Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Gauss's Law: Point Form

Maxwell's Eqs:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$ Gauss

$$\nabla \cdot \mathbf{B} = 0$$ No Monopoles

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$ Faraday

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$ Ampere

Gauss's Law: Integral Form

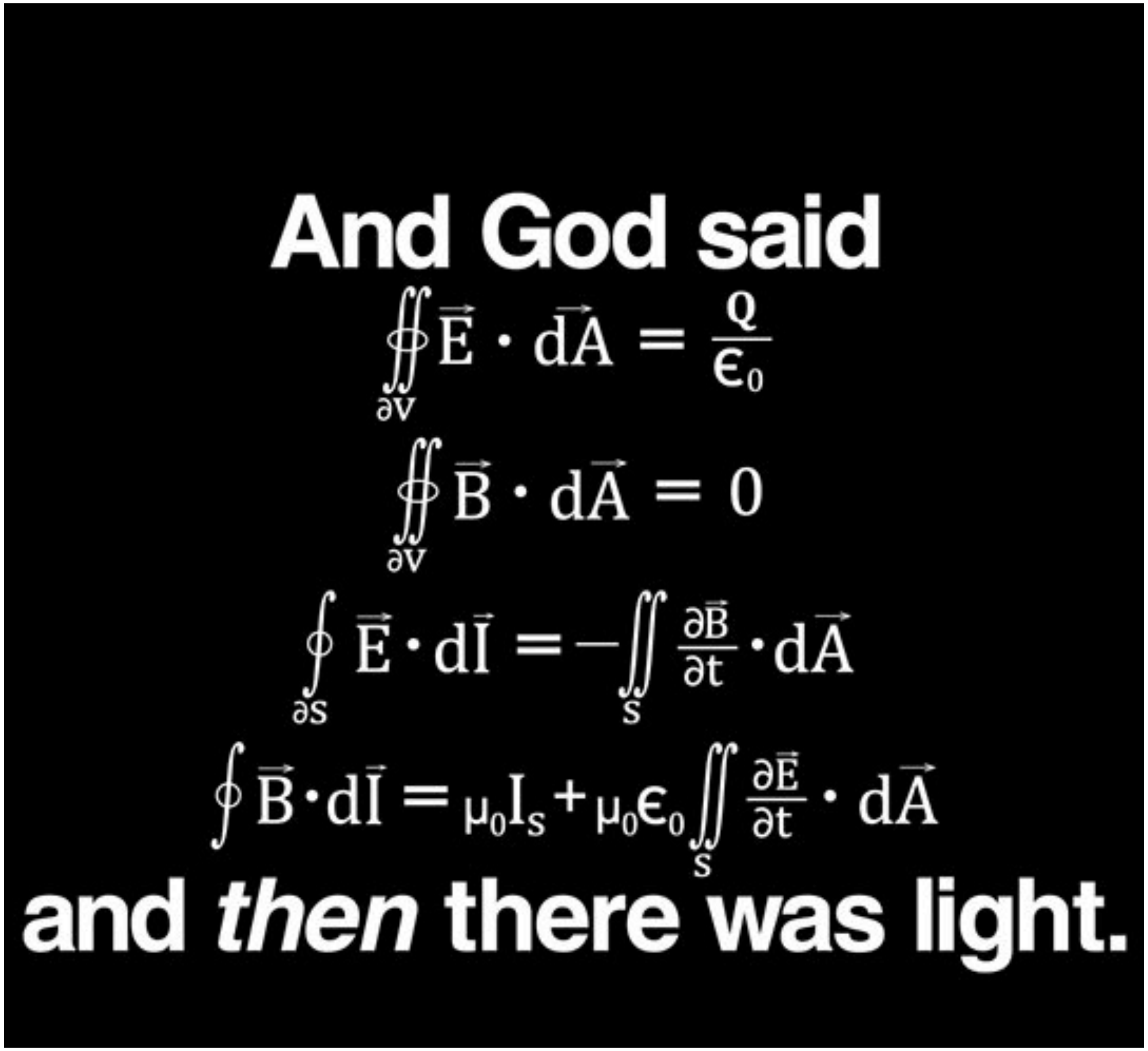
$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_{\mathrm{B}}}{dt}$$

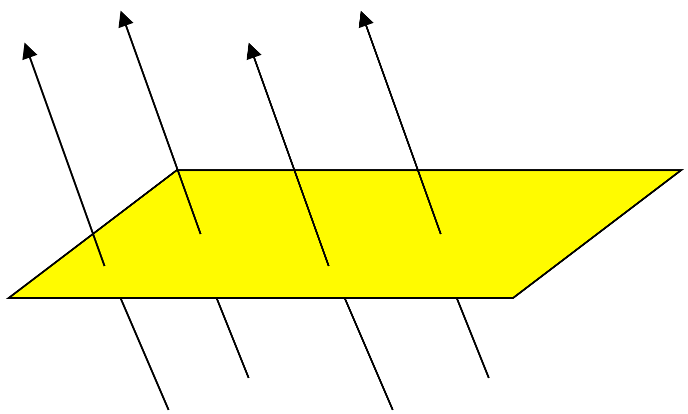
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_{\mathrm{E}}}{dt} + \mu_0 i_{enc}$$

Gauss's Law: Nerdy T-Shirt Form



Electric Flux

New concept: Electric Flux Φ_E through a surface



Surface with some area A has some electric flux
(E-field lines) through it.

Define **surface vector**

$$\vec{A} = A\hat{n}$$

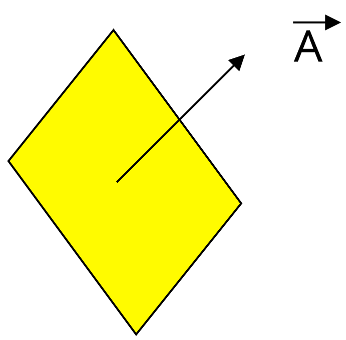
Surface Vector

Define **surface vector**

$$\vec{A} = A\hat{n}$$

$A = |\vec{A}|$ = magnitude of the area of the surface [m^2]

$\hat{n}$ = direction of $\vec{A}$ = direction perpendicular to the surface



* Note that there are two possible directions for $\vec{A}$

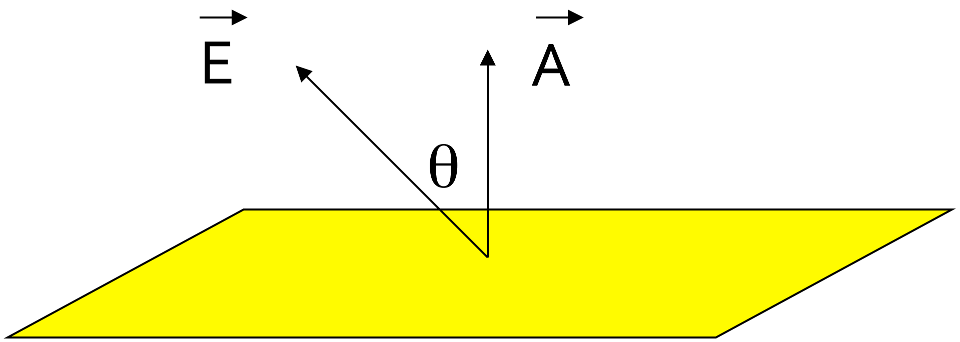
Electric Flux

$$\Phi_E = \vec{E} \bullet \vec{A}$$

(Vector Dot Product)

$$\Phi_E = \vec{E} \bullet \vec{A} = |\vec{E}||\vec{A}|\cos\theta$$

* In special case of a single flat surface.

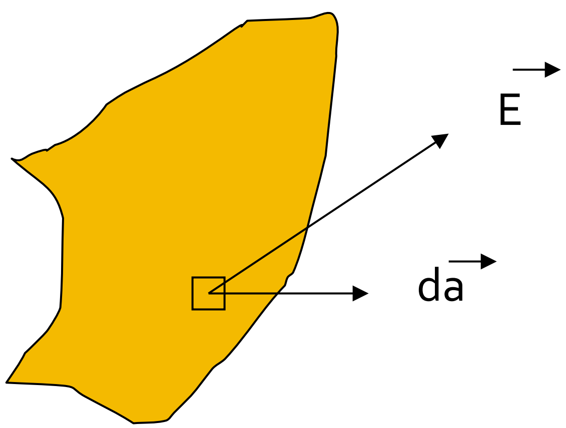


Integral Form

One more complication…

What if our surface is not flat?

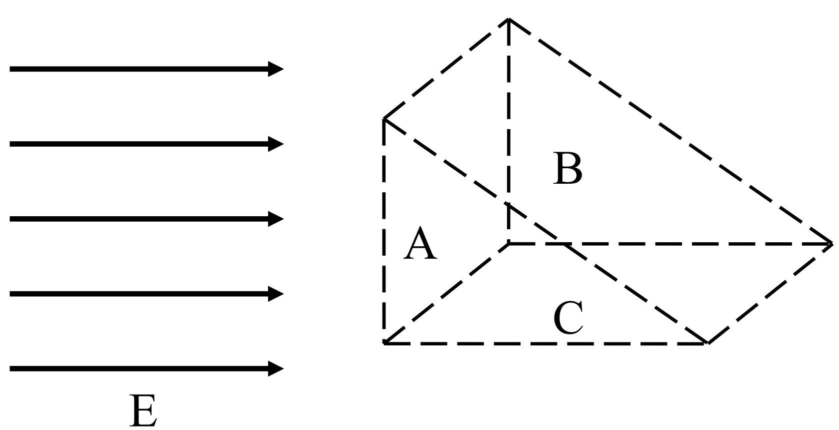
$$\Phi_E = \int \vec{E} \bullet d\vec{a}$$



Break up surface into many tiny segments of area da, which must be flat in the infinitesimal limit.
<u>Integral is over the surface.</u>

Concept Check

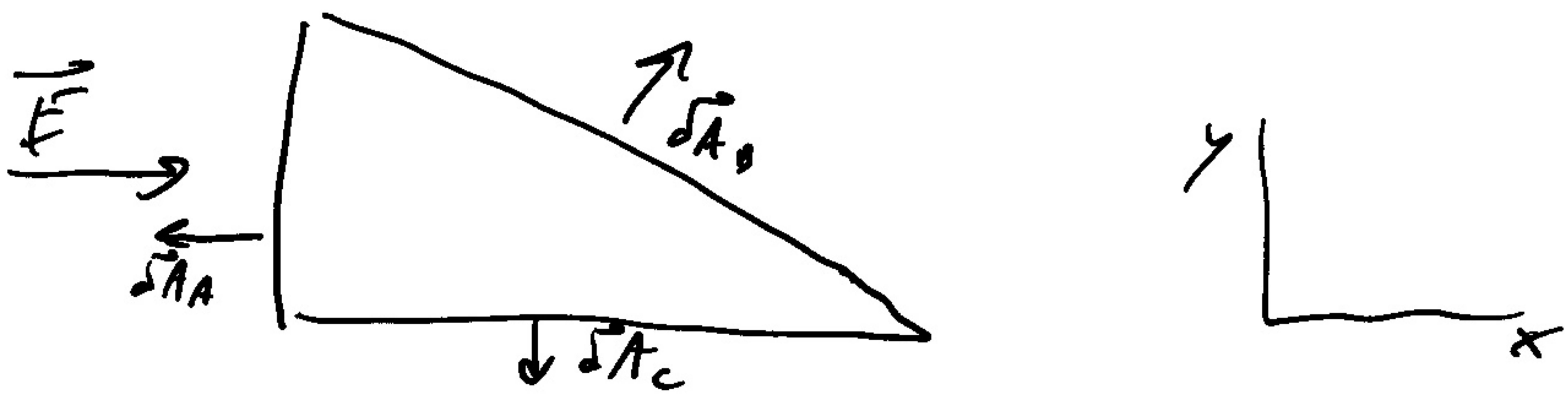
A prism-shaped closed surface is in a constant, uniform electric field **E**, filling all space, pointing right. The 3 rectangular faces of the prism are labeled A, B, and C. Face A is perpendicular to the E-field. The bottom face C is parallel to **E**. Face B is the leaning face.



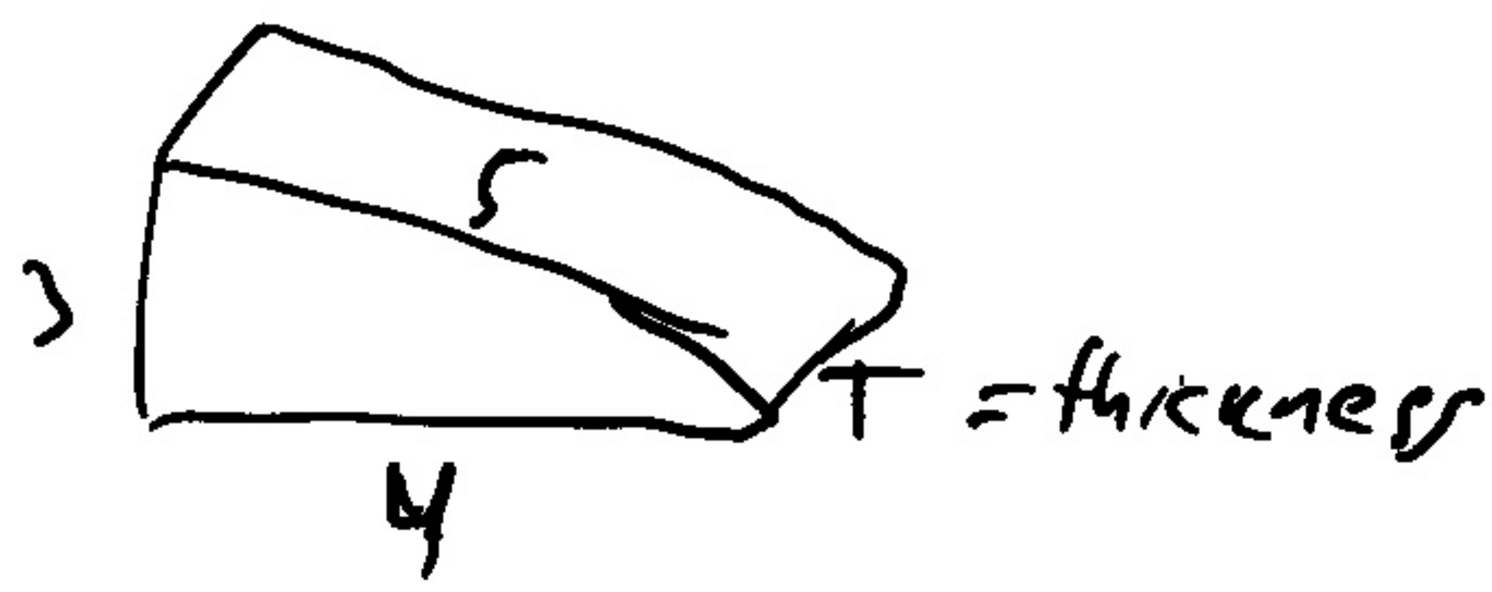
Which face has the largest magnitude electric flux through it?

A) A B) B C) C

D) A and B have the same magnitude flux



- Say we have a 3-4-5 triangle



$$\vec{E} = E\hat{i}$$

$$d\vec{A}_A = dA\,(-\hat{i})$$
$$d\vec{A}_B = dA\,(3\hat{i} + 4\hat{j})/5$$
$$d\vec{A}_C = dA\,(-\hat{j})$$

$$\int_A \vec{E}\cdot d\vec{A}_A = \int_A E\,dA\,(\hat{i}\cdot -\hat{i})$$
$$= -\int_A E\,dA$$
$$= -E A_A$$
$$= -E\cdot 3T = -3ET$$

$$\int_B \vec{E}\cdot d\vec{A}_B = \int_B E\,dA\,(\hat{i}\cdot(3\hat{i}+4\hat{j})/5)$$
$$= \int_B E\,dA \cdot 3/5$$
$$= 3/5\, E A_B$$
$$= 3/5\, E\cdot 5T$$
$$= 3ET$$

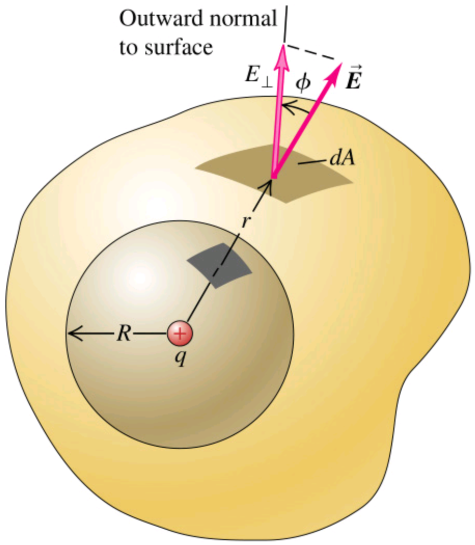
$$\int_C \vec{E}\cdot d\vec{A}_C = \int_C E\,dA\,(\hat{i}\cdot -\hat{j}) = 0$$

Flux through sides also zero, so $\oint \vec{E}\cdot d\vec{A} = 0$ as expected

Gauss's Law

Gauss's Law

$$\Phi_E = \oint \vec{E} \bullet d\vec{a} = \frac{Q_{inside}}{\varepsilon_0}$$



Circle = integral over a closed surface!
For a closed surface, $\vec{da}$ is always outward.

The electric flux thru any **closed surface** S is a constant ($1/\varepsilon_0$) times the **net charge enclosed** by S.

Concept Check

Three closed surfaces enclose a point charge. The three surfaces are a small cube, a small sphere, and a larger sphere – all centered on the charge.

Which surface has the largest flux through it?

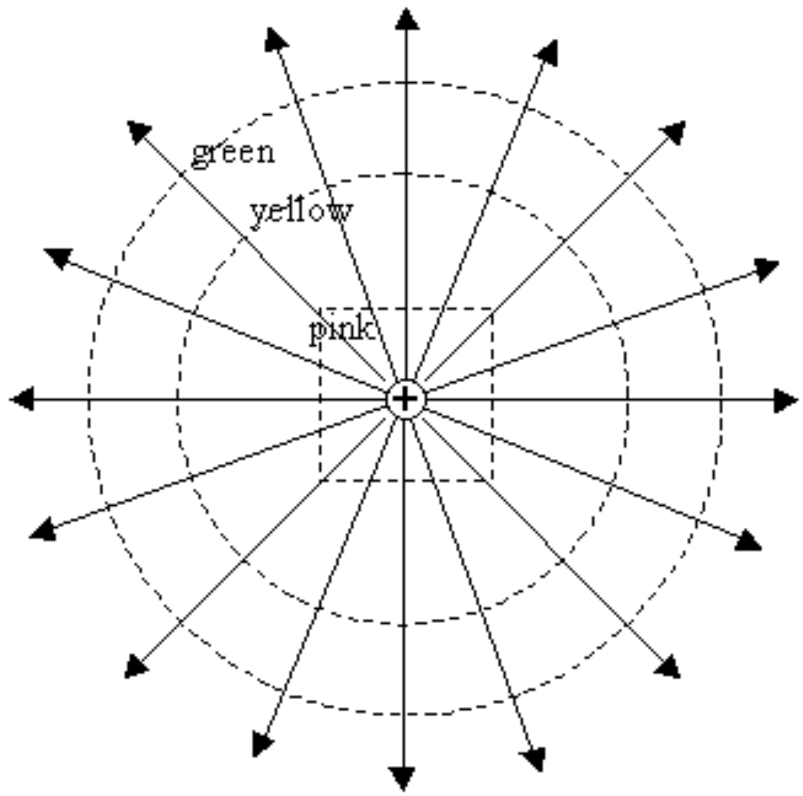
A) Small cube

B) smaller sphere

C) larger sphere

D) Impossible to tell without more information

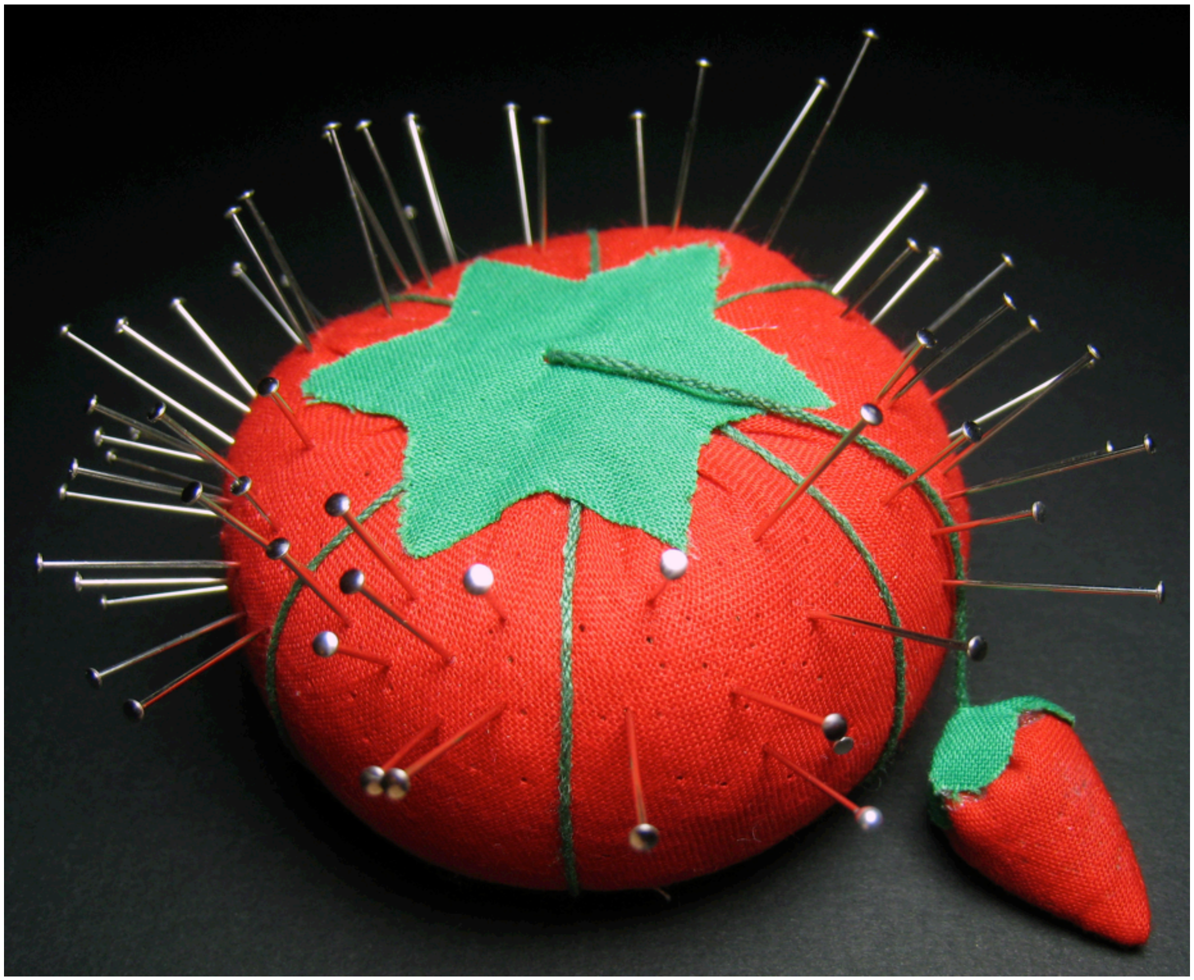
E) All three have the same flux.



Gauss's Law Conceptually

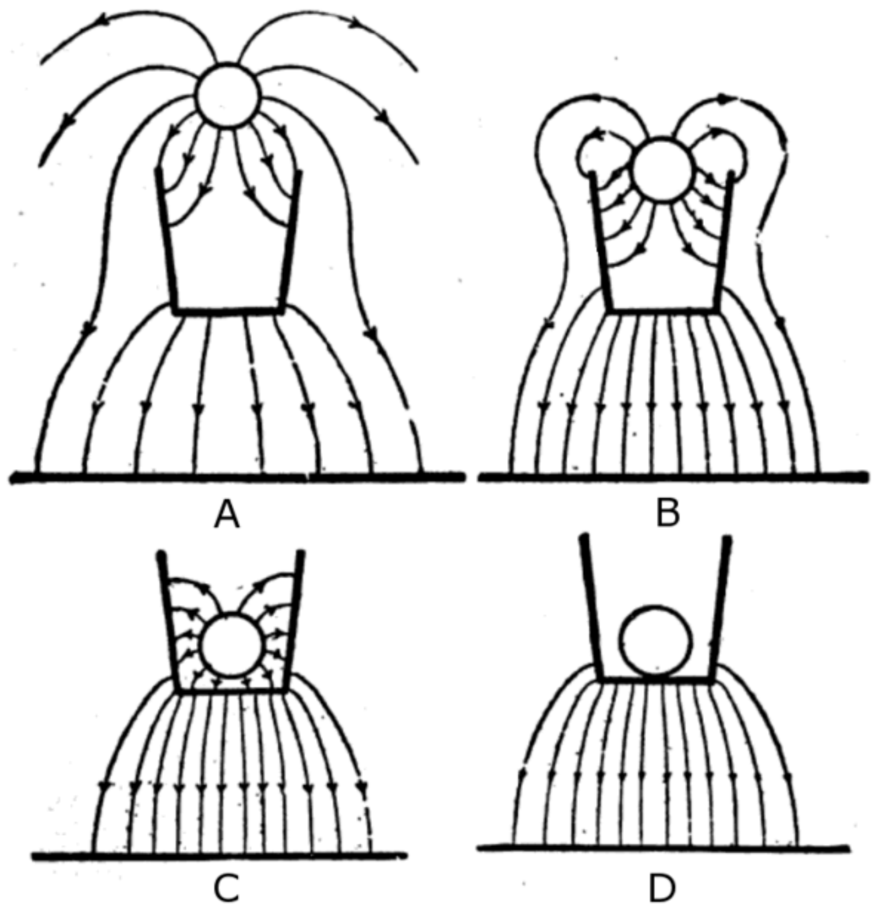
- Gauss's law is really about counting up how many field lines go through your surface
  - Remember field lines only start and end at charges, so the number of field lines depends on the number (and sign) of charges inside a volume
  - How do you define how many field lines there are coming out from a given charge?
    - This is really a convention based upon experiment, and depends upon what units we use

Gauss's Law: Pincushion Model

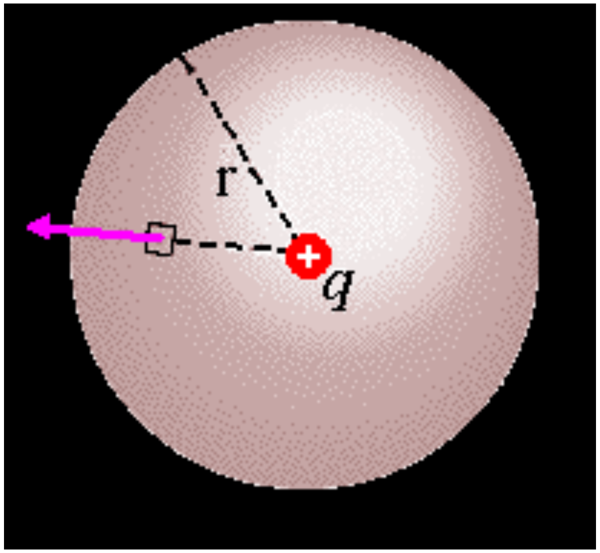


Gauss's Law: Hellraiser Model

Ice Bucket Explanation: V1



Gauss Vs. Coulomb

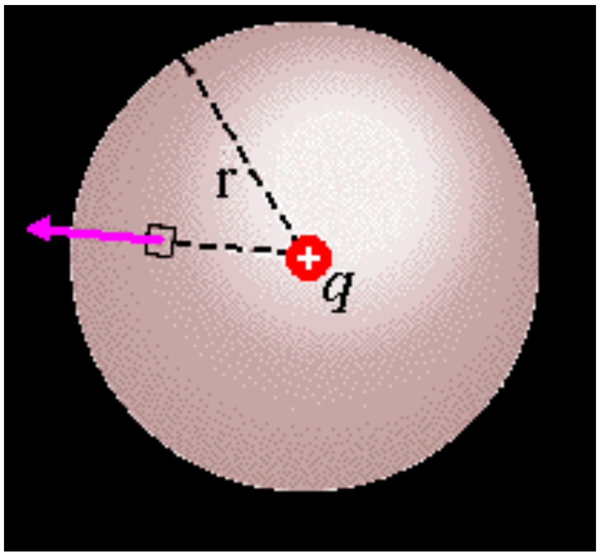


Gauss's Law

$$\oint_{surf} \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\varepsilon_0}$$

1. $Q_{inside} = +q$
2. $\vec{E}$ is always radially outward.
3. $\vec{da}$ is always radially outward.
4. Thus $\vec{E}.\vec{da} = |\vec{E}||\vec{da}|$ everywhere!

Gauss Vs. Coulomb



$$\oint_{surf} \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\varepsilon_0}$$

$$\oint_{surf} |\vec{E}||d\vec{a}| = \frac{+q}{\varepsilon_0}$$

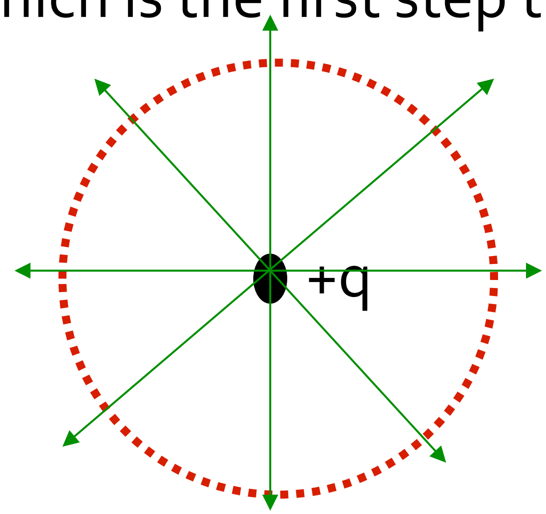
$$|\vec{E}| \oint_{surf} |d\vec{a}| = \frac{+q}{\varepsilon_0}$$

$$|\vec{E}|(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\boxed{|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

Concept Check

Consider a spherical Gaussian surface with a source charge +q at the center.
If we move the charge slightly to the right, which is the first step that is wrong?



A) $\oint_{surf} \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\varepsilon_0}$

B) $\oint_{surf} |\vec{E}|| d\vec{a}| = \frac{+q}{\varepsilon_0}$

C) $|\vec{E}| \oint_{surf} | d\vec{a}| = \frac{+q}{\varepsilon_0}$

D) $|\vec{E}|(4\pi r^2) = \frac{q}{\varepsilon_0}$

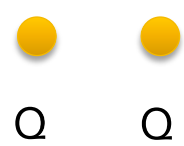
E) All correct.

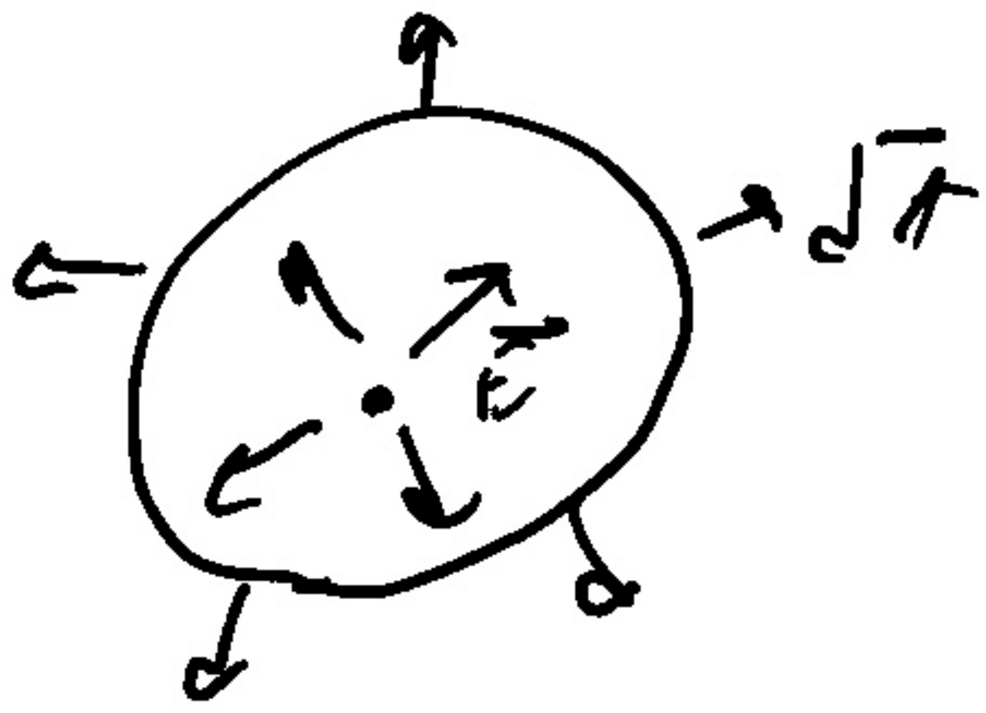
Gauss's Law

- Easier than Coulomb's law, but not always
  - Only easy if you have symmetry!

Easy

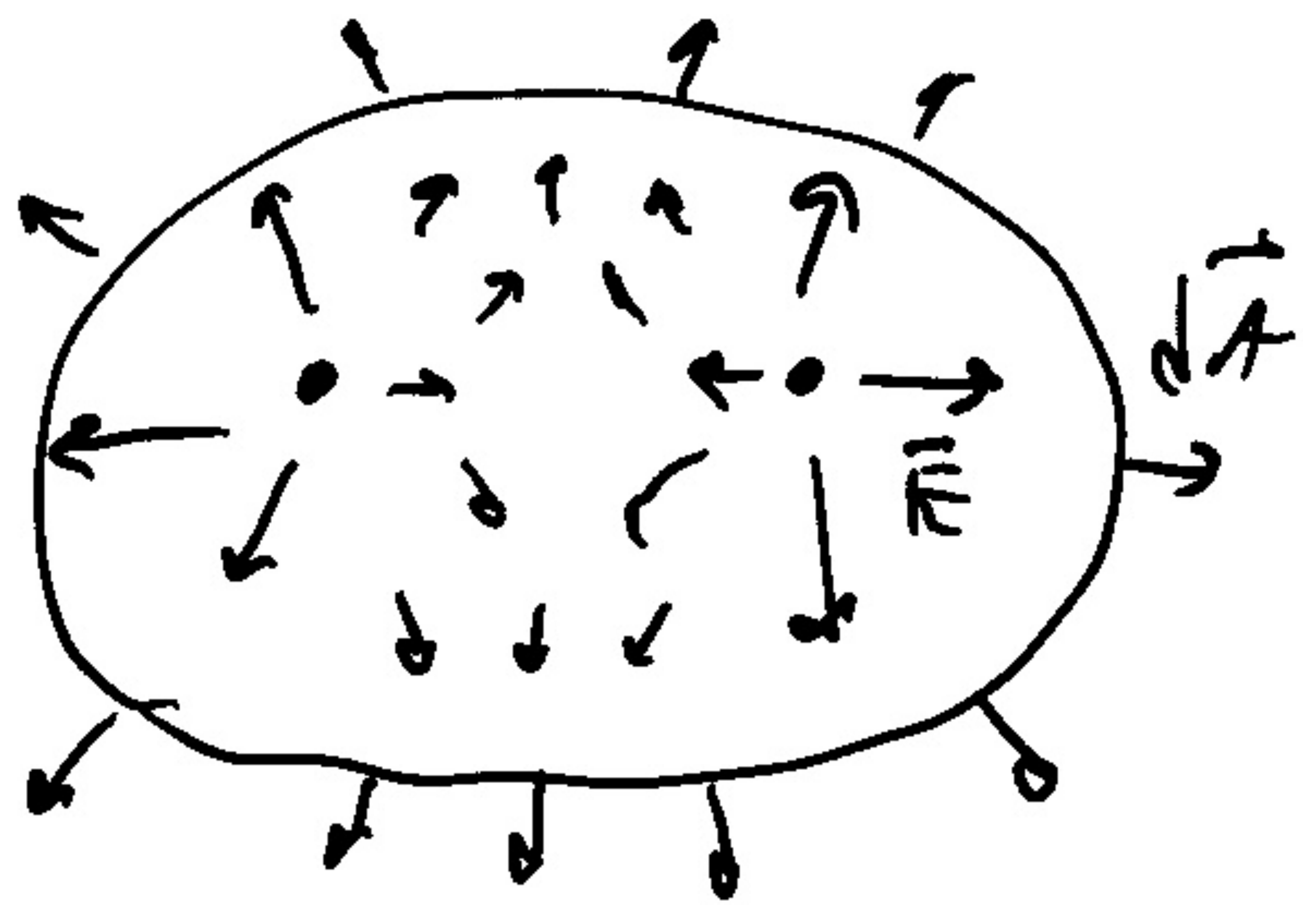
Not Easy



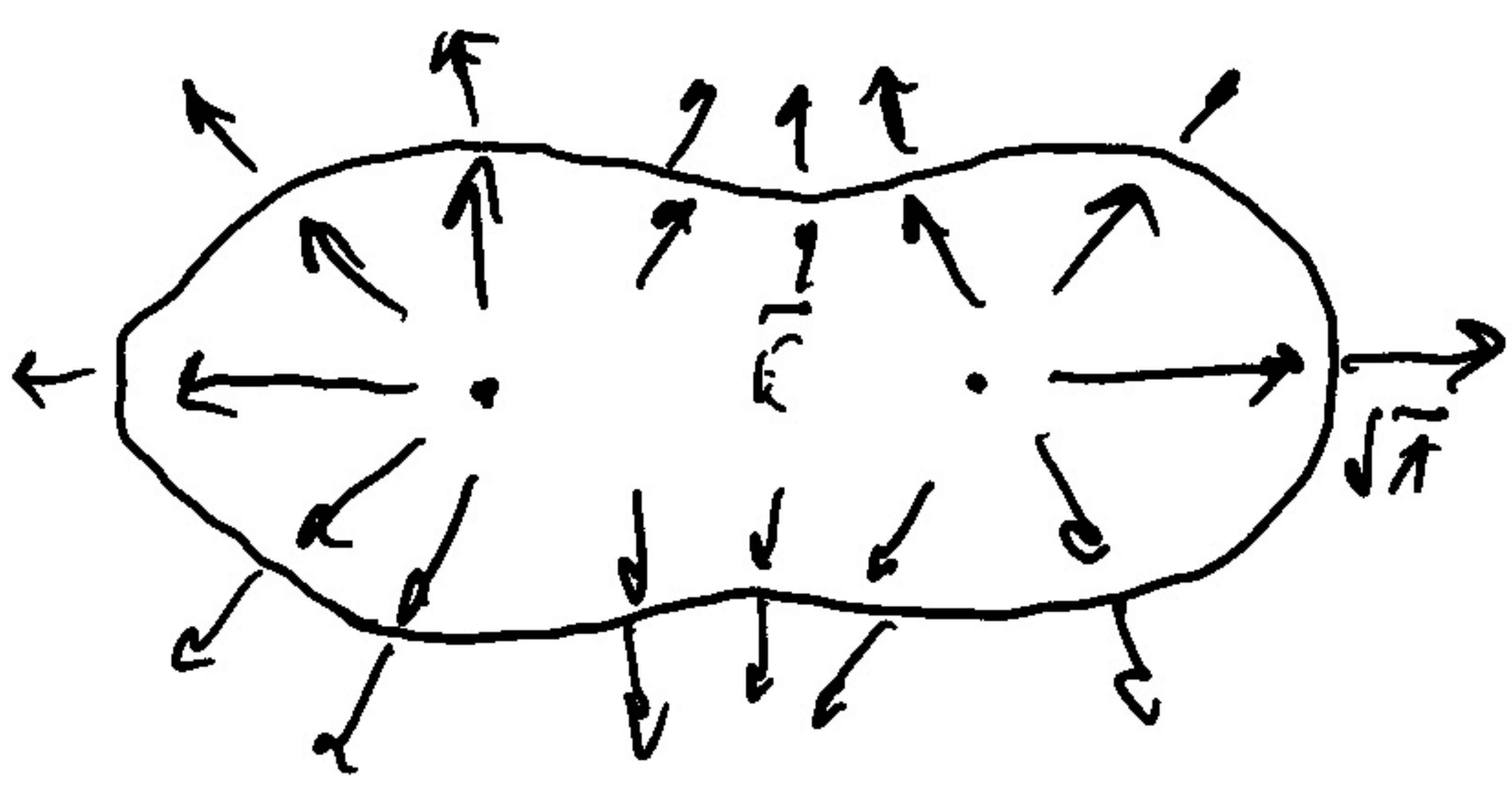


$\vec{E} \parallel d\vec{A}$

and $d\vec{A}$ simple to integrate



$\vec{E} \nparallel d\vec{A}$



$\vec{E} \parallel d\vec{A}$

but $d\vec{A}$ not easy to integrate!!