HW1: For each Gaussian surface:

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$$

$$= E \cdot 4\pi r^2$$

$$\Rightarrow E = q_{enc}/4\pi\varepsilon_0 r^2$$

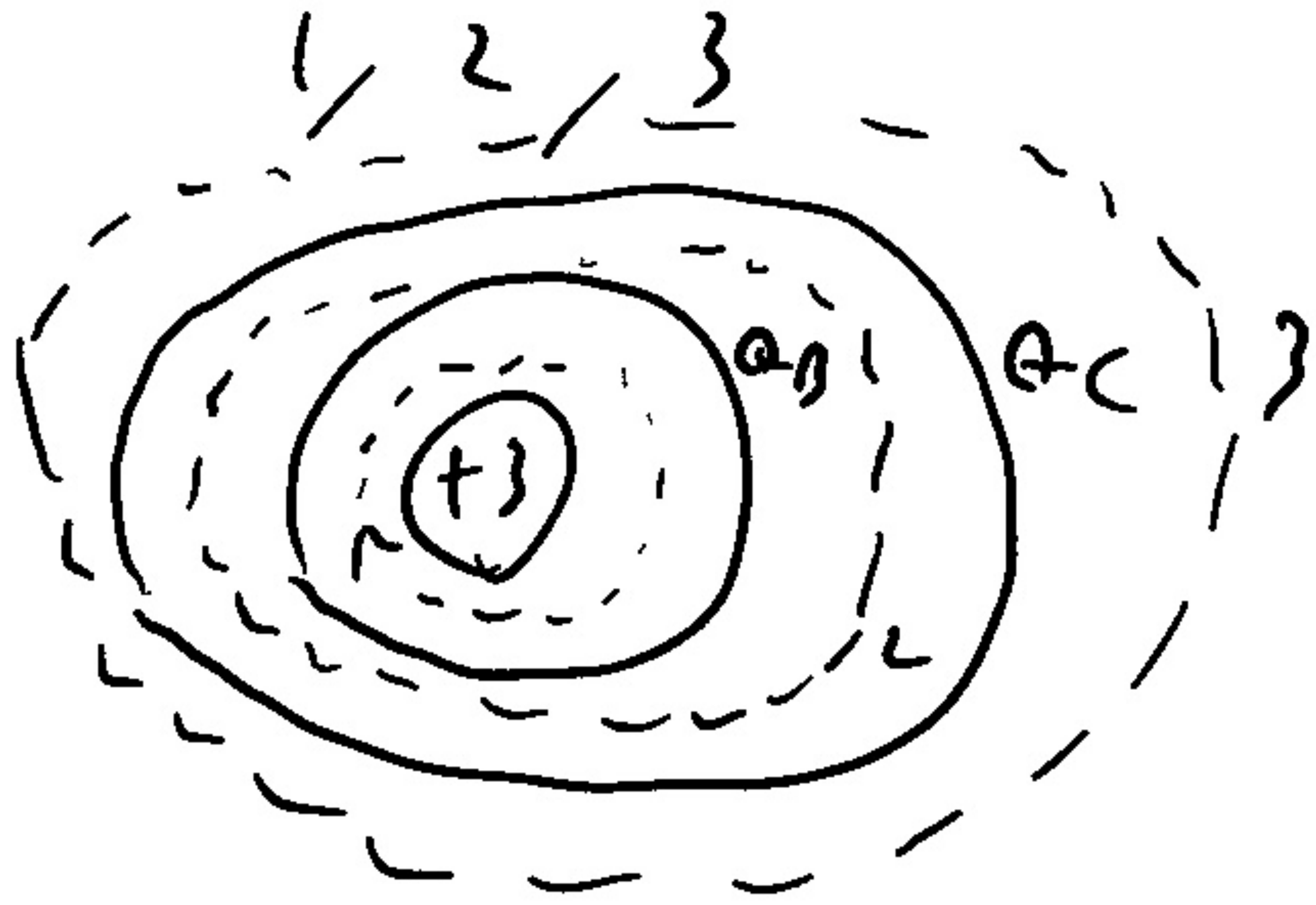
Inner: $E = Q/4\pi\varepsilon_0 R^2$

Middle: $E = (Q+3Q)/4\pi\varepsilon_0 (2R)^2 = Q/4\pi\varepsilon_0 R^2$

Outer: $E = (Q+3Q+5Q)/4\pi\varepsilon_0 (3R)^2 = Q/4\pi\varepsilon_0 R^2$

$\boxed{\text{All Equal}}$

HW2: Look @ cylindrical Gaussian surfaces w/ radii of points




$q_{enc-1} = +3q$

$q_{enc-2} = 3q + Q_B$

$q_{enc-3} = 3q + Q_B + Q_C$

- Can't cancel field @ point 1
- Put $Q_B = -3q$ to cancel field at points 2 and 3

HW3: $\rho = Q/V = Q/\tfrac{4}{3}\pi R^3$

$\rho_a > \rho_b > \rho_c > \rho_d$

$E = \frac{q_{enc}}{4\pi\varepsilon_0 r^2}$ from Gauss's Law

$E_a = E_b > E_c > E_d$

HW4: Say $\Delta x$ from top to bottom

$|\vec{E}| = \Delta V / \Delta x$

$E_1 = 80/\Delta x$    so    1

$E_2 = 40/\Delta x$    2

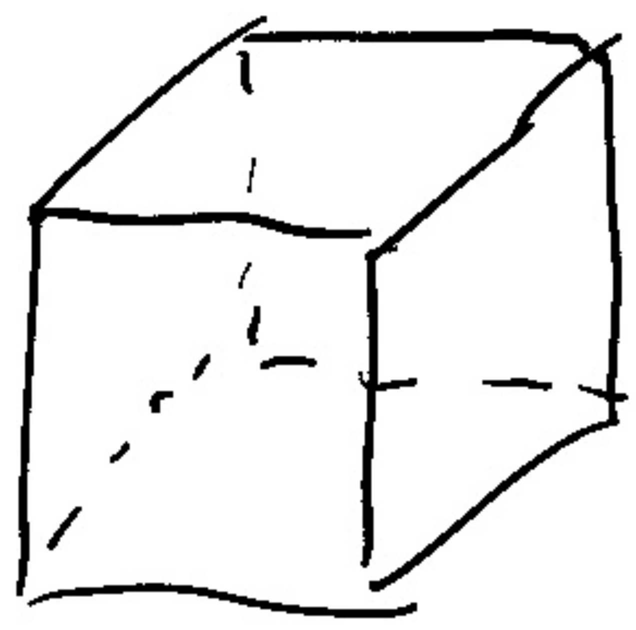
$E_3 = 40/\Delta x$    2

E from high to low so in case 3 E is $\downarrow$

HW5: steepest change in V corresponds to largest $|\vec{E}|$

so $2 > 4 > 1 = 3 = 5$

HW6:



$$\oint \vec{E}\cdot d\vec{A} = {+e}/{\varepsilon_0}$$

so $\boxed{\Phi_{square} = {+e}/{6\varepsilon_0}}$

HW7: $q_{enc} = 0$,

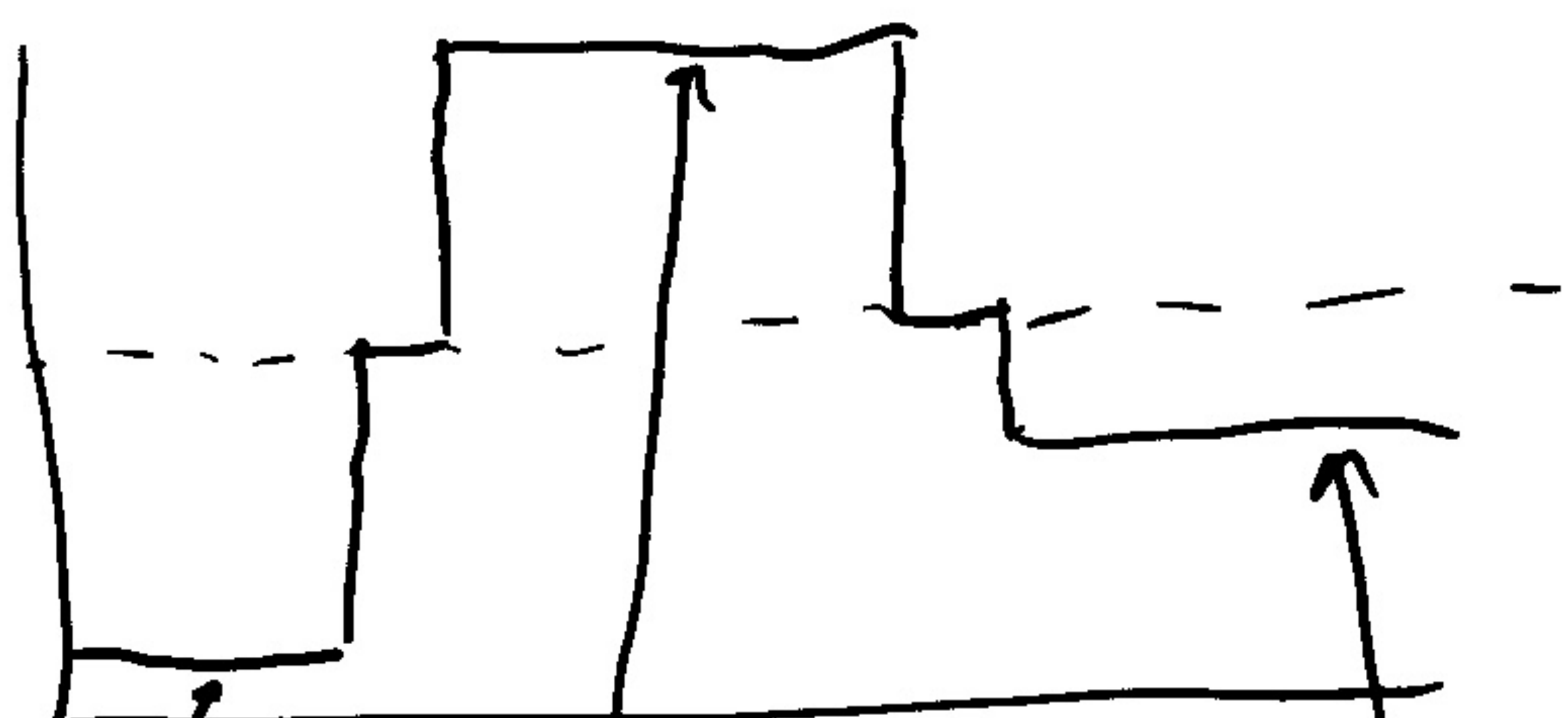
so $\oint \vec{E}\cdot d\vec{A} = 0$

$\int \vec{E}\cdot d\vec{A}$ through rim

$= EA = E\cdot 4\pi a^2$

so $\oint \vec{E}\cdot d\vec{A}$ through net $= \boxed{-E\cdot 4\pi a^2}$

HW8: $\Phi = \oint \vec{E}\cdot d\vec{A} = q_{enc}/\varepsilon_0$




HW9: a, b, c all 0 since $q_{enc}=0$

d. $q_{enc} = \rho\cdot\left(\frac{4}{3}\pi\cdot(1.5a)^3 - \frac{4}{3}\pi a^3\right)$

$E = q_{enc}/4\pi\varepsilon_0(1.5a)^2$

e. $q_{enc} = \rho\cdot\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)$

$E = q_{enc}/4\pi\varepsilon_0 b^2$

f. $q_{enc}$ same as e

$$E = \frac{q_{enc}}{4\pi\varepsilon_0 (3b)^2}$$

HW 10: a. $F = qE \Rightarrow E = F/q_e$

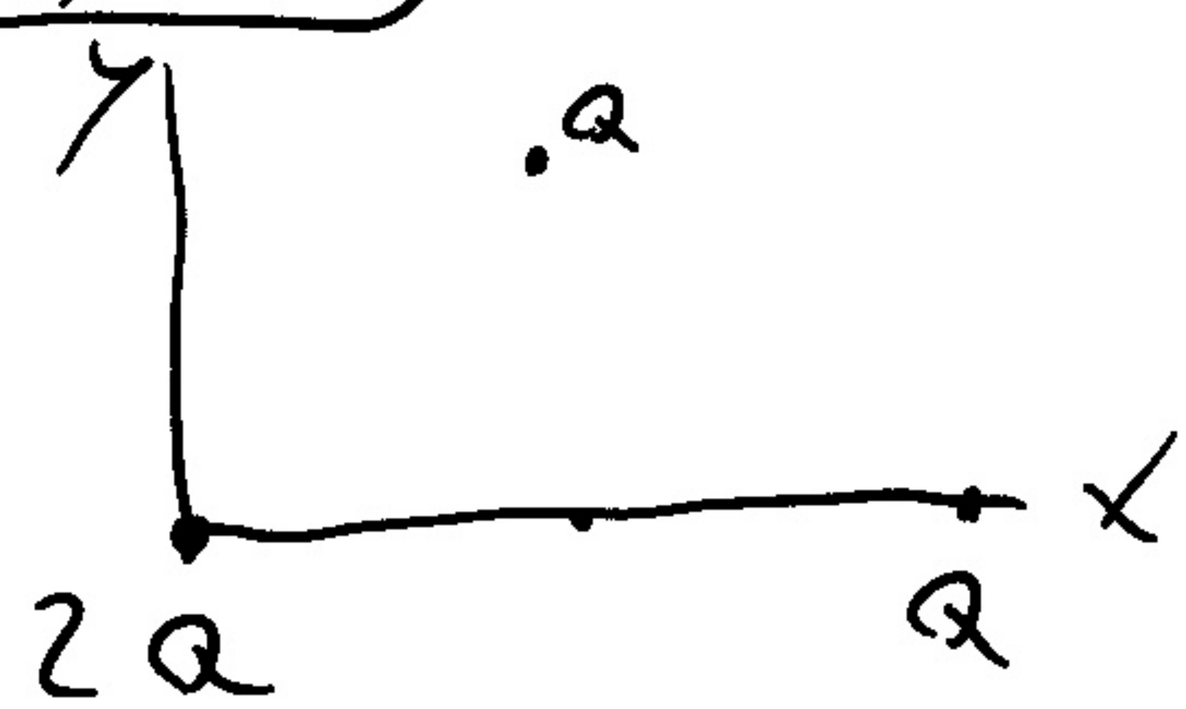
$$= F/e$$

b. $\Delta V = -\int \vec{E} \cdot d\vec{x}$

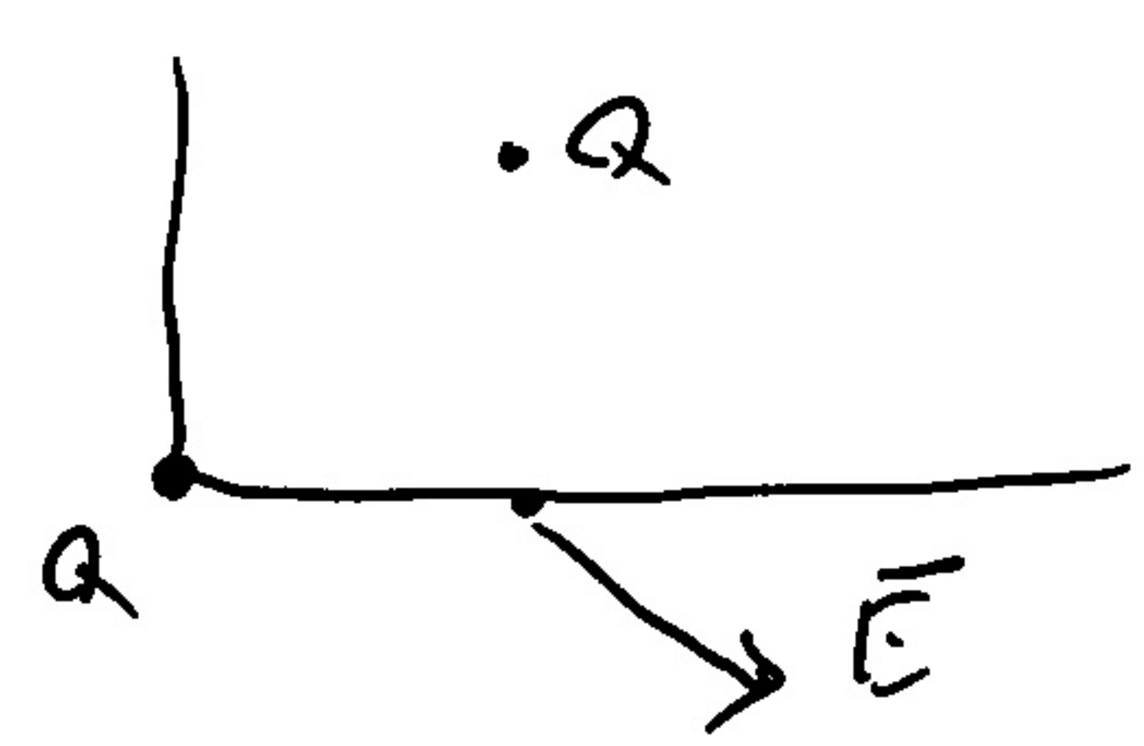
$$= -E\,\Delta x$$

# Sample Midterm Questions

$\boxed{\text{Charges}}$




Equivalent to




45° below x-axis

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 L^2}\hat{i} - \frac{Q}{4\pi\varepsilon_0 L^2}\hat{j}$$

$$|\vec{E}| = \frac{\sqrt{2}\,Q}{4\pi\varepsilon_0 L^2}$$

## Charged rods

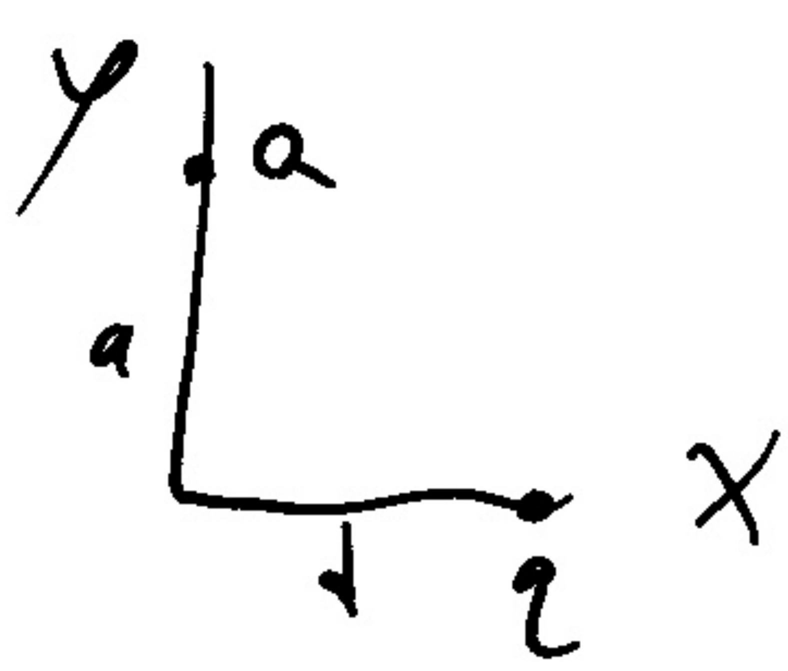
A. charge must be +

B. charge must be − or neutral

C. Insulator polarizes, dipoles align with field, rods attract because − closer than +

D. Charge flows to cancel $\vec{E}$ in conductor, rods attract because − closer than +

## Force between charges




$$F = kqQ/(a^2+d^2)$$

$$F_x = F\cos\theta$$
$$= F \cdot d/\sqrt{a^2+d^2}$$
$$= kqQd/(a^2+d^2)^{3/2}$$

$$dF/dd = \frac{kqQ}{(a^2+d^2)^{3/2}} - \frac{kqQd \cdot 3d}{(a^2+d^2)^{5/2}}$$

$$= \frac{kqQ((a^2+d^2) - 3d^2)}{(a^2+d^2)^{5/2}}$$

$0$ if $a^2+d^2-3d^2 = 0$

or $a^2 = 2d^2$ or $d = a/\sqrt{2}$

## $\boxed{\text{Torque on Dipole}}$

$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$|\vec{\tau}| = p E \sin\theta$$

$\tau_1 = 0$ 4

$\tau_2 \cong pE \cdot \sqrt{3}/2$ 2

$\tau_3 = pE$ 1

$\tau_4 \cong pE \cdot \sqrt{3}/2$ 2

## $\boxed{\text{Gaussian Cube}}$

$$\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0 \quad \text{symmetric}$$

so $\int_{\text{Face}} \vec{E} \cdot d\vec{A} = \boxed{q/6\varepsilon_0}$

## $\boxed{\text{Charged Balls}}$

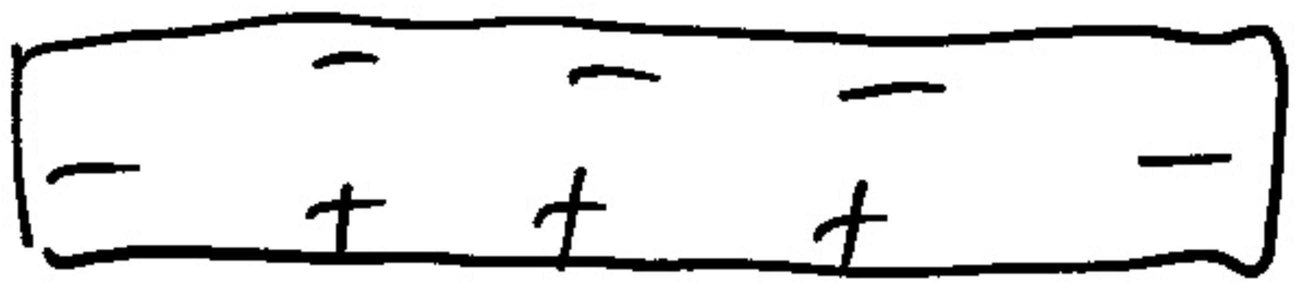
Force from rod smaller


- y components cancel
- total force less

## Charge & bar

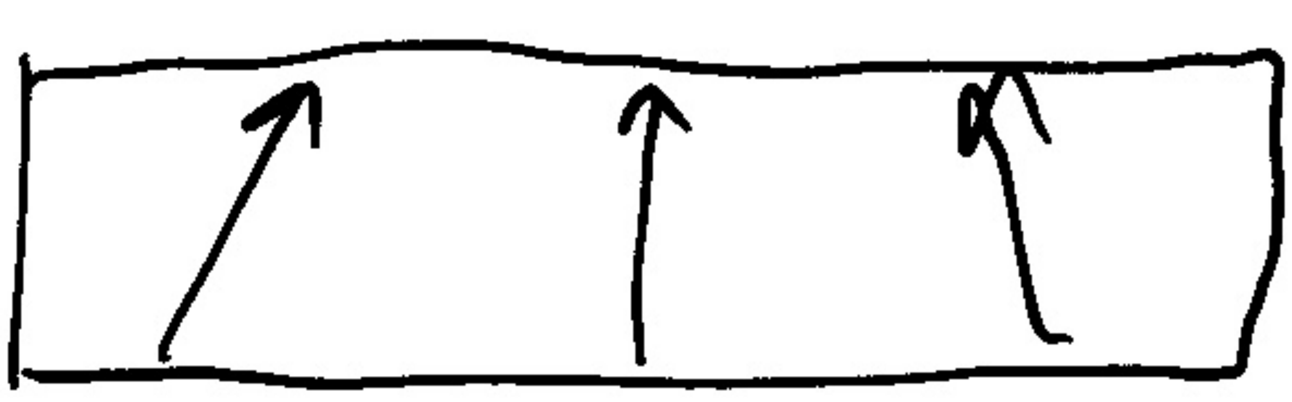
a. $|Q_{bar}| < Q$ since net flux (field lines out − field lines in) smaller

b.



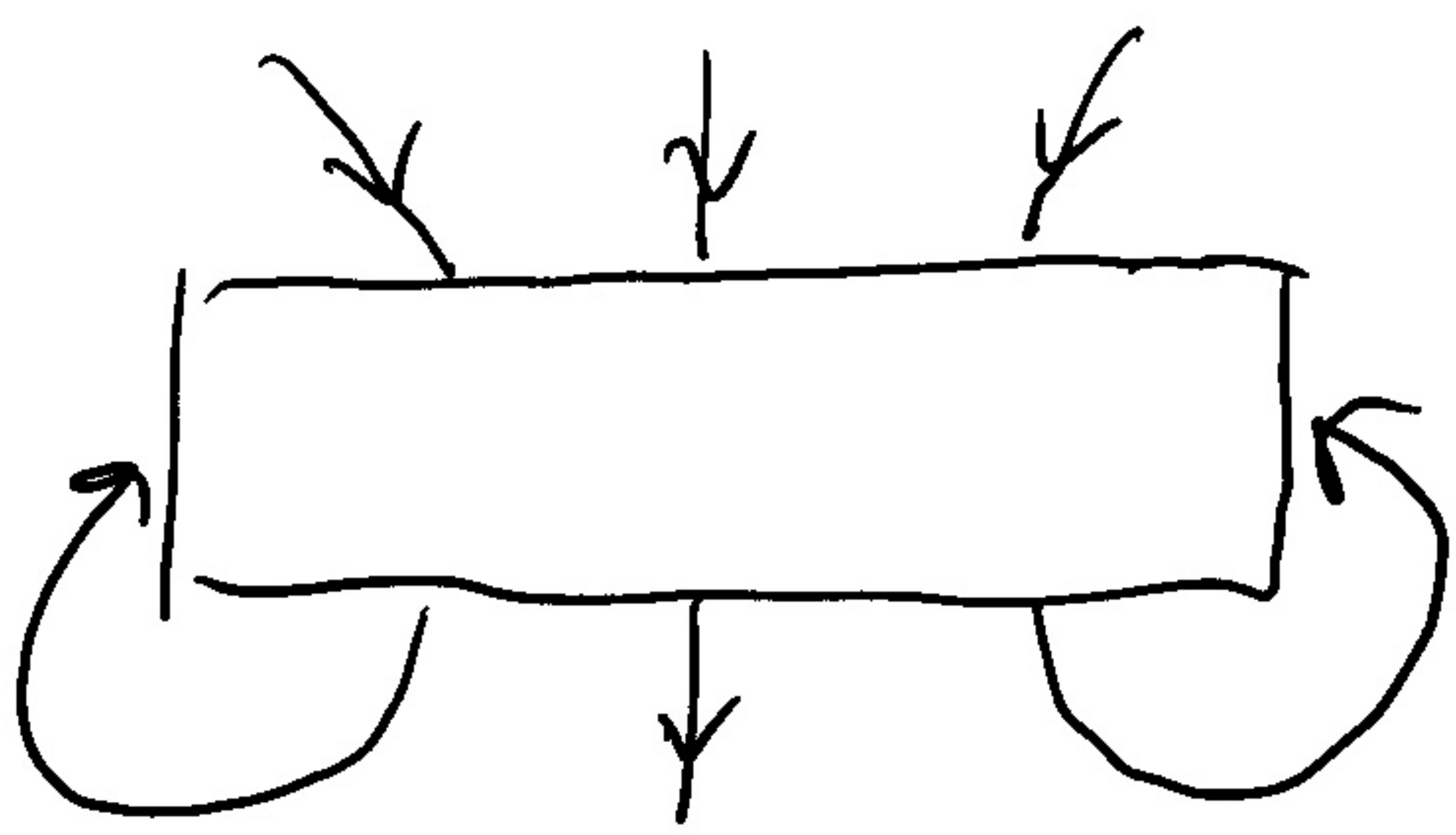
$\vec{E}$ in to −, out from +

c.

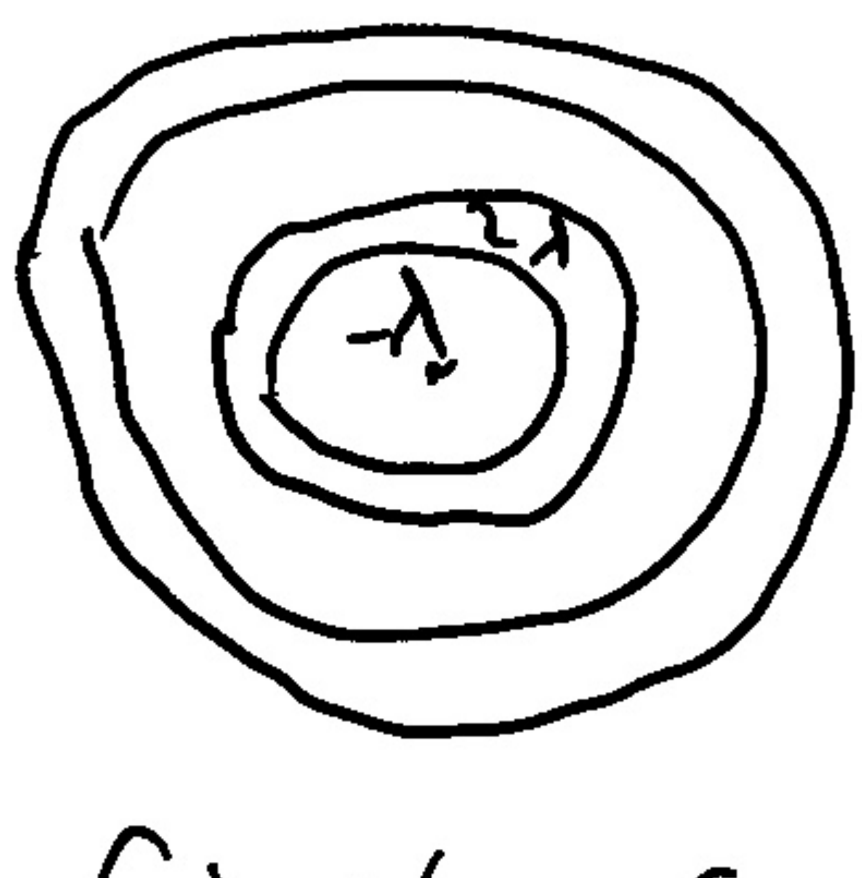


must cancel field of charge Q

d



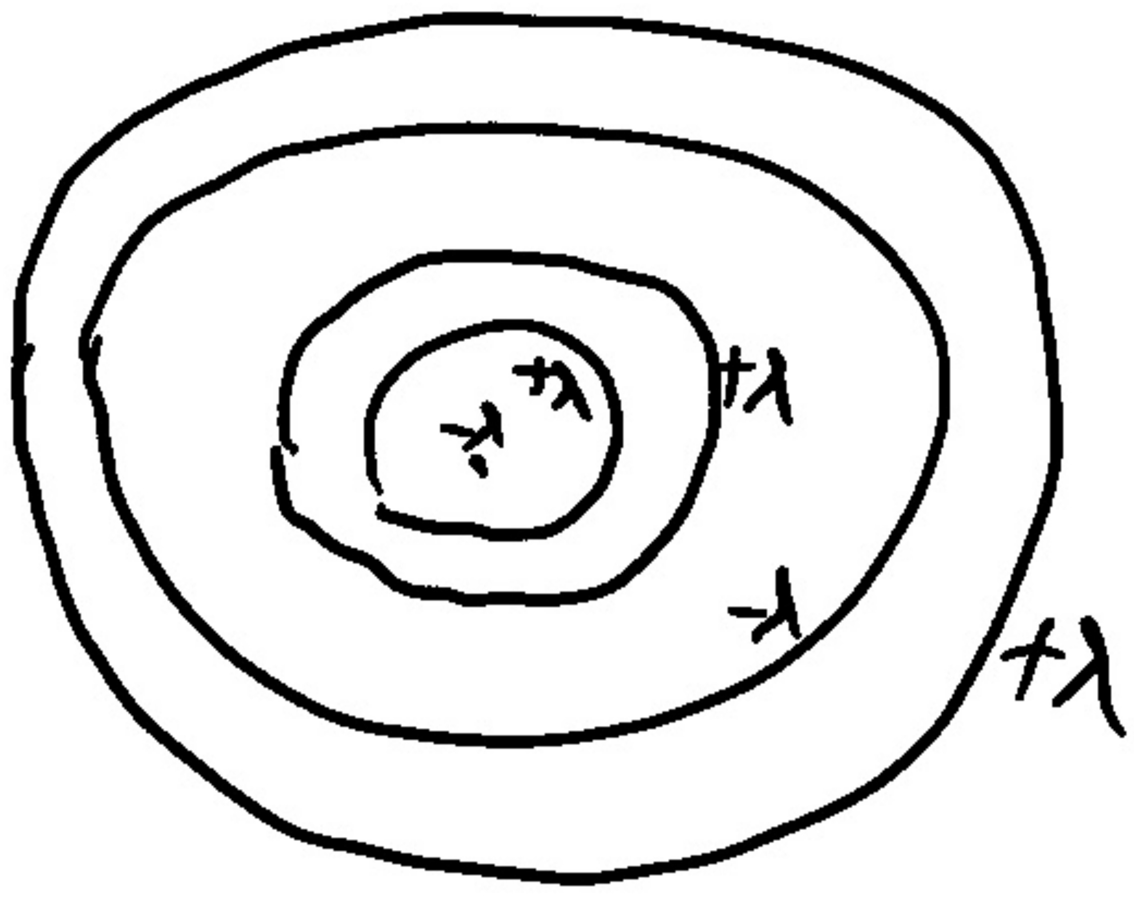
## Cylinders




- Field in first conductor must cancel, so $+\lambda$ on inner, leaves $+\lambda$ on outer.

- To cancel field in outer conductor, must have $-\lambda$ on inner, $+\lambda$ on outer

- so:




- use Gauss's law w/ cylinder
- Inside both:

$$\int \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$$

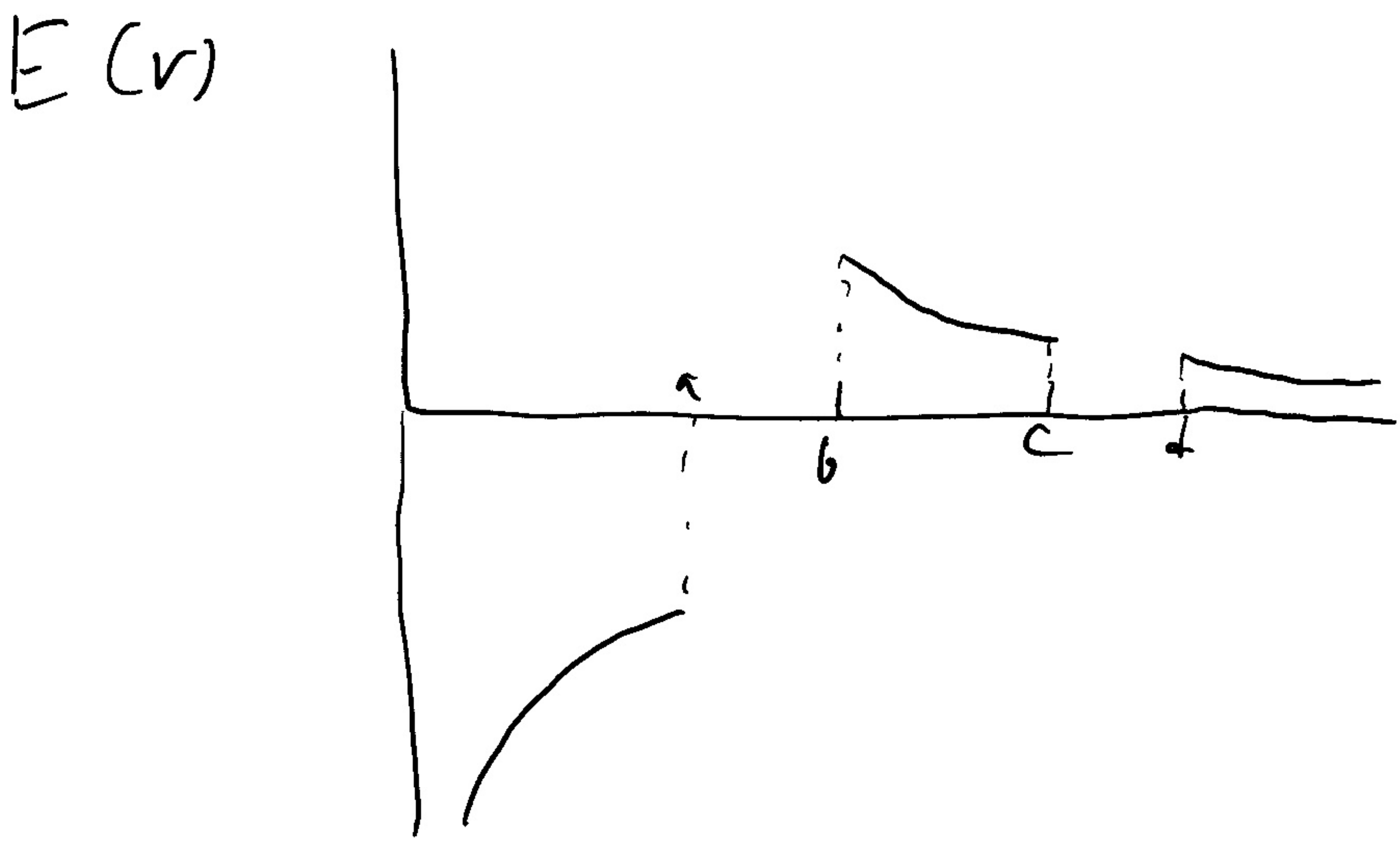
$$E \cdot 2\pi r L = -\lambda L/\varepsilon_0$$

$$E = -\lambda/2\pi r \varepsilon_0$$

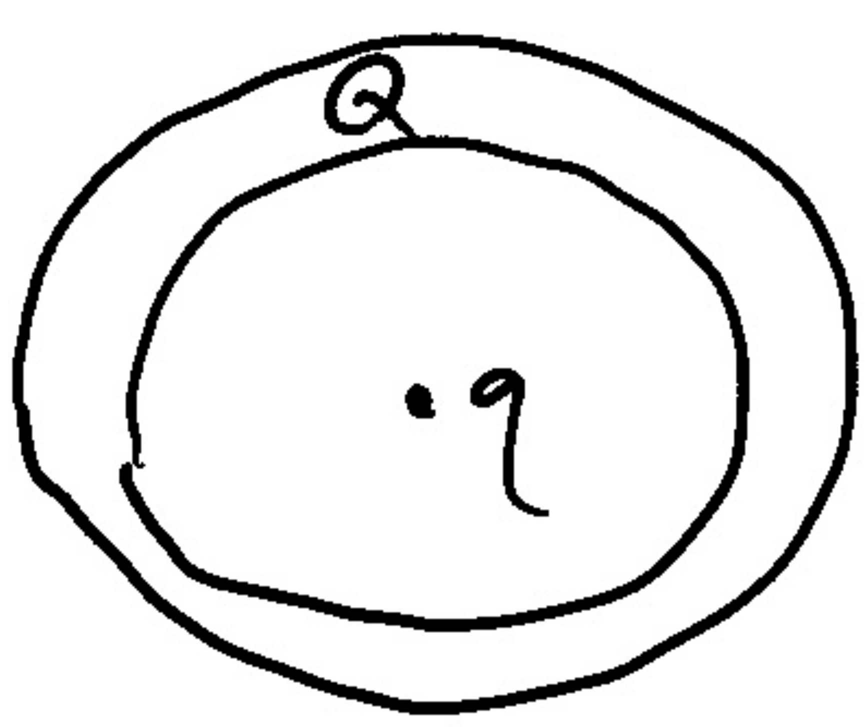
- In inner, $q_{enc} = 0$
  so $E = 0$

- Between conductors, $q_{enc} = \lambda L$
  so $E = +\lambda/2\pi r \varepsilon_0$

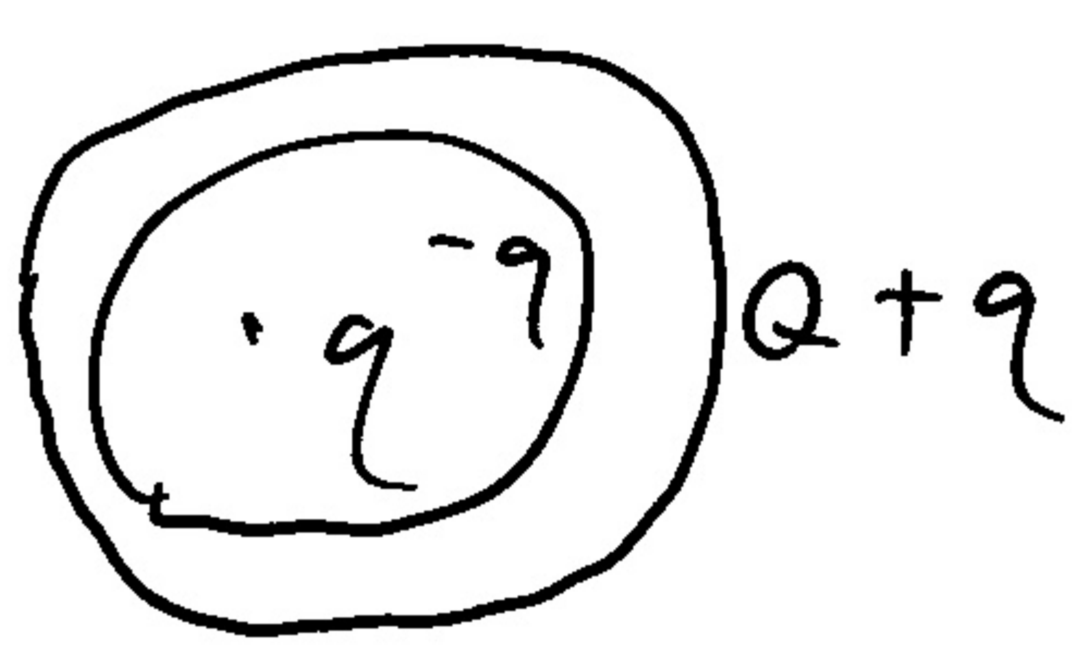
- In outer, $q_{enc} = 0$, so $E = 0$
- Outside both, $q_{enc} = \lambda L$
  so $E = \lambda/2\pi r \varepsilon_0$




## $\boxed{\text{Spherical shells}}$

a. 


- Must have $\vec{E} = 0$ for $R_1 < r < R_2$
- so $-q$ on inner
- leaves $Q+q$ on outer

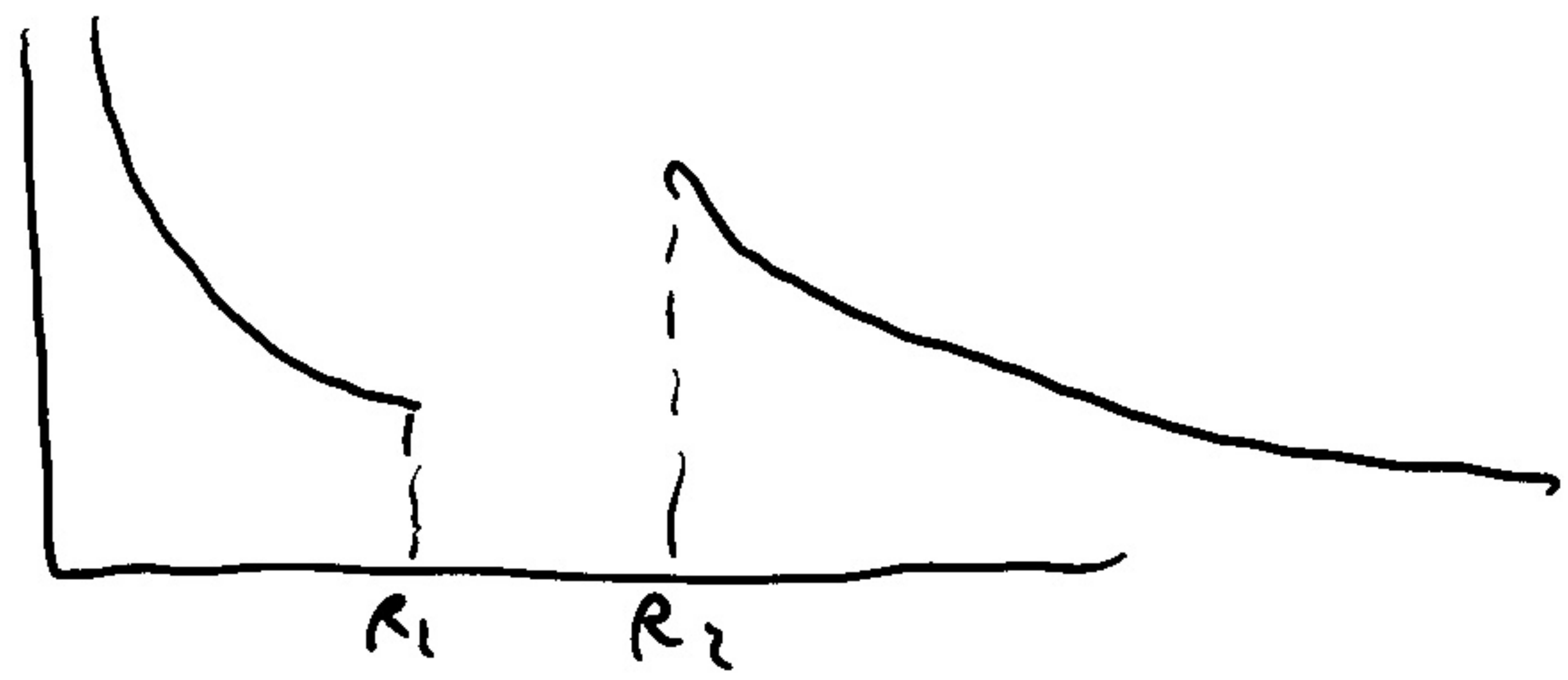



b. $E = q / 4\pi\varepsilon_0 r^2$

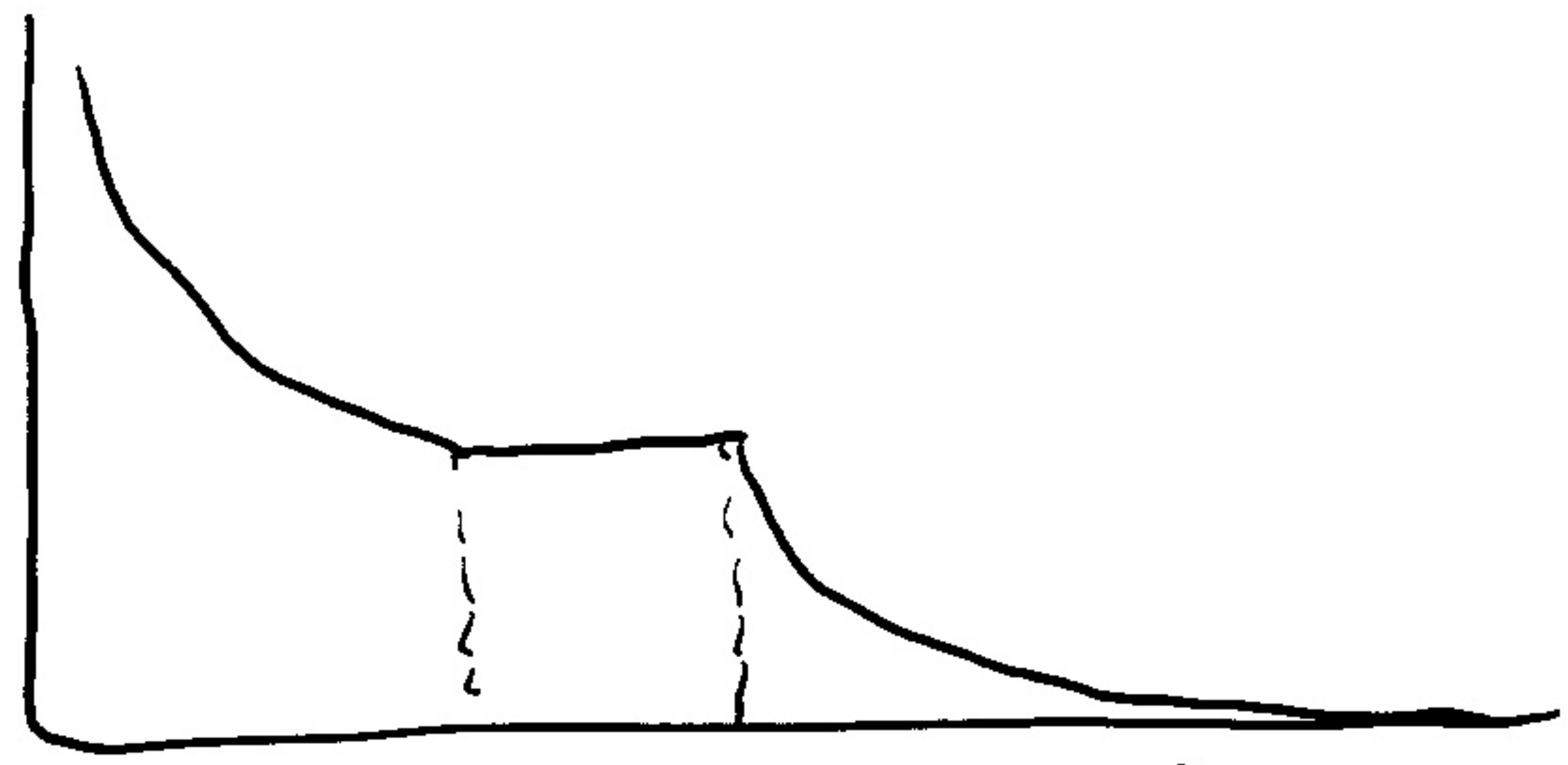
c. $E = (q+Q) / 4\pi\varepsilon_0 r^2$

d. $E = 0$

e. $E(r)$




f. $V(r)$



verify $E(r) = -\partial V / \partial r$

voltage in uniformly charged sphere

Find $E(r)$ from Gauss

$$E \cdot 4\pi r^2 = \rho \cdot \tfrac{4}{3}\pi r^3 \quad r < R$$

$$\Rightarrow \quad E = \frac{Qr}{4\pi\varepsilon_0 R^3} \quad r < R$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \quad r > R$$

Integrate from infinity:

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r} \quad r > R$$

$$V(r) = -\frac{Qr^2}{8\pi\varepsilon_0 R^3} + \text{const.} \quad r < R$$

$V(R)$ continuous so

$$V(r) = -\frac{Qr^2}{(8\pi\varepsilon_0 R^3)} + \frac{3Q}{8\pi\varepsilon_0 R}$$

$$= \frac{Q}{(8\pi\varepsilon_0 R)}\left[3 - \frac{r^2}{R^2}\right] \quad r < R$$