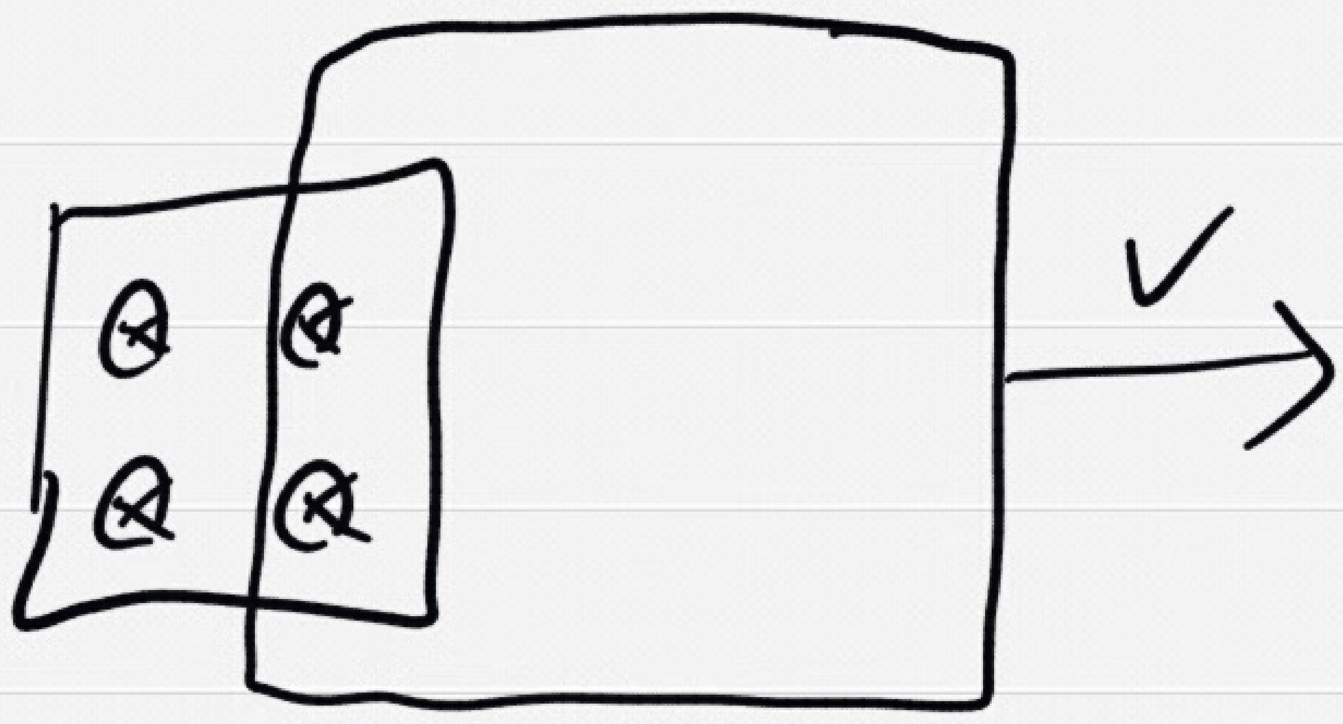
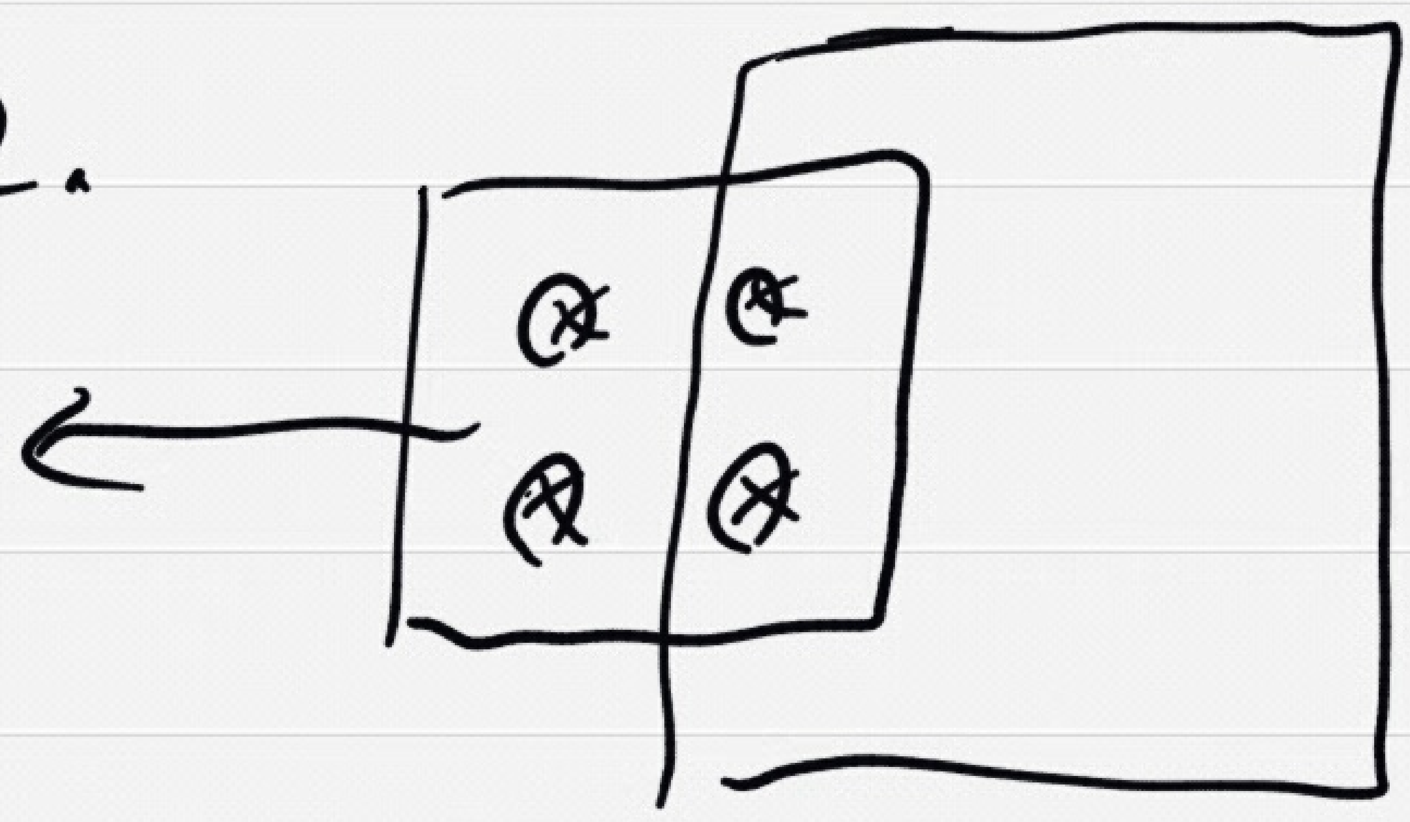
An issue w/ motional EMF

1. 

Move loop near magnet

2. 

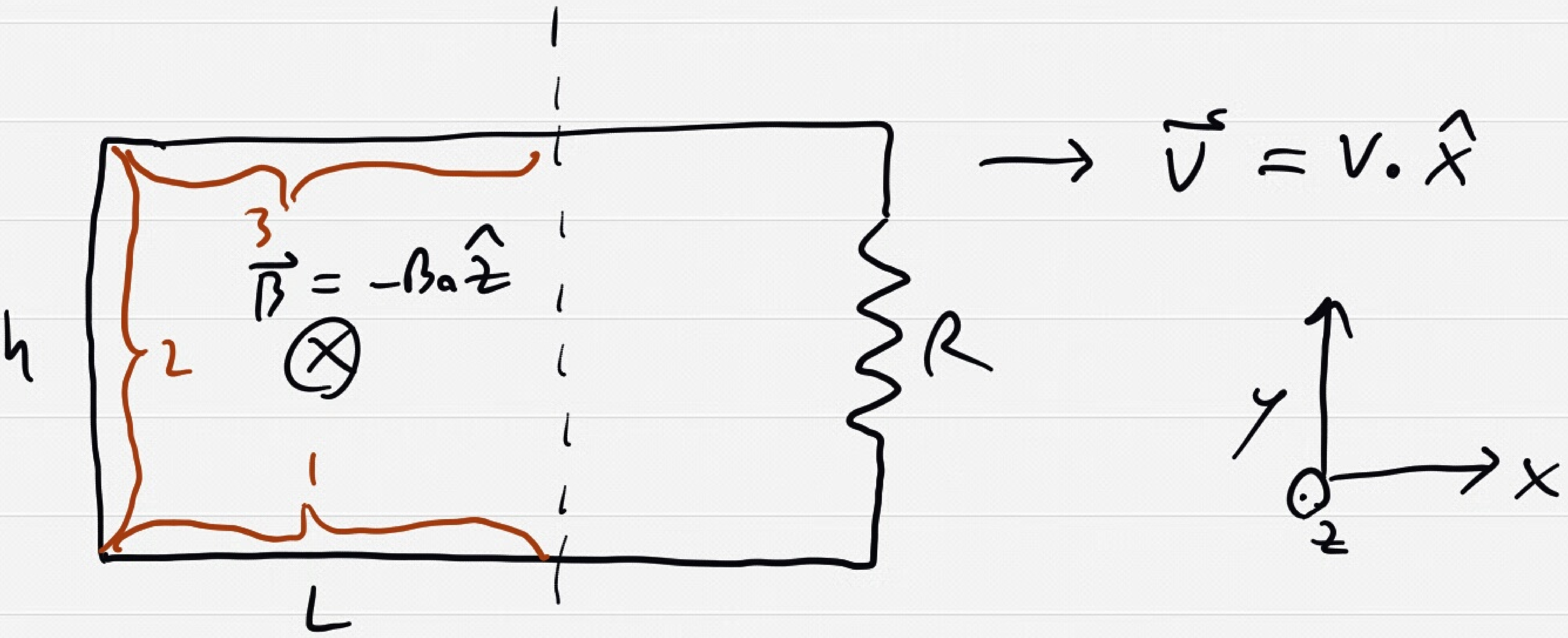
Move magnet near loop

- Same thing happens!
- But charge is stationary in Ex. 2, so $q\vec{v} \times \vec{B} = 0$
- Need relativity to reconcile this
- In the meantime, let's express motional EMF in a different way (which will avoid this problem)

Re-express Motional EMF

- Define magnetic flux

$$\Phi_B = \int \vec{B} \cdot d\vec{a}$$



- For this example $\Phi_B = B_0 h \cdot L$
  (if $d\vec{a} = -\hat{z}\, da$)
- Look at

$$d\Phi_B/dt = B_0 h \, dL/dt$$

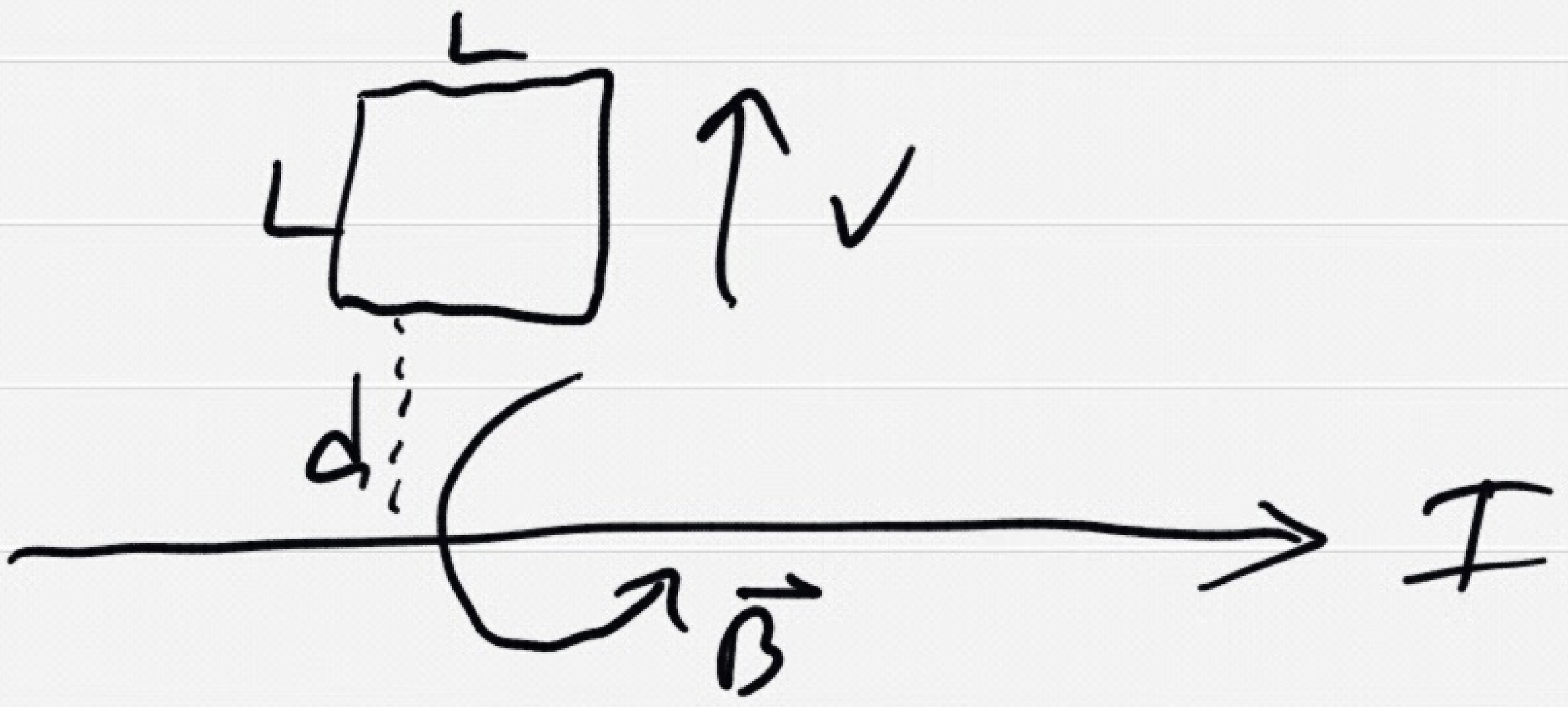
$$= B_0 \cdot h \cdot -v \quad \text{(L decreasing)}$$

$$= -\mathcal{E}$$

$\Rightarrow$ $\boxed{\mathcal{E} = - d\Phi_B/dt}$ at least for this case (it always does)

- But this formula works if instead the magnet moves!

Example



$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\varphi}$$

$$\Phi_B = \int \vec{B}\cdot d\vec{a}$$

$$= \int_0^L \int_d^{d+L} \frac{\mu_0 I}{2\pi s}\, ds\, dz$$

$$= \frac{\mu_0 I L}{2\pi}\left[\ln(d+L) - \ln(d)\right]$$

$$= \frac{\mu_0 I L}{2\pi}\ln\left(\frac{d+L}{d}\right)$$

$$\mathcal{E} = -d\Phi_B/dt$$

$$= -\frac{\mu_0 I L}{2\pi}\, d/dt\, \ln(1 + L/d)$$

$$= -\frac{\mu_0 I L}{2\pi}\,\frac{1}{1+L/d}\cdot -\frac{L}{d^2}\cdot dd/dt$$

$$= \boxed{\frac{\mu_0 I L^2}{2\pi d\,(L+d)}\, v}$$

positive, so CCW

Note: $\mathcal{E}$ drives I that tries to increase B (as B decreases)

Ch. 7.2	Electromagnetic Induction

Faraday's Law

$$\mathcal{E} = \oint \vec{E}\cdot d\vec{l} = -d\Phi_B/dt$$

$$= -d/dt \int \vec{B}\cdot d\vec{a}$$

$$= -\int \partial\vec{B}/\partial t \cdot d\vec{a}$$

But Stokes' Thm. says

$$\oint \vec{E}\cdot d\vec{l} = \int (\nabla\times\vec{E})\cdot d\vec{a}$$

True for any surface

$$\Rightarrow \boxed{\nabla\times\vec{E} = -\partial\vec{B}/\partial t}$$

= Differential form of Faraday's Law

Note! Now $\vec{E} \neq -\nabla V$!

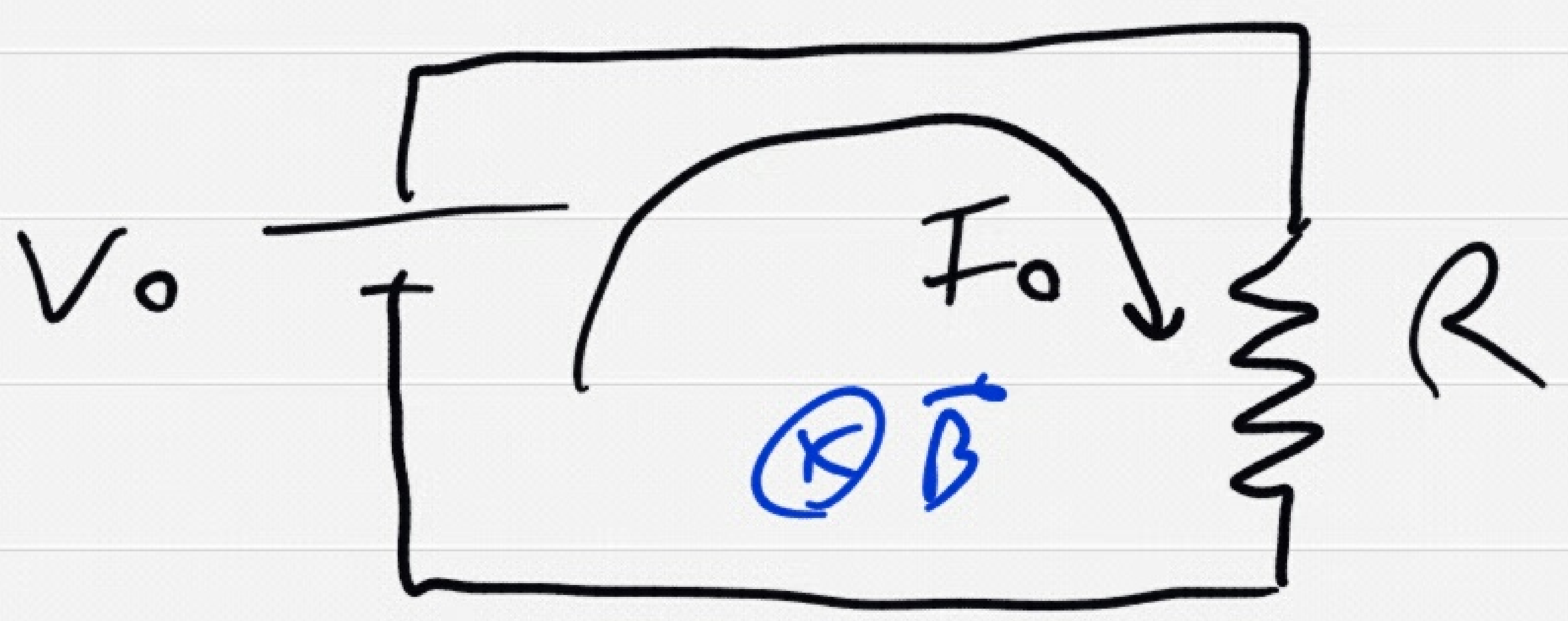
$\boxed{\text{Lenz's Law}}$

"Induced EMF always opposes <u>change</u> in flux"

$$d\Phi_B/dt \rightarrow \mathcal{E} \rightarrow I = \mathcal{E}/R$$

I will produce $\vec{B}$
that opposes change

Back EMF



$I_0 = V_0/R$

- Increase V_0 → increase I_0
  → increase $|\vec{B}|$

- Induced EMF $\mathcal{E}_{ind}$ opposes change in magnetic flux

  - $\mathcal{E}_{ind}$ is CCW
  - drives $I_{ind} = \mathcal{E}_{ind}/R$
  - opposes I_0

- Decrease V_0 → Decrease I_0
  → Decrease $|\vec{B}|$

  - Now $\mathcal{E}_{ind}$ is CW
  - drives $I_{ind} = \mathcal{E}_{ind}/R$
  - reinforces I_0

- Back EMF can be important at switch-on and switch-off

Calculating Induced Electric Field

For purely induced $\vec{E}$ (no charge)

$$\nabla \cdot \vec{E} = 0$$
$$\nabla \times \vec{E} = -\partial \vec{B} / \partial t$$

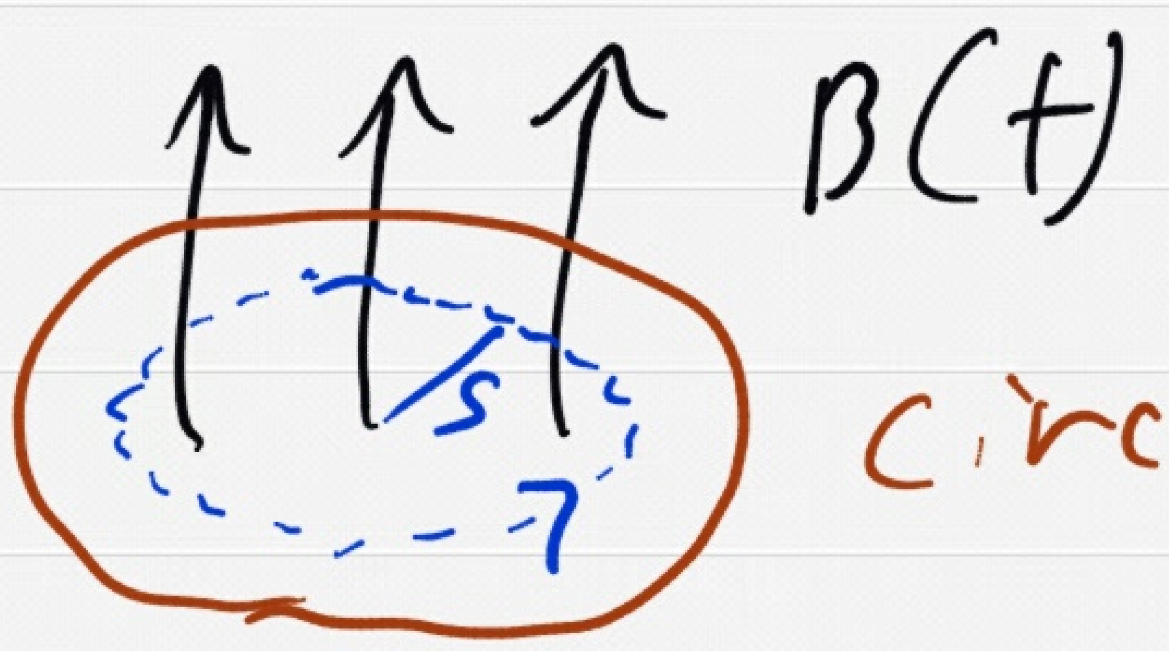
Looks like

$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Solve like Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$
$$\Rightarrow \oint \vec{E} \cdot d\vec{l} = -d\Phi_B / dt$$

Example



$$\int \vec{E} \cdot d\vec{l} = E \cdot 2\pi s$$
$$= -d\Phi_B / dt$$
$$= -d/dt\,(\pi s^2 B) = -\pi s^2 \, dB/dt$$
$$\Rightarrow \boxed{\vec{E} = -\frac{s}{2}\frac{dB}{dt}\hat{\varphi}}$$