C1:

 i_{ind} $\Rightarrow$ ⊗ B_{ind}

$\Rightarrow$ $d\varphi_B/dt$ ⊙

A goes up, so B ⊙

C2: a. i_{1ind}  $\Rightarrow$ B_{ind} = ⊙

A increasing, so B ⊗

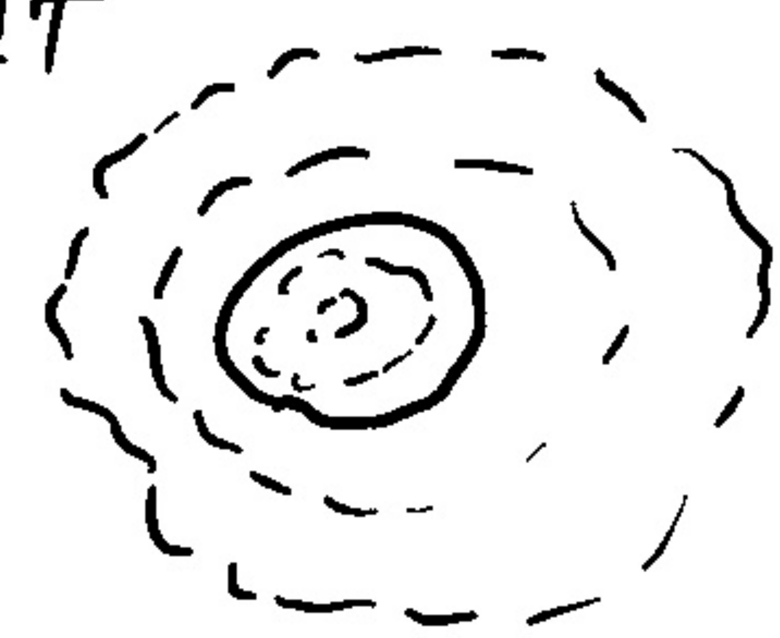
b. A also increasing, so i_{2ind} CCW

c. 

↑ dA/dt bigger, so $\mathcal{E}_1 > \mathcal{E}_2$

C3: $\oint \vec{E}\cdot d\vec{s} = d\varphi_B/dt = A\, dB/dt$

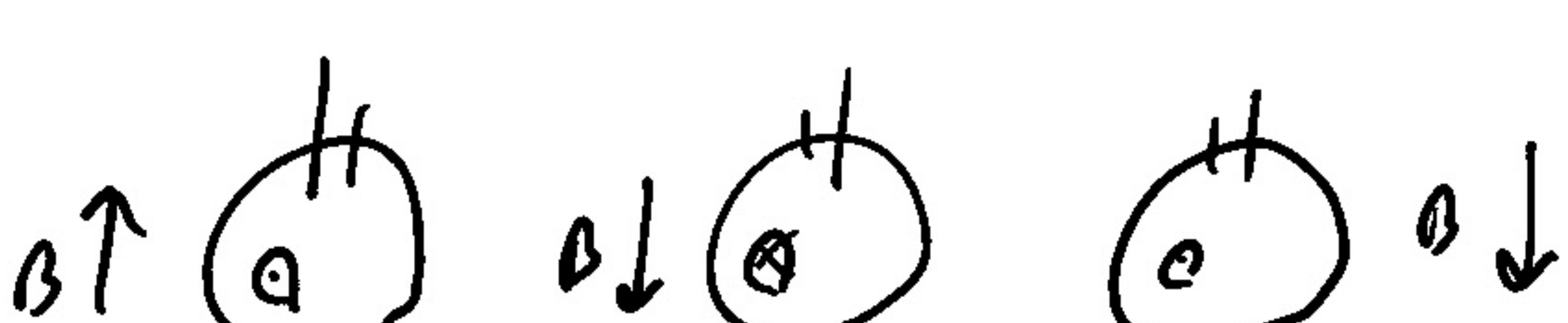
so $\mathcal{E}_C = \mathcal{E}_D > \mathcal{E}_D > \mathcal{E}_A$



C4: a. $\mathcal{E}_{ind}$ CW so opposite $\mathcal{E}_{bat}$

b. $\mathcal{E}_{ind}$ CW so same as $\mathcal{E}_{bat}$ ✓

c. $\mathcal{E}_{ind}$ CCW so opposite $\mathcal{E}_{bat}$

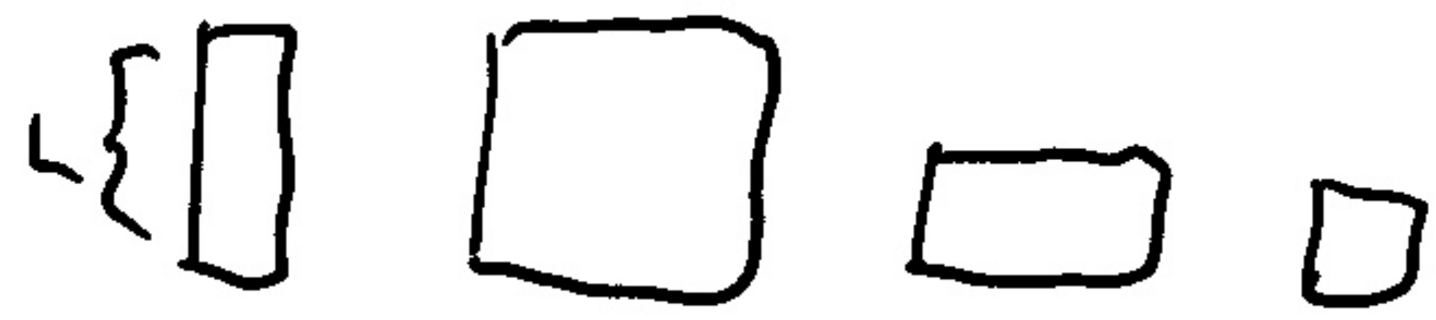


CS: $\varepsilon = d\phi_B/dt$

$= -B\, dA/dt$

$= BL\, dx/dt$

$= BLv$



$F = ILB = \varepsilon/R \cdot LB$

$= B^2L^2v/R$

$R = \rho L_w/A_w$

so $F = B^2L^2vA_w/\rho L_w$

$\propto L^2/L_w$

$1 > 2 > 4 > 3$

$P = I^2R = B^2L^2v^2/R$

$\propto L^2/L_w$

$1 > 2 > 4 > 3$

M1: $\varepsilon = -d\varphi_B/dt$

$= -A\, dB/dt$

M2: $C_w = \pi D$

$A_w = \pi (d/2)^2$

$R = \rho\, C_w/A_w$

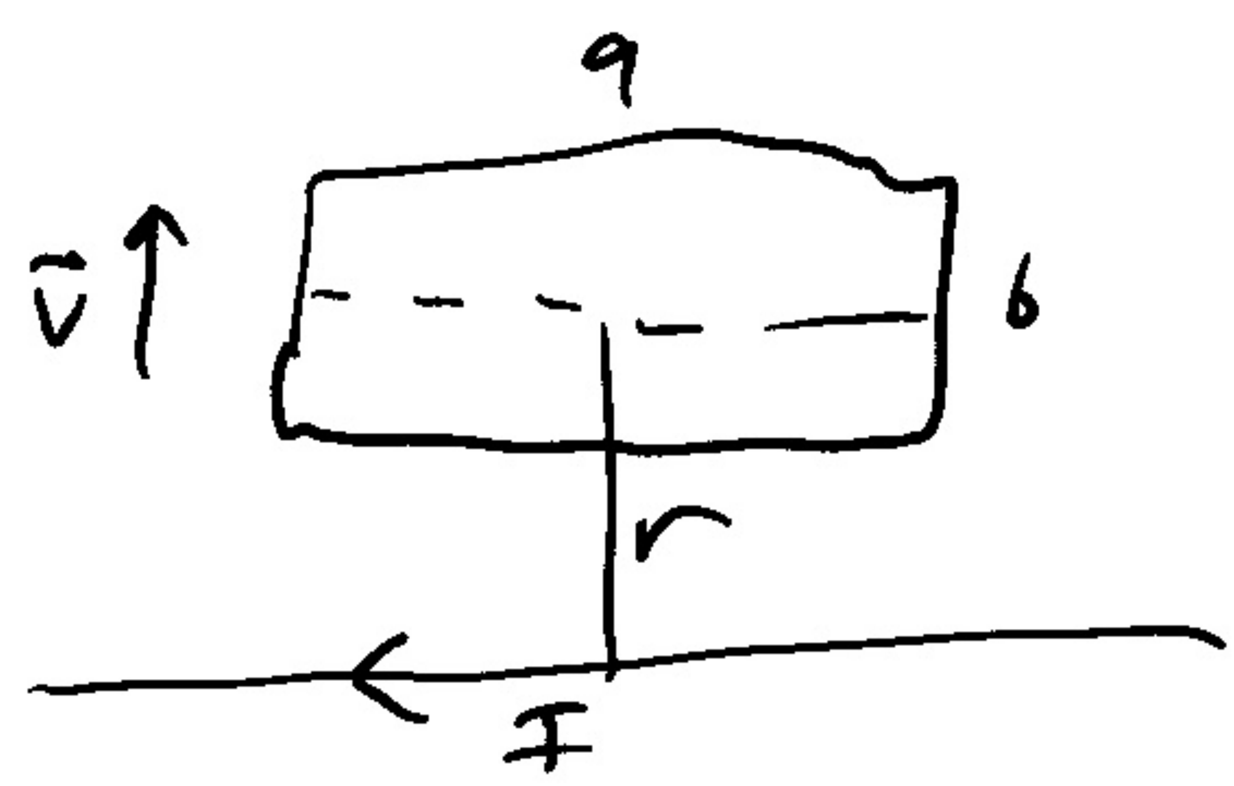
$I_{ind} = \varepsilon/R = \frac{1}{R} d\varphi_B/dt$

$= 1/R\, A\, dB/dt$

$= A_w/\rho C_w \cdot \pi (D/2)^2 \cdot dB/dt$

M3: $B_{wire} = \mu_0 I / 2\pi d$

$\varphi_B = \int \vec{B} \cdot d\vec{A}$



$\int \frac{\mu_0 I}{2\pi y}\, dA$

$= \int_{r-b/2}^{r+b/2} \int_0^a \frac{\mu_0 I}{2\pi y}\, dx\, dy$

$= \int_{r-b/2}^{r+b/2} \frac{\mu_0 I a}{2\pi y}\, dy$

$= \frac{\mu_0 I a}{2\pi} \ln(y) \Big|_{r-b/2}^{r+b/2}$

$= \frac{\mu_0 I a}{2\pi} \left[\ln(r + b/2) - \ln(r - b/2) \right]$

$$i_{ind} = \mathcal{E}/R$$

$$= \frac{1}{R} d\varphi_B/dt$$

$$= 1/R \; d/dt \left[\frac{\mu_0 I a}{2\pi} \left(\ln(r + b/2) - \ln(r - b/2) \right) \right]$$

$$= \frac{\mu_0 I a}{2\pi R} \left[\frac{1}{r + b/2} \, dr/dt - \frac{1}{r - b/2} \, dr/dt \right]$$

$$= \frac{\mu_0 I a v}{2\pi R} \left[\frac{1}{r + b/2} - \frac{1}{r - b/2} \right]$$

$$= \frac{\mu_0 I a v}{2\pi R} \left[\frac{-b}{r^2 - b^2/4} \right]$$

M4: $dB/dt = \Delta B/\Delta t$

$$\mathcal{E} = d\varphi_B/dt = A \, \Delta B/\Delta t$$

$$i = \mathcal{E}/R$$

$$P = i^2 R = \mathcal{E}^2/R$$

$$\Delta W = P \Delta t = \mathcal{E}^2/R \; \Delta t$$

$$= \frac{A^2 \Delta B^2}{R \Delta t}$$

M5: V_t when F_B balances F_g

$$F_B = ILB = \mathcal{E}/R \, LB$$

$$= BLV/R \cdot LB = B^2 L^2 V/R$$

$$F_g = mg$$

$$\Rightarrow v = mgR/(B^2 L^2)$$

$$\frac{d^2 v}{dt^2} = -\omega^2 v$$

- comes up often in physics

two solutions $\cos(\omega t)$, $\sin(\omega t)$

General solution

$$v(t) = A\cos(\omega t) + B\sin(\omega t)$$

- solve for A, B to match initial conditions

Another solution

$$e^{i\omega t}, \ e^{-i\omega t} \quad (i = \sqrt{-1})$$

$$\frac{d}{dt}\left(e^{i\omega t}\right) = i\omega e^{i\omega t}$$

$$\frac{d^2}{dt^2}\left(e^{i\omega t}\right) = (i\omega)^2 e^{i\omega t} = -\omega^2 e^{i\omega t}$$

$\cos\omega t$, $\sin\omega t$ related to $e^{i\omega t}$, $e^{-i\omega t}$

$$\cos\omega t = \left[e^{i\omega t} + e^{-i\omega t}\right]/2$$

$$\sin\omega t = \left[e^{i\omega t} - e^{-i\omega t}\right]/2i$$

– Advantage in dealing w/ phase shifts

$$\cos(\omega t + \varphi)$$

$$= \cos \omega t \cos \varphi - \sin \omega t \sin \varphi$$

$$\sin(\omega t + \varphi)$$

$$= \sin \omega t \cos \varphi + \cos \omega t \cos \varphi$$

$$e^{i(\omega t + \varphi)} = e^{i\omega t} e^{i\varphi}$$

$$e^{-i(\omega t + \varphi)} = e^{-i\omega t} e^{-i\varphi}$$

– Shifting phase mixes cosines and sines

– Does not mix $e^{\pm i\omega t}$