\vec{A} + \mu_0\varepsilon_0\frac{\partial\phi}{\partial t}\right] = -\mu_0\vec{J}$$

↑ Ampere's Law

Lienard-Wiechert Potentials

$$V(\mathbf{r},t)=\frac{1}{4\pi\varepsilon_0}\frac{qc}{|\mathbf{r}-\mathbf{w}(t_r)|c-(\mathbf{r}-\mathbf{w}(t_r))\cdot\mathbf{v}(t_r)}$$

implicit equation for *retarded time* t_r

$$c(t-t_r)=|\mathbf{r}-\mathbf{w}(t_r)|$$

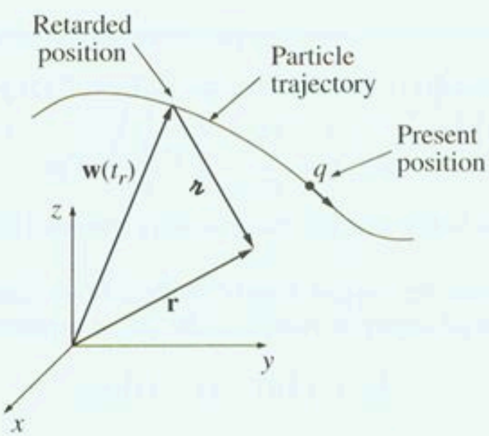


Figure 10.6

$$\mathbf{A}(\mathbf{r},t)=\frac{\mu_0}{4\pi}\frac{qc\mathbf{v}(t_r)}{|\mathbf{r}-\mathbf{w}(t_r)|c-(\mathbf{r}-\mathbf{w}(t_r))\cdot\mathbf{v}(t_r)}$$

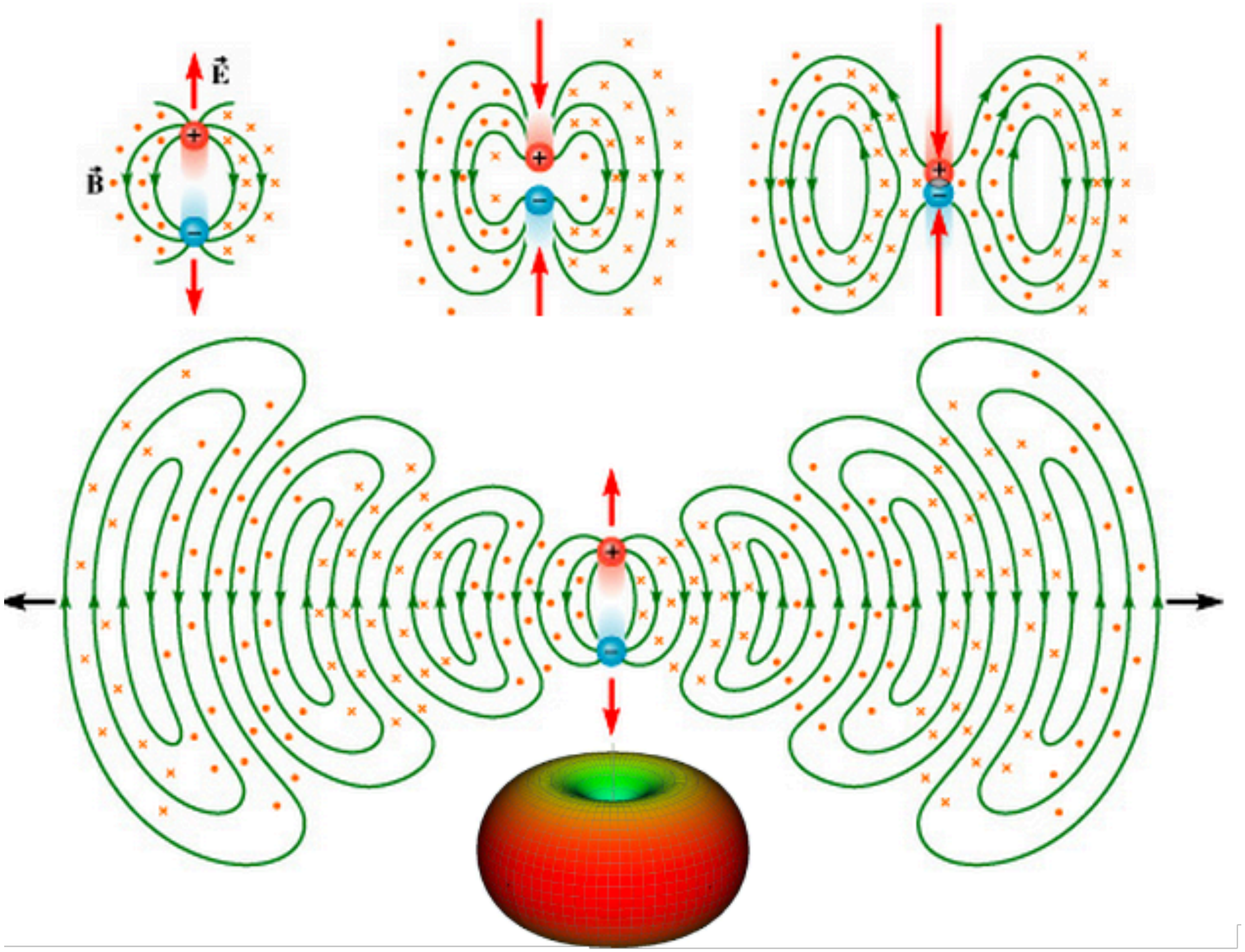
$$=\frac{\mathbf{v}(t_r)}{c^2}V(\mathbf{r},t)$$

Special Case: Constant Velocity

$$V(\mathbf{r},t)=\frac{1}{4\pi\epsilon_0}\frac{q}{R\sqrt{1-v^2\sin^2\theta/c^2}},$$

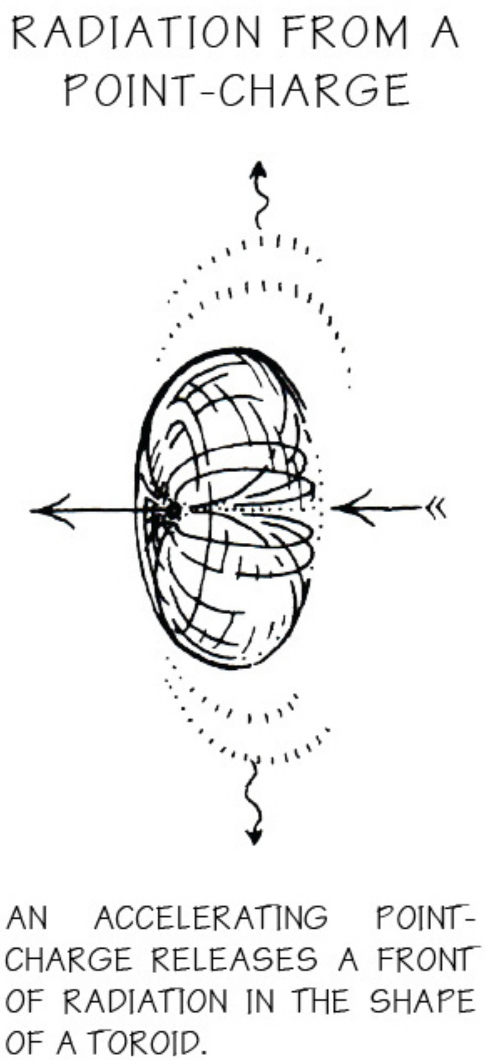
where $\mathbf{R}\equiv\mathbf{r}-\mathbf{v}t$ is the vector from the *present (!)* position of the particle to the field point $\mathbf{r}$, and θ is the angle between $\mathbf{R}$ and $\mathbf{v}$

Ch. 11: Radiation

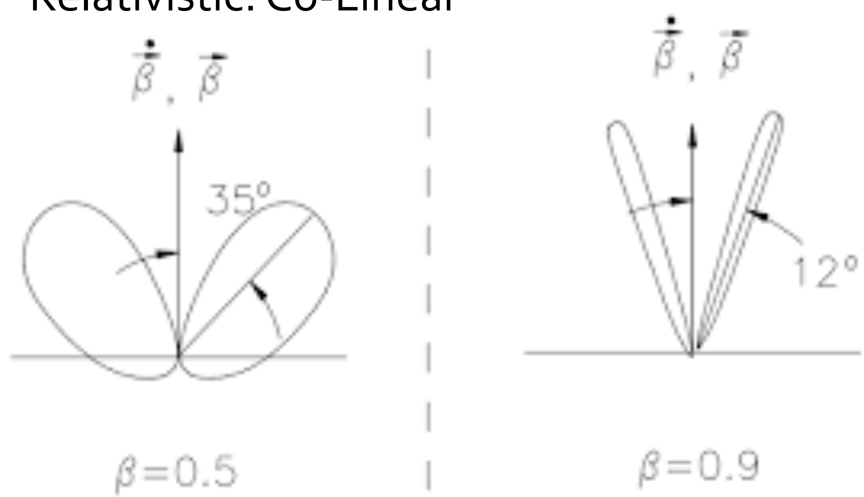


Electromagnetic Radiation from Oscillating Dipole

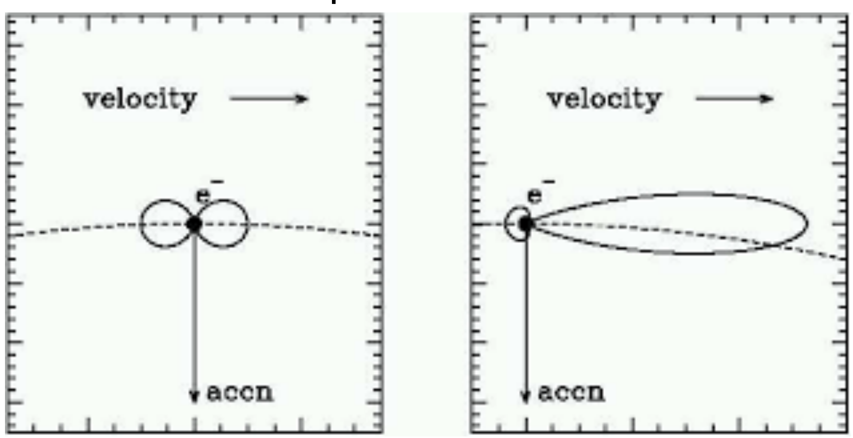
Radiation from Point Charges



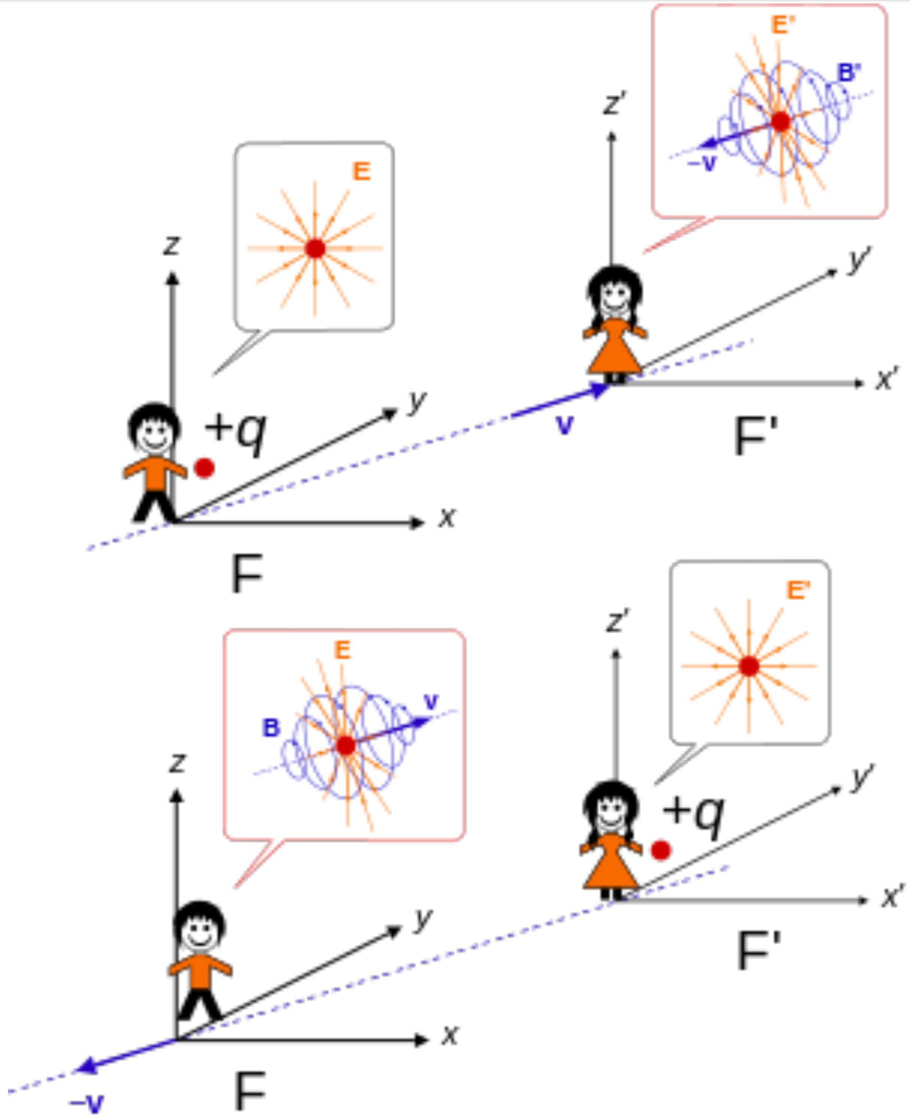
Relativistic: Co-Linear



Relativistic: Perpendicular



Ch. 12: Electrodynamics and Relativity



$$E'_x = E_x$$

$$E'_y = \gamma\left(E_y - vB_z\right)$$

$$E'_z = \gamma\left(E_z + vB_y\right)$$

$$B'_x = B_x$$

$$B'_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right)$$

$$B'_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$$

4-vectors and Lorentz Transformations

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma ct - \beta\gamma x \\ -\beta\gamma ct + \gamma x \\ y \\ z \end{bmatrix} \qquad \beta = \frac{\text{v}}{\text{c}} \qquad \gamma = \frac{1}{\sqrt{1 - \frac{\text{v}^2}{\text{c}^2}}} \qquad \Leftrightarrow \qquad \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$$\eta^\mu = \frac{\partial x_\mu}{\partial \tau} = \gamma \frac{\partial x_\mu}{\partial t} = \gamma \frac{\partial}{\partial t}(ct, \vec{x}) = \gamma(c, \frac{\partial \vec{x}}{\partial t})$$

$$J^\mu = \rho_0 \eta^\mu = (c\rho_0, \rho_0 v_x, \rho_0 v_y, \rho_0 v_z)/\sqrt{1 - v^2/c^2} = (c\rho, J_x, J_y, J_z)$$

$$A^\mu = (V/c, A_x, A_y, A_z)$$