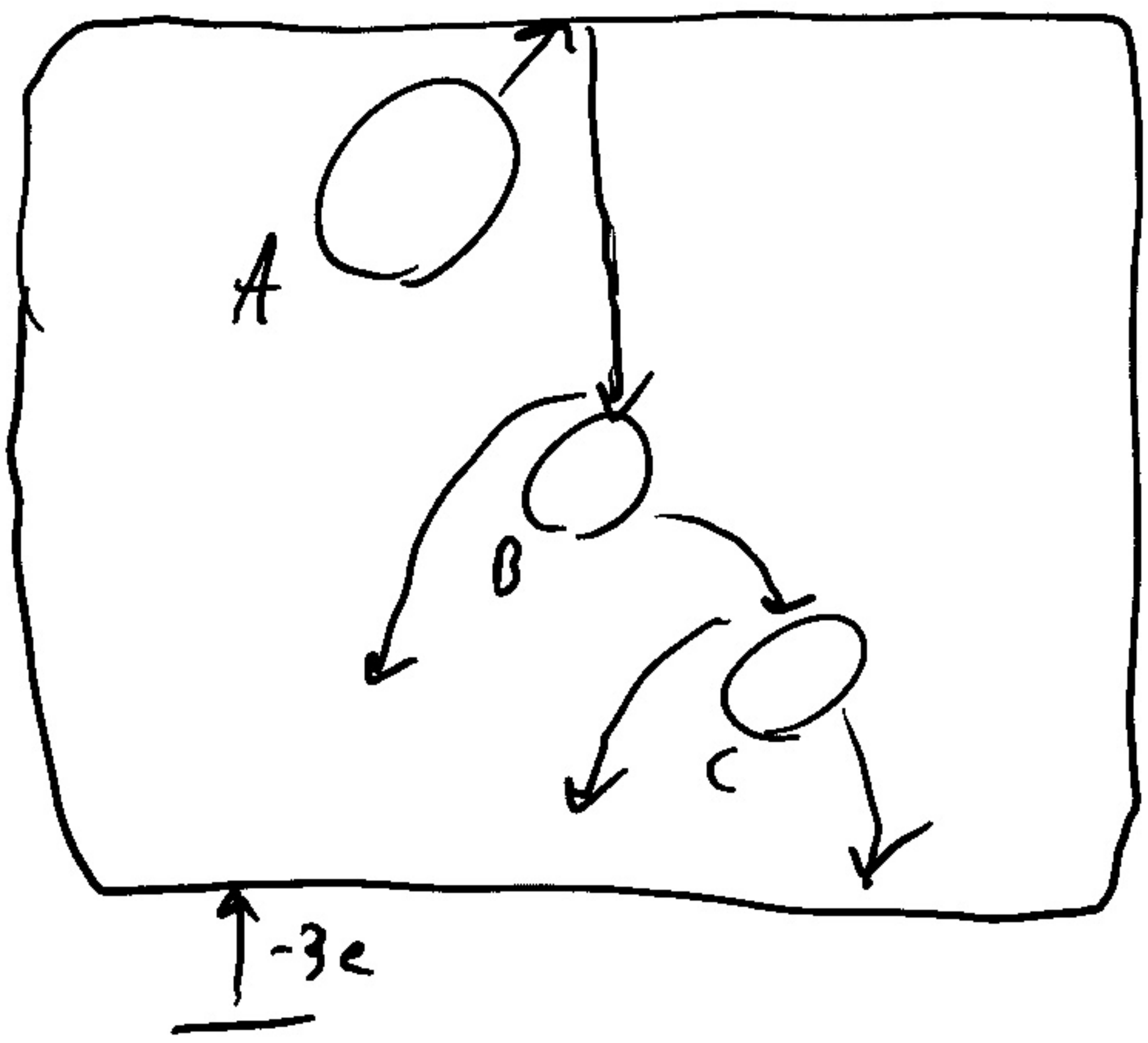
C5:



All start w/ Q_0

① A loses Q_0 hitting ceiling

$Q_A = 0$, $Q_B = Q_0$, $Q_C = Q_0$

② A and B share charge

$Q_A = Q_0/2$, $Q_B = Q_0/2$, $Q_C = Q_0$

③ B and C share charge

$Q_A = Q_0/2$, $Q_B = 3Q_0/4$, $Q_C = 3Q_0/4$

④ Q_C lost $= 3e \Rightarrow Q_0 = 4e$

Next A loses $Q_0/2 = 2e$

B loses $3Q_0/4 = 3e$

Total transfer $= 12e$

M2: Remember from class, $\vec{g}$ along axis is

$$= \frac{-GMx}{(x^2+R^2)^{3/2}}\hat{x}$$

$$\vec{F} = m\vec{g} = \frac{-GMmx\hat{x}}{(x^2+R^2)^{3/2}}$$

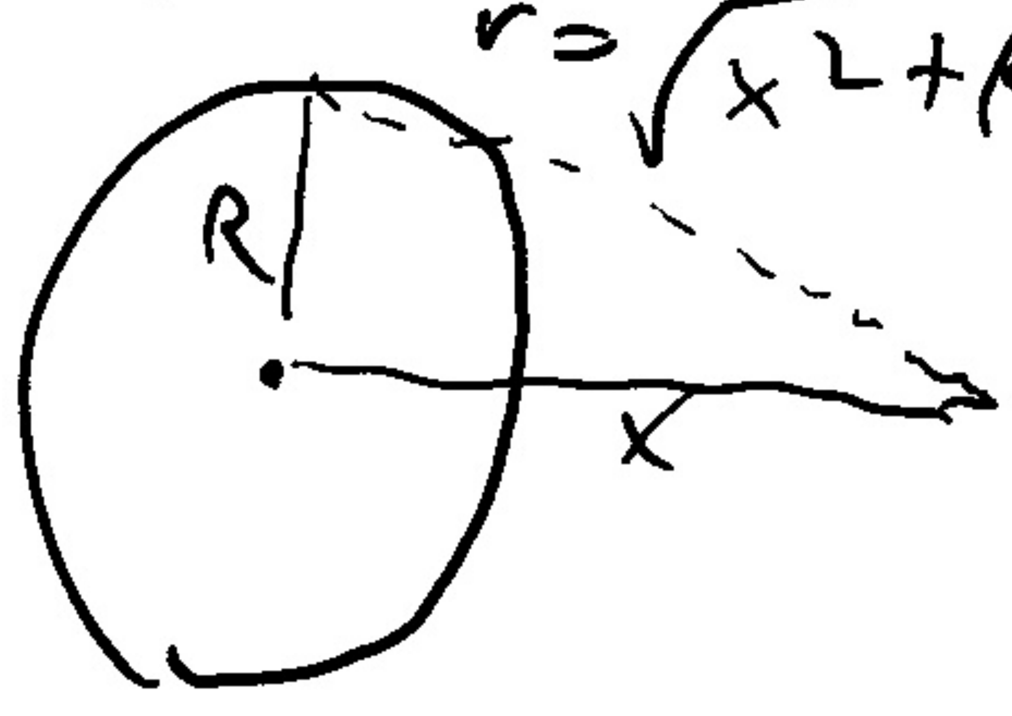
$$\vec{F} = m\vec{a} = m\,d\vec{v}/dt = m\,dv/dx\,dx/dt\,\hat{x}$$

- could solve a diff. eq.
- or, could use potential energy

$$U = -\frac{GMm}{r}$$ for point mass

- to compute U for extended source, add potential from all parts

$$dU = \frac{-Gm\,dM}{r}$$



All points at same distance, so $U = \int dU$

$$-\frac{GMm}{\sqrt{x^2+R^2}}$$ along axis

$$\Delta E = \Delta KE + \Delta U = 0$$

$$\Delta KE = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_0^2$$
$$= \tfrac{1}{2} m v_f^2 \quad \text{starting from rest}$$

$$= -\Delta U$$
$$= U_0 - U_f$$

$$= \frac{-GMm}{\sqrt{R^2 + x_0^2}} - \left(-\frac{GMm}{\sqrt{R^2}} \right)$$

$$= GMm \left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + x_0^2}} \right)$$

$$\text{so} \quad v_f^2 = 2MG \left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + x_0^2}} \right)$$

$$v_f = \sqrt{2MG \left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + x_0^2}} \right)}$$

- Note potential was easy to calculate because we added scalars instead of vectors

- Could calculate U, then use that to get $\vec{F}$

$$U(x) = \frac{-GMm}{\sqrt{R^2+x^2}} \quad \text{on axis}$$

$$\vec{F} = -\nabla U$$

$$= -\left[\partial/\partial x, \partial/\partial y, \partial/\partial z\right] U$$

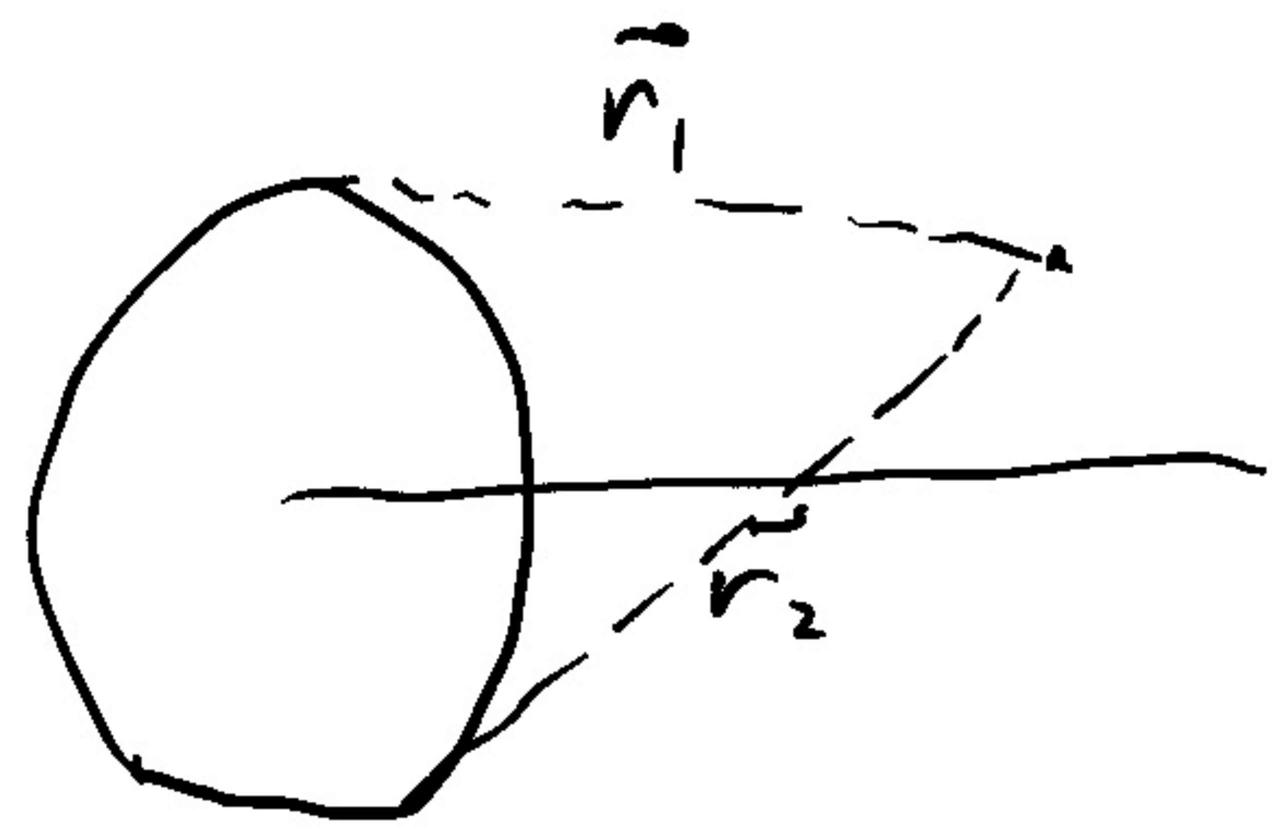
$$= -\partial/\partial x \, U(x)\, \hat{x}$$

$$= -\left(-GMm \frac{-1/2}{(R^2+x^2)^{3/2}} \cdot 2x\right)\hat{x}$$

$$= \boxed{\frac{-GMmx\,\hat{x}}{(R^2+x^2)^{3/2}}}$$

Much easier!

- Note if you were off the axis it wouldn't be so easy

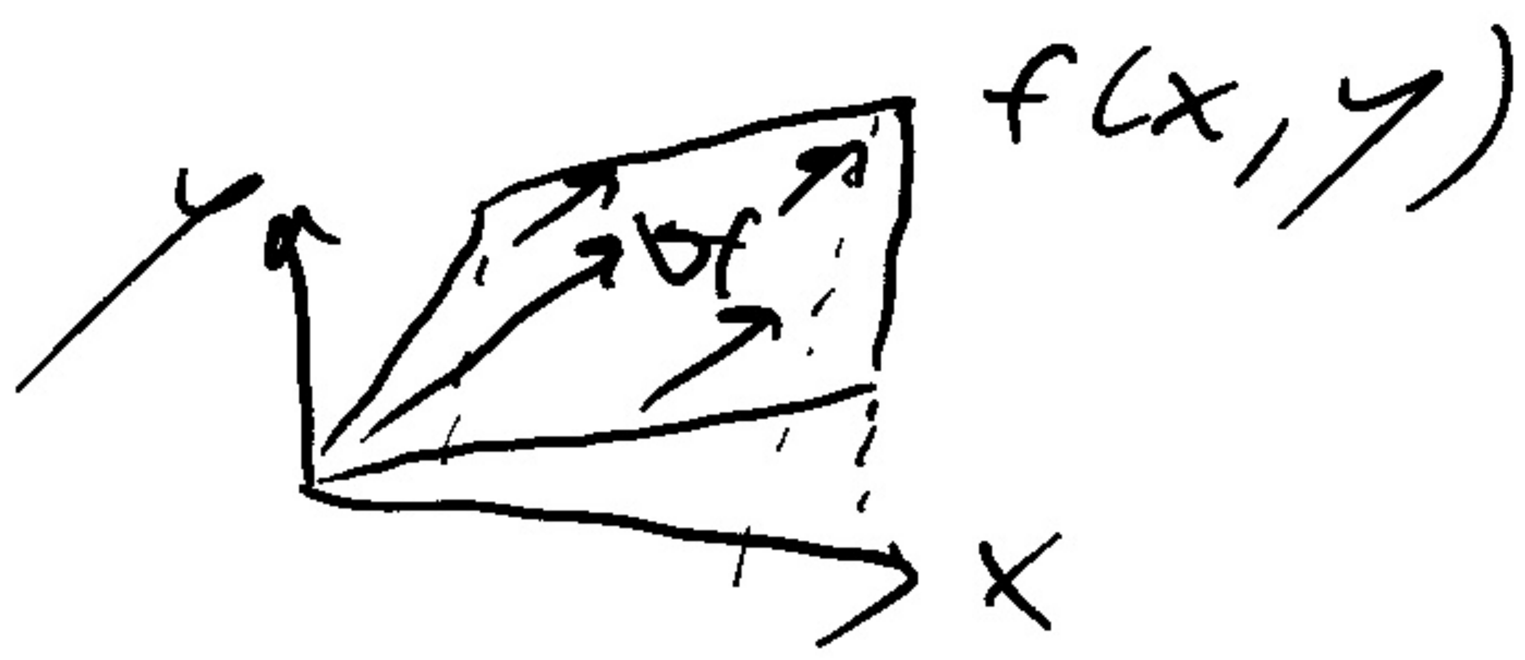


$|\vec{r}_1| \neq |\vec{r}_2|$
so $dU(\vec{r}_1) \neq dU(\vec{r}_2)$

More on gradient.

Have seen gradient in 1-d. What about a true multi-dimensional function?

Say $f(\vec{r}) = x + y$
(flat tilted plane)

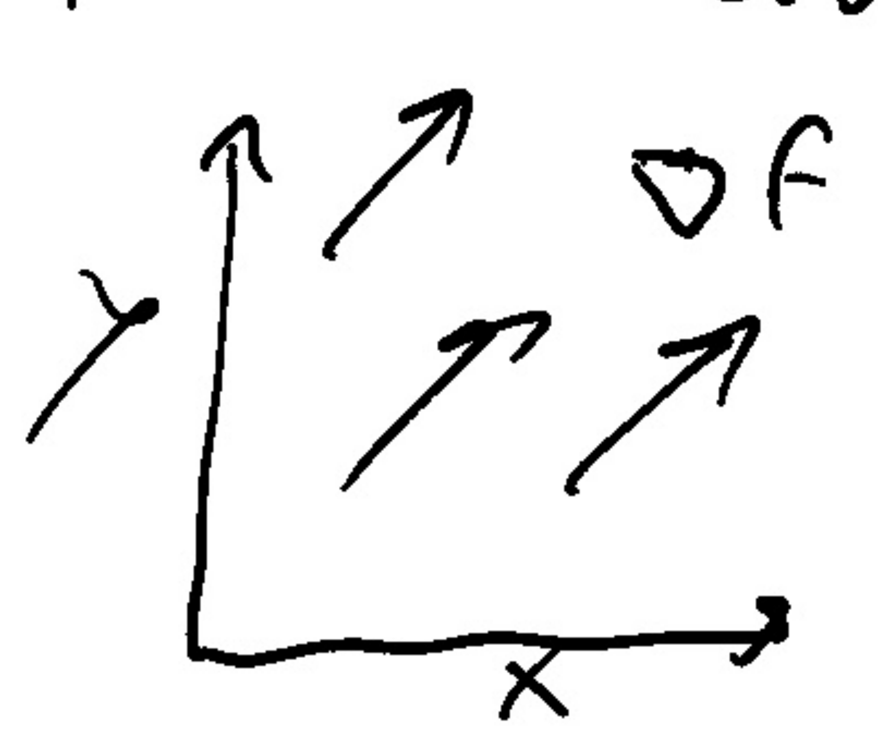


$$\nabla f = \partial f/\partial x \, \hat{i} + \partial f/\partial y \, \hat{j}$$

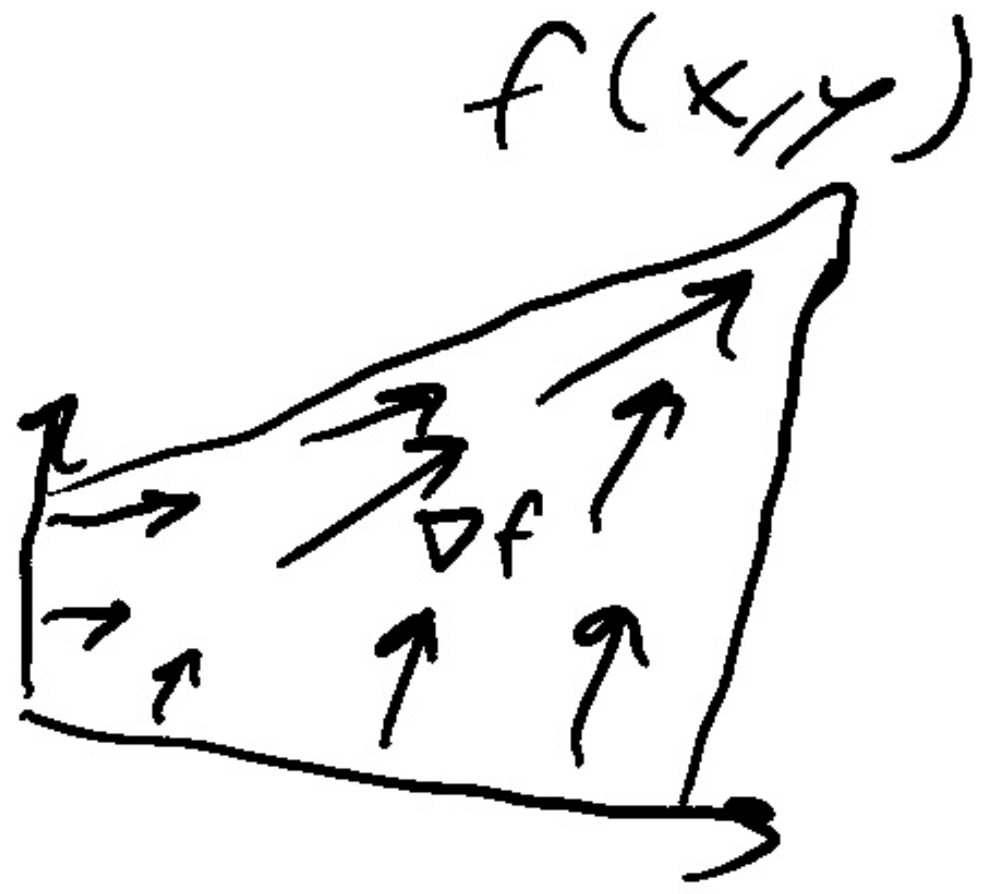
$$= \hat{i} + \hat{j}$$

- constant magnitude, points directly uphill (same direction everywhere)

View from above



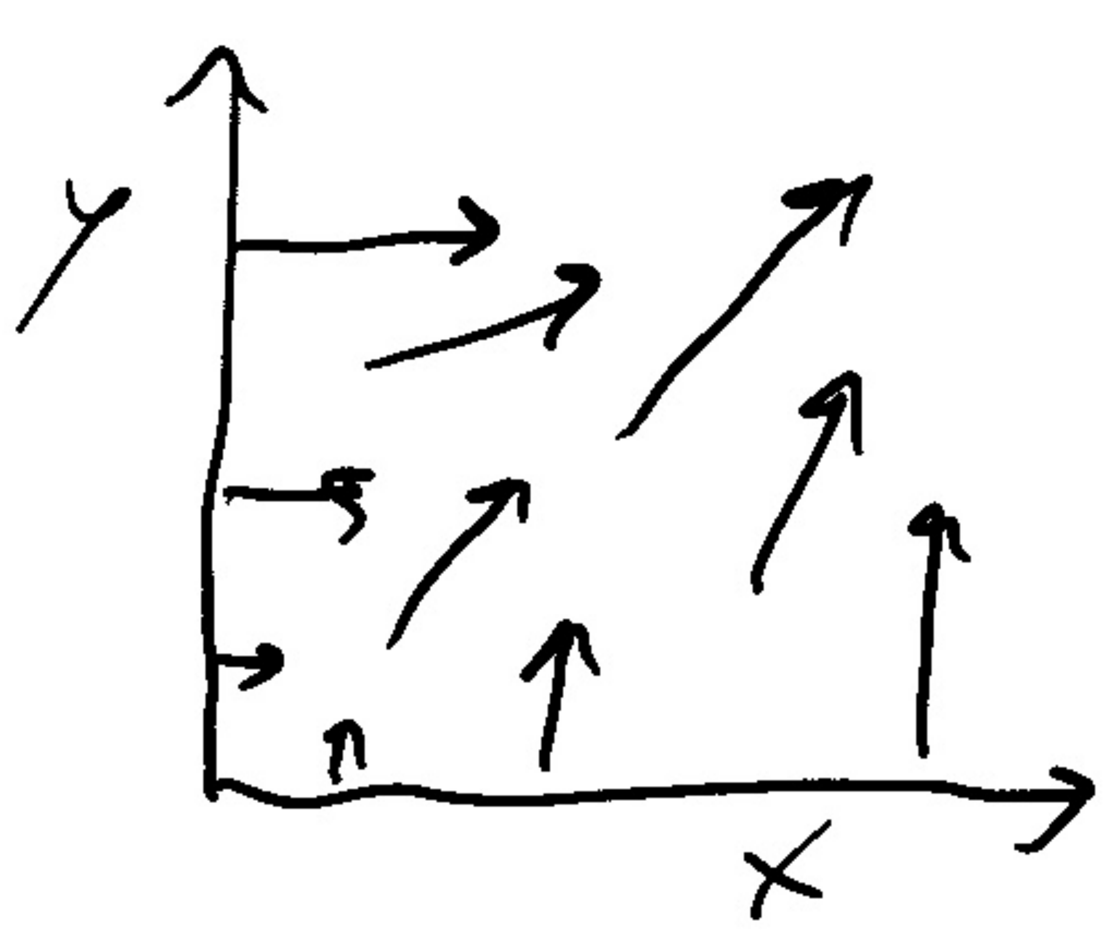
Say $f(\vec{r}) = xy$



pyramidal shape

$\nabla f = y\hat{i} + x\hat{j}$

View from above



∇f uphill towards crest of pyramid

- Note this function gets really interesting outside of $[x>0, y>0]$ quadrant