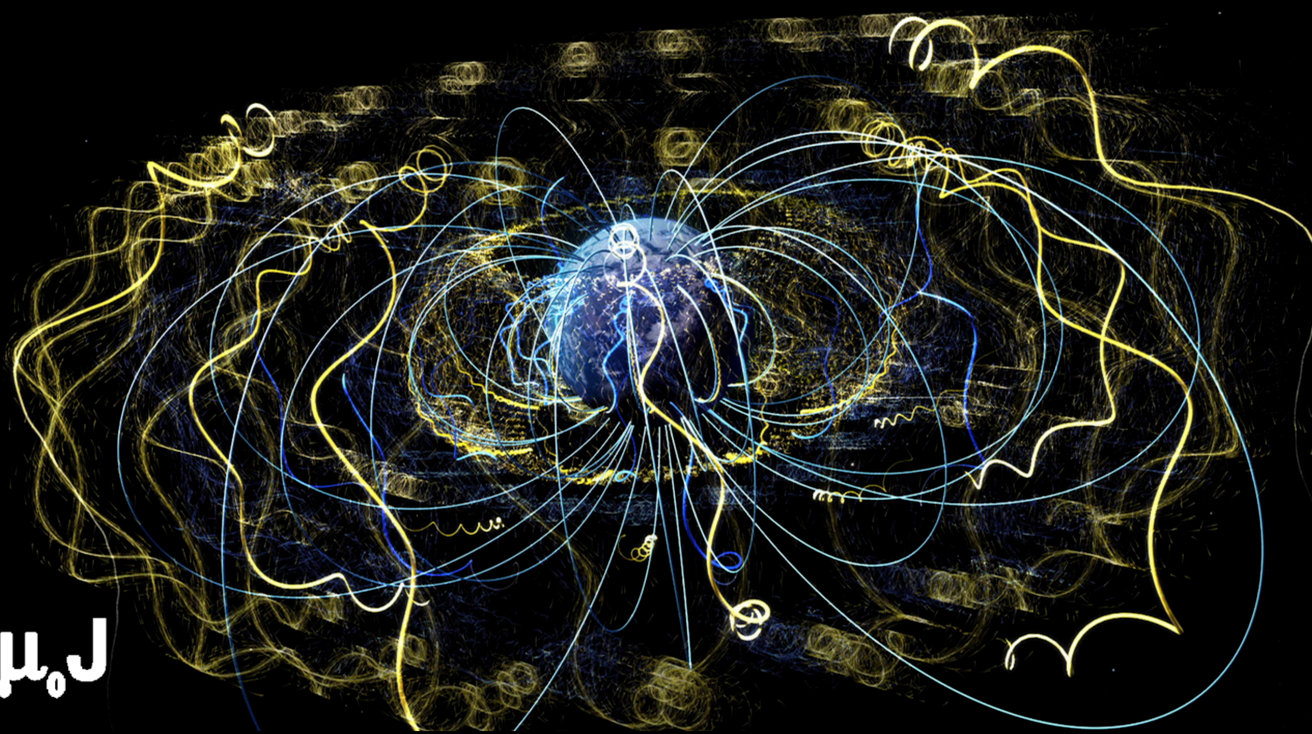
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mu_0 \mathbf{J}$$



Electricity and Magnetism II: 3812

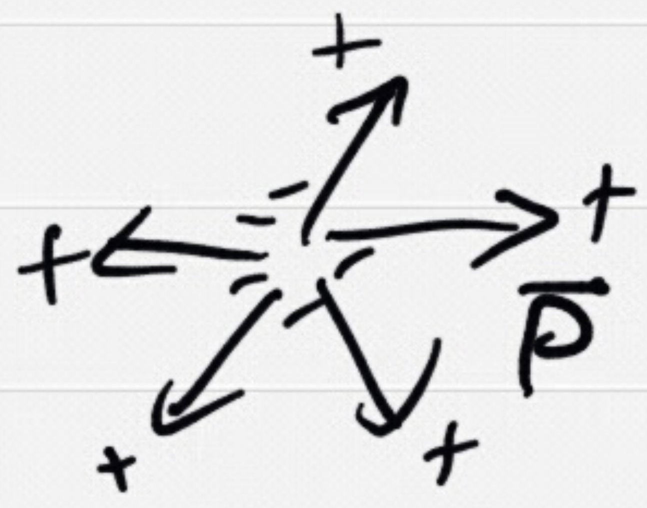
Professor Jasper Halekas
Van Allen 70
MWF 9:30-10:20 Lecture

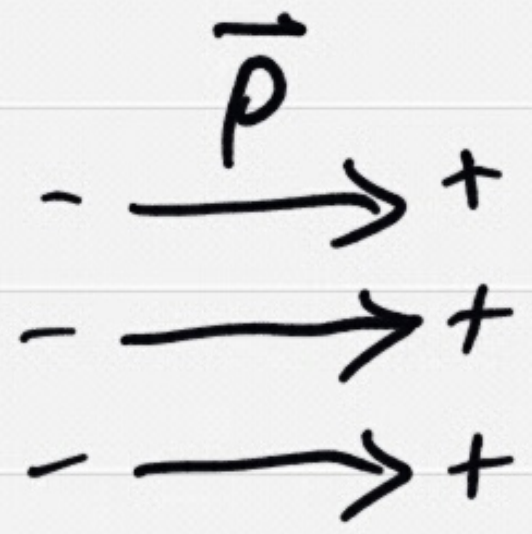
Maxwell's Equations in Matter

"special" kinds of charge & current

Bound Charge

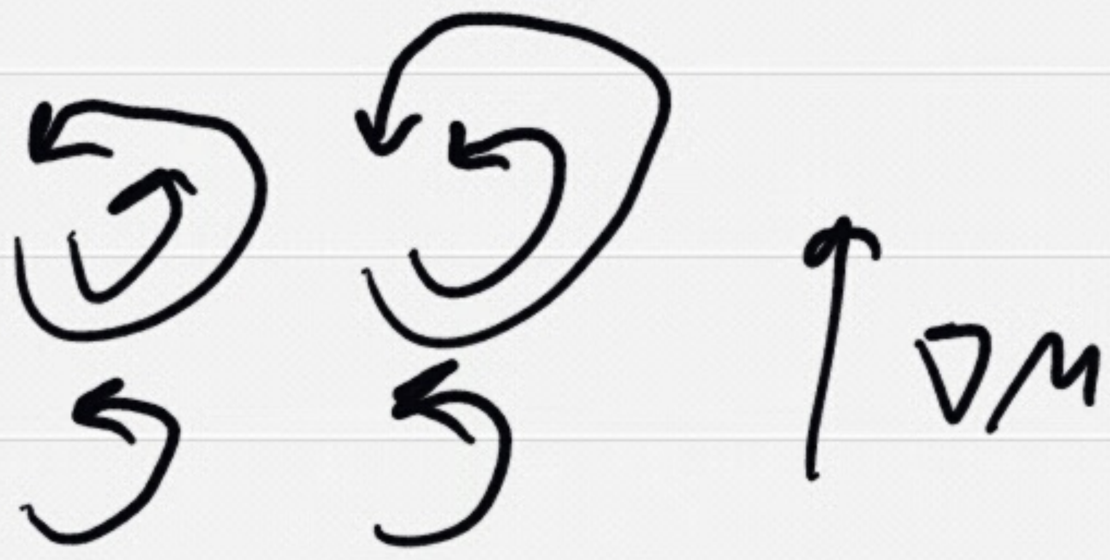
$$\rho_b = -\nabla \cdot \vec{P} \quad , \quad \sigma_b = \vec{P} \cdot \hat{n}$$





Bound current

$$\vec{J}_b = \nabla \times \vec{M} \quad , \quad \vec{K}_b = \vec{M} \times \hat{n}$$



Polarization Current (New)

$$\vec{J}_p = \partial \vec{P} / \partial t$$

$\partial \vec{P}/\partial t > 0$  $= \xrightarrow{\vec{J}_p}$

$\partial \vec{P}/\partial t < 0$  $= \xleftarrow{\vec{J}_p}$

Continuity of Bound Charge

$$\begin{aligned} \partial \rho_b / \partial t &= \partial/\partial t \left(-\nabla \cdot \vec{P}\right) \\ &= -\nabla \cdot \left(\partial \vec{P}/\partial t\right) \\ &= -\nabla \cdot \vec{J}_p \end{aligned}$$

$\vec{J}_p$ balances change in ρ_b as polarization changes

Bound Charge & Current

bound charge

$$\rho_b = -\nabla \cdot P$$

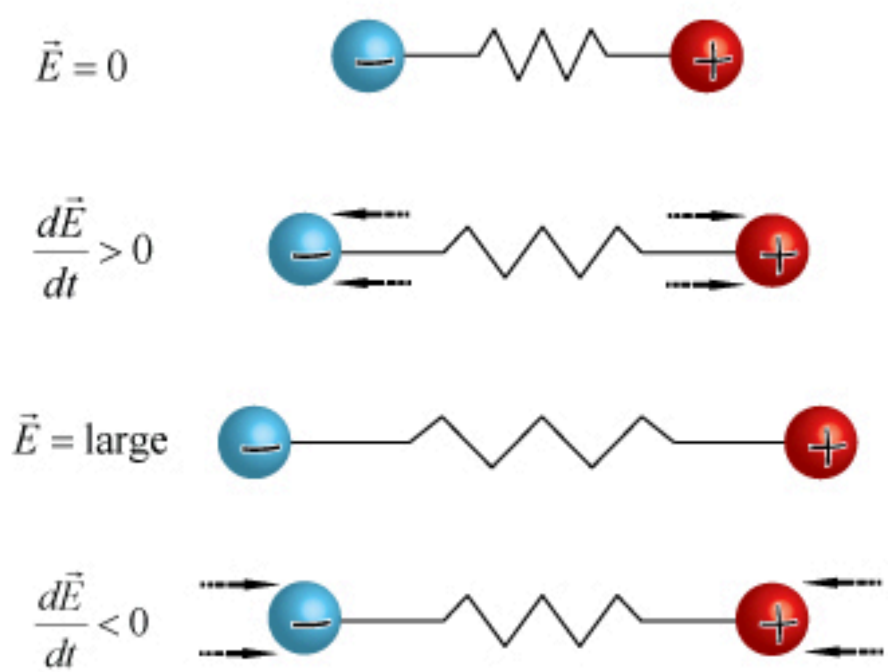
$$\frac{\partial \rho_b}{\partial t} = -\nabla \cdot \underbrace{\frac{\partial P}{\partial t}}_{\vec{J}_P} \quad \text{polarization current}$$

$$\frac{\partial \rho_b}{\partial t} + \nabla \cdot \vec{J}_P = 0$$

bound current

$$\vec{J}_b = \nabla \times \vec{M}$$

$$\nabla \cdot \vec{J}_b = 0 \quad \therefore \text{no corresponding } \rho$$



Total Charge Density

$$\rho = \rho_f + \rho_b$$
$$= \rho_f - \nabla \cdot \vec{P}$$

Total Current Density

$$\vec{J} = \vec{J}_f + \vec{J}_p + \vec{J}_b$$
$$= \vec{J}_f + \partial \vec{P}/\partial t + \nabla \times \vec{M}$$

Gauss's Law

$$\nabla \cdot \vec{E} = \frac{1}{\varepsilon_0}(\rho_f - \nabla \cdot \vec{P})$$

$$\nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\boxed{\nabla \cdot \vec{D} = \rho_f \quad \text{w/} \quad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}}$$

Ampere's Law

$$\nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \nabla \times \vec{M} + \partial \vec{P}/\partial t \right) + \mu_0 \varepsilon_0 \, \partial \vec{E}/\partial t$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f + \varepsilon_0 \, \partial \vec{E}/\partial t + \partial \vec{P}/\partial t$$

$$\boxed{\begin{aligned} \nabla \times \vec{H} &= \vec{J}_f + \partial \vec{D}/\partial t \\ \text{w/ } \vec{H} &= \frac{\vec{B}}{\mu_0} - \vec{M} \end{aligned}}$$

Others

$$\left.\boxed{\begin{aligned} \nabla \times \vec{E} &= -\partial \vec{B}/\partial t \\ \nabla \cdot \vec{B} &= 0 \end{aligned}}\right\} \text{unchanged}$$

- Mix of $\vec{B}$, $\vec{H}$, $\vec{E}$, $\vec{D}$

Note: $\vec{J}_d = \partial \vec{D}/\partial t = \vec{J}_p + \varepsilon_0 \, \partial \vec{E}/\partial t$

= "displacement current" in matter

Maxwell's Equations in Matter

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho_{\mathrm{free}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{\mathrm{free}} + \frac{\partial \vec{D}}{\partial t}$$

Integral Form

$$\oint \vec{D} \cdot d\vec{a} = Q_{f\_enc}$$

$$\oint \vec{B} \cdot d\vec{a} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -d/dt \int \vec{B} \cdot d\vec{a}$$

$$\oint \vec{H} \cdot d\vec{l} = I_{f\_enc} + d/dt \int \vec{D} \cdot d\vec{a}$$

Linear Media

To solve in matter, need relationship between $\vec{D}$ & $\vec{E}$, $\vec{B}$ & $\vec{H}$

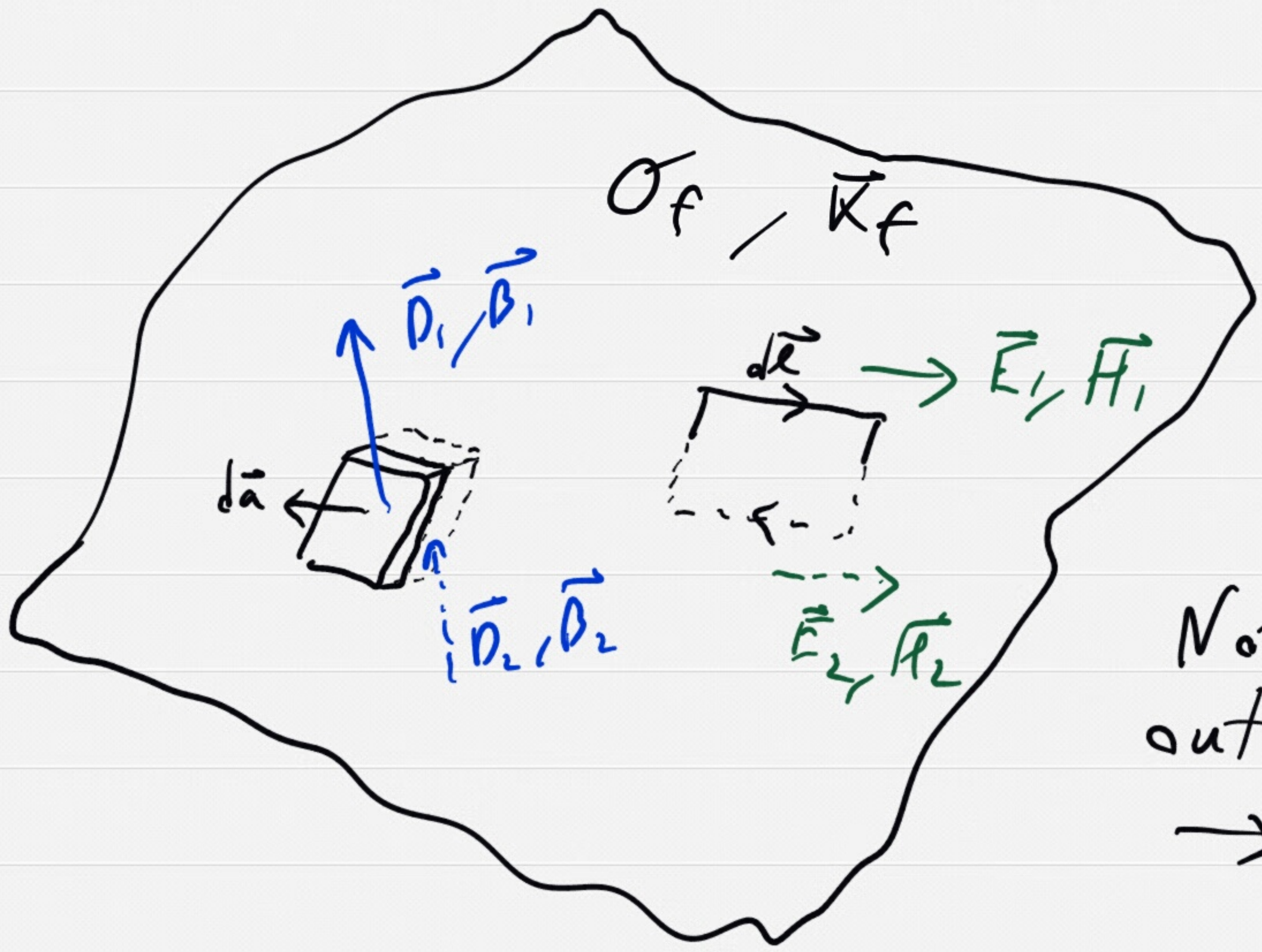
In linear media only:

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}, \qquad \vec{M} = \chi_m \vec{H}$$

$$\Rightarrow \vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon \vec{E}$$

$$\vec{H} = \vec{B}/\mu \quad \text{w/ } \mu = \mu_0 (1 + \chi_m)$$

Boundary Conditions



Note: thickness out of plane $\to 0$

$$\oint \vec{D} \cdot d\vec{a} = \vec{D}_1 \cdot \vec{a} - \vec{D}_2 \cdot \vec{a} = \sigma_f \cdot a$$

$$\Rightarrow \boxed{\Delta D_\perp = \sigma_f}$$

$$\oint \vec{B} \cdot d\vec{a} = \vec{B}_1 \cdot \vec{a} - \vec{B}_2 \cdot \vec{a} = 0$$

$$\Rightarrow \boxed{\Delta B_\perp = 0}$$

$$\oint \vec{H} \cdot d\vec{\ell} = \vec{H}_1 \cdot \vec{\ell} - \vec{H}_2 \cdot \vec{\ell} = I_{fenc} + d/dt \int \vec{D} \, d\vec{a}$$

$$\to I_{fenc} = (\vec{K}_f \times \hat{n}) \cdot \vec{\ell}$$

$$\Rightarrow \boxed{\Delta \vec{H}_\parallel = \vec{K}_f \times \hat{n}}$$

$$\oint \vec{E} \cdot d\vec{\ell} = \vec{E}_1 \cdot \vec{\ell} - \vec{E}_2 \cdot \vec{\ell} = -d/dt \int \vec{B} \cdot d\vec{a}$$

$$\to 0$$

$$\Rightarrow \boxed{\Delta \vec{E}_\parallel = 0}$$

Linear Media

$$\Delta(\varepsilon E_\perp) = \sigma_f$$
$$\Delta \vec{E}_\parallel = 0$$
$$\Delta B_\perp = 0$$
$$\Delta\left(\frac{1}{\mu}\vec{B}_\parallel\right) = \vec{K}_f \times \hat{n}$$

If $\sigma_f, \vec{K}_f = 0$

$$\varepsilon_1 E_{\perp 1} = \varepsilon_2 E_{\perp 2}$$
$$\vec{E}_{1\parallel} = \vec{E}_{2\parallel}$$
$$B_{1\perp} = B_{2\perp}$$
$$\vec{B}_{1\parallel}/\mu_1 = \vec{B}_{2\parallel}/\mu_2$$

$\Rightarrow$ reflection & refraction