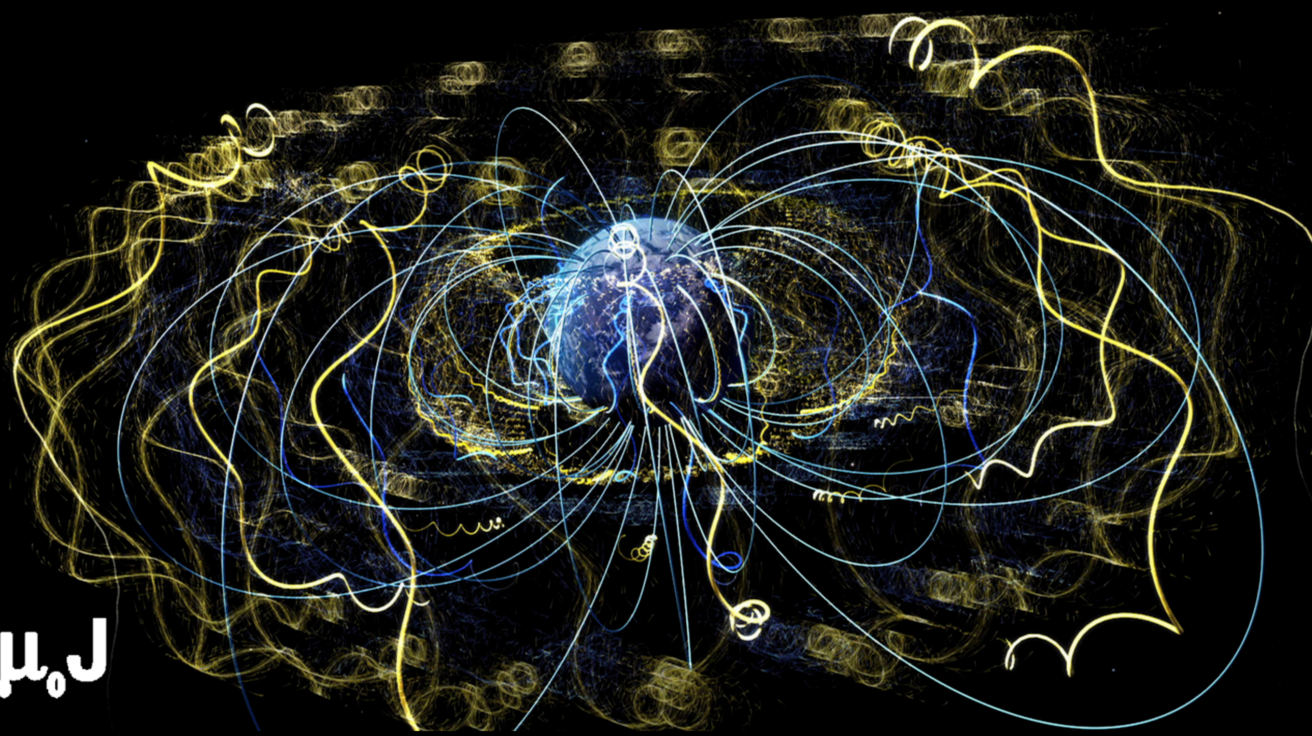
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

Inductance & Energy

- In a circuit, energy is required to overcome inductive effects

$$W/q = -\mathcal{E}_{ind} = \text{work done against induced EMF}$$

$$dW/dt = dW/dq \cdot dq/dt$$
$$= -\mathcal{E}_{ind} \cdot I$$
$$= -(-L\, dI/dt) \cdot I$$
$$= LI\, dI/dt$$
$$= \frac{L}{2}\, d/dt\, (I^2)$$

$$W = \int dW/dt\; dt = \boxed{\tfrac{1}{2} L I^2}$$

Example: solenoid

$$L = \mu_0 \cdot n^2 \cdot \pi r^2 \cdot l$$
$$= \mu_0 n^2 \cdot \text{Volume}$$

$$W = \tfrac{1}{2} \mu_0 n^2 \cdot I^2 \cdot \text{Volume}$$

$$B = \mu_0 n I \Rightarrow \boxed{W = \frac{1}{2\mu_0} \cdot B^2 \cdot \text{Volume}}$$

$\boxed{\text{Magnetic Energy}}$

$$L = \Phi_B / I \Rightarrow \Phi_B = LI$$

$$\Phi_B = \int \vec{B} \cdot d\vec{a} = \int (\nabla \times \vec{A}) \cdot d\vec{a}$$
$$= \oint \vec{A} \cdot d\vec{l}$$

so $W = \frac{1}{2} L I^2 = \frac{1}{2} I \cdot \Phi_B$
$$= \frac{1}{2} I \oint \vec{A} \cdot d\vec{l}$$
$$= \frac{1}{2} \oint \vec{A} \cdot \vec{I}\, dl$$ since I along $d\vec{l}$

- For general volume current

$$W \to \frac{1}{2} \int_V \vec{A} \cdot \vec{J}\, d\tau$$

In static case $\nabla \times \vec{B} = \mu_0 \vec{J}$

$$\Rightarrow W = \frac{1}{2\mu_0} \int_V \vec{A} \cdot (\nabla \times \vec{B})\, d\tau$$

$$\vec{A} \cdot (\nabla \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \nabla \cdot (\vec{A} \times \vec{B})$$
$$= B^2 - \nabla \cdot (\vec{A} \times \vec{B})$$

$$\Rightarrow W = \frac{1}{2\mu_0} \left[\int B^2 d\tau - \int \nabla \cdot (A \times \vec{B})\, d\tau \right]$$

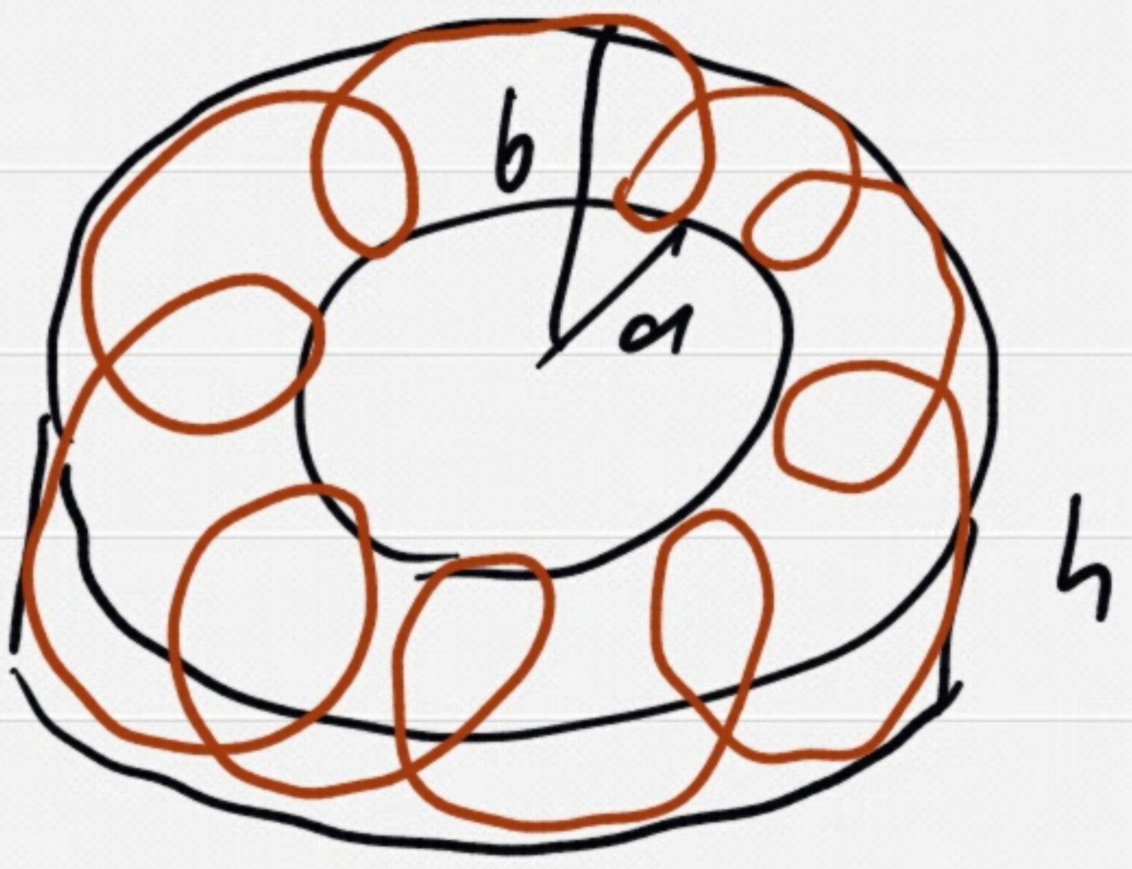
to zero for large volume

$$\boxed{W = \frac{1}{2\mu_0} \int B^2 d\tau}$$

Electromagnetic Energy

$$W = \frac{1}{2}\varepsilon_0 \int E^2 \, d\tau + \frac{1}{2\mu_0}\int B^2 \, d\tau$$

Example: Magnetic Energy in Toroid



$$\vec{B} = \frac{\mu_0 N I}{2\pi s}\hat{\varphi} \quad \text{inside}$$

$$B^2 = \frac{\mu_0^2 N^2 I^2}{4\pi^2 s^2}$$

$$W = \int_0^h \int_a^b \int_0^{2\pi} \frac{\mu_0 N^2 I^2}{8\pi^2 s^2} \cdot d\varphi \cdot s\,ds \cdot dz$$

$$= \int_0^h \int_a^b \frac{\mu_0 N^2 I^2}{4\pi s} \, ds \, dz$$

$$= \int_0^h \frac{\mu_0 N^2 I^2}{4\pi} \ln(s)\Big|_a^b \, dz$$

$$= \boxed{\frac{\mu_0 N^2 I^2}{4\pi} \ln(b/a) \cdot h}$$

but $W = \frac{1}{2} L I^2$

$$\Rightarrow \boxed{L = \frac{\mu_0 N^2 h}{2\pi} \ln(b/a)}$$

Electrostatics/Magnetostatics => Electrodynamics (Not quite Maxwell's Equations)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = 0$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

7.3. | Maxwells Equations

Consistency (or inconsistency) of E & M:

$$\nabla \times \vec{E} = -\partial \vec{B}/\partial t$$

$$\nabla \cdot (\nabla \times \vec{E}) = \nabla \cdot (-\partial \vec{B}/\partial t)$$

$$= -\partial/\partial t\,(\nabla \cdot \vec{B})$$

$$0 = 0 \quad //$$

But $\nabla \times \vec{B} = \mu_0 \vec{J}$

$$\nabla \cdot (\nabla \times \vec{B}) = \nabla \cdot (\mu_0 \vec{J})$$

$$0 = \;?$$

D'oh!

$\nabla \cdot \vec{J} \neq 0$ if current accumulates charge

e.g. charging capacitor