- Imagine a 220 kg satellite in orbit 640 km above the Earth.

- What are initial speed & period?

$$mv^2/r = GMm/r^2$$

$$\Rightarrow v^2 = GM/r$$

$$\Rightarrow v = \sqrt{GM/r}$$

$$= \sqrt{6.7\times10^{-11} \cdot 6\times10^{24}/(6378000 + 640000)}$$

$$\sim \sqrt{40\times10^{13}/7\times10^{6}}$$

$$\sim \sqrt{6\times10^{7}}$$

$$\sim 8000 \text{ m/s}$$

$$= \boxed{8 \text{ km/s}}$$

$$T = 2\pi r/v = \frac{2\pi\cdot 7\times10^{6}}{8000}$$

$$\sim \frac{40\times10^{6}}{8\times10^{3}} = 5000 \text{ s}$$

$$\equiv \boxed{90 \text{ min}}$$

- If satellite loses mechanical energy of $1.4 \times 10^5$ J per orbit, but maintains a circular orbit, what happens after 1500 orbits?

In circular orbit $K = GMm/2r$
$= -U/2$

$K_{init} = \frac{1}{2} \cdot 220 \cdot 8000^2$
$= 110 \cdot 64 \times 10^6$
$\cong 7 \times 10^9$ J

$\Delta K = 1.4 \times 10^5 \cdot 1500$
$= 2100 \times 10^5$
$= 2.1 \times 10^8$

$K_{final} = K_{init} + \Delta K$
$= 7.2 \times 10^9$

$$\frac{r_{final}}{r_{init}} = \frac{K_{init}}{K_{final}} \cong 0.97$$

$\boxed{\text{altitude} \sim 420 \text{ km}}$

V increases slightly
and T decreases slightly

- What is magnitude of retarding force?

$$W = \Delta KE$$
$$= F \Delta s$$

$$\Delta s = 2\pi r$$
$$\cong 40 \times 10^6 \text{ m}$$

$$\Delta KE = 1.4 \times 10^5$$

$$\Rightarrow F = \Delta KE / \Delta s$$
$$= 1.4 \times 10^5 / 4 \times 10^7$$
$$= \boxed{3.5 \times 10^{-3} \text{ N}}$$

- Very small drag force will deorbit a satellite

1500 orbits @ 90 min/orbit
$\Rightarrow$ 135 000 min
= 2250 hr
< 100 days

- In one-dimension:

$$\Delta U_{ab} = -\int_a^b F(x)\,dx$$

with inverse operation

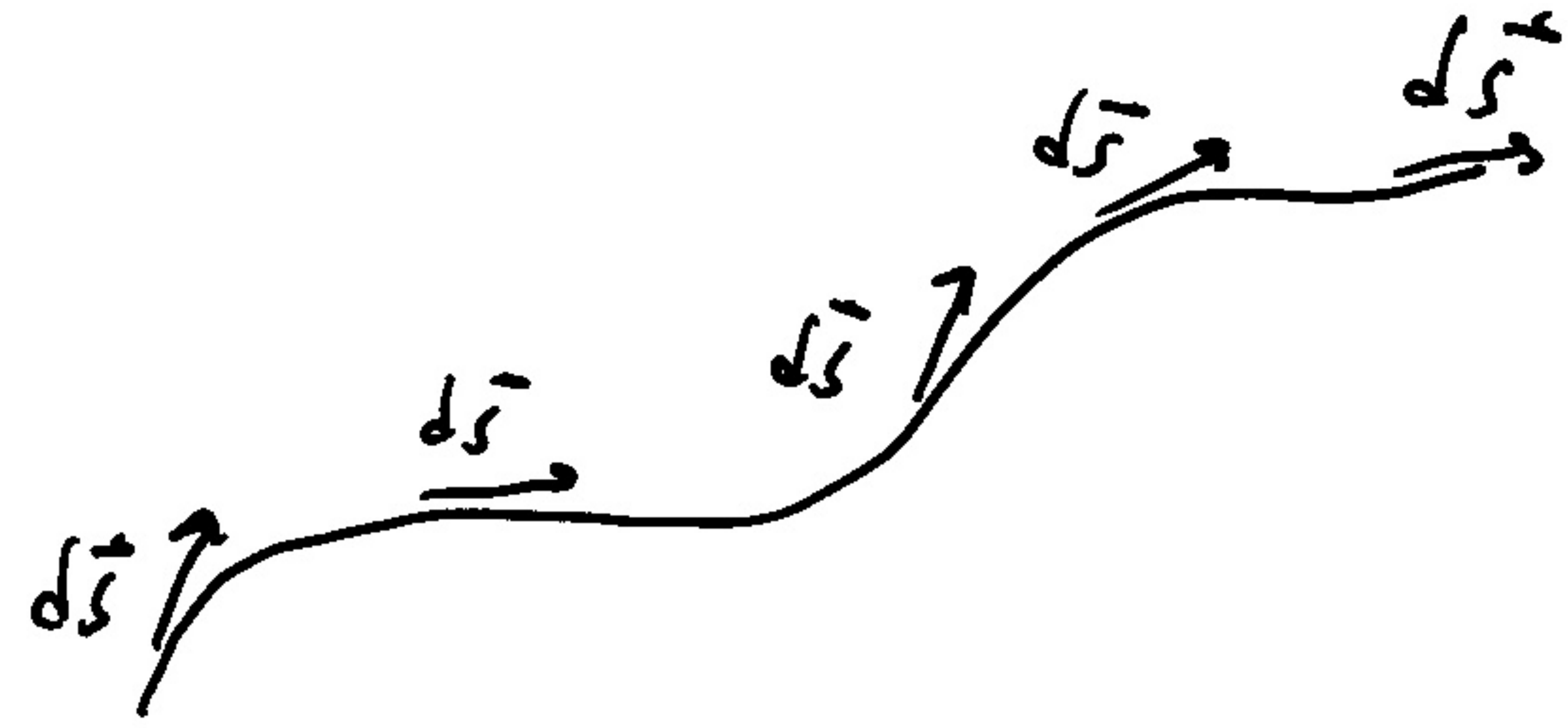
$$F(x) = -dU/dx$$

- In three-dimensions:

we have seen that

$$\Delta U = -\int_a^b \vec{F}(\vec{r}) \cdot d\vec{s}$$

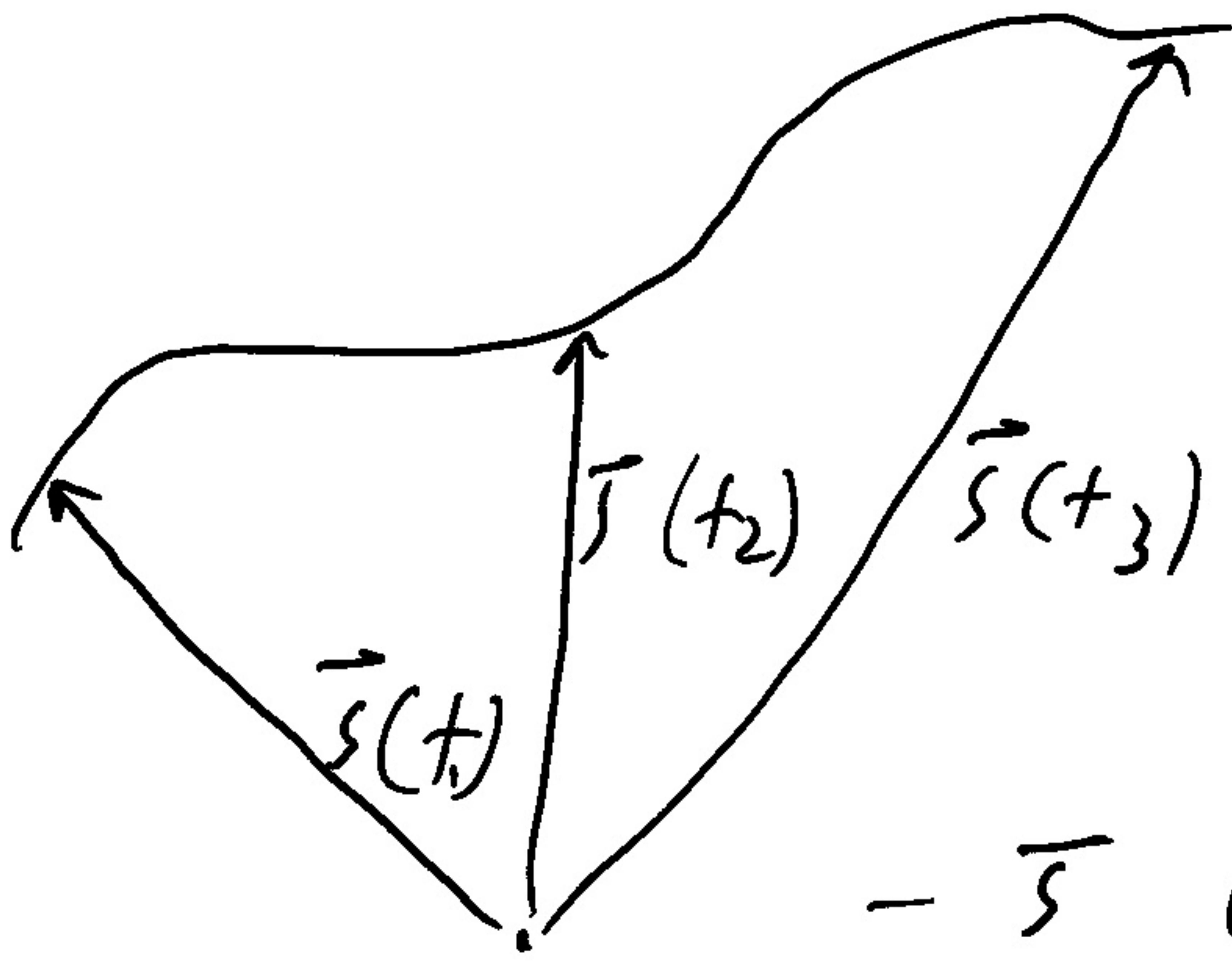
where $d\vec{s}$ is tangent to a curve




What is the inverse?

$$\vec{F}(\vec{r}) = -\nabla U = -\left[\partial U/\partial x,\ \partial U/\partial y,\ \partial U/\partial z\right]$$

- First let's parameterize our curve as a function of a dummy variable $t$




- $\vec{s}$ is a position vector

$\vec{s}\,'(t)$ is a velocity vector tangent to the curve

$$\vec{s}\,'(t) = [dx/dt,\ dy/dt,\ dz/dt]$$

where $\vec{s}(t) = [x(t), y(t), z(t)]$

so $-\int \vec{F}(\vec{r}) \cdot d\vec{s}$

$$= -\int \vec{F}(\vec{s}(t)) \cdot \vec{s}\,'(t)\, dt$$

$$-\int \vec{F}(\vec{s}(t)) \cdot \vec{s}\,'(t)\, dt$$

$$= -\int [\underbrace{F_x \, dx/dt + F_y \, dy/dt + F_z \, dz/dt}_{\text{Need this to be a perfect differential}}]\, dt$$

Use chain rule on $U(\vec{r})$

$$dU(\vec{r})/dt = dU(x(t), y(t), z(t))/dt$$

$$= \partial U/\partial x \; dx/dt + \partial U/\partial y \; dy/dt + \partial U/\partial z \; dz/dt$$

$$= [\partial U/\partial x, \partial U/\partial y, \partial U/\partial z] \cdot \vec{s}\,'(t)$$

$$= \nabla U \cdot \vec{s}\,'(t)$$

$$\int_a^b dU/dt \; dt = U(b) - U(a)$$

$$= \Delta U_{ab}$$

$$= \int_a^b \nabla U \cdot \vec{s}\,'(t)\, dt$$

$$= \int_a^b \nabla U \cdot d\vec{s} = -\int_a^b \vec{F} \cdot d\vec{s}$$

$$\Rightarrow \vec{F} = -\nabla U$$

$\nabla U$ = grad $U$

= gradient of $U$

- What is physical meaning?

- Just like $dU/dx$ in 1-d tells you the slope in 1-d, $\nabla U$ tells you the slope in 3-d.

$\partial U/\partial x$ is slope in x-direction
$\partial U/\partial y$ " " y-direction
$\partial U/\partial z$ " " z-direction

- Where does $\nabla U$ point?

It points uphill!

so $-\nabla U = \vec{F}$ points downhill