Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

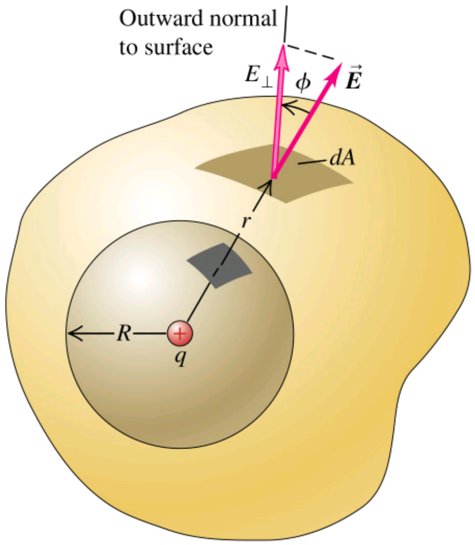
Announcements

- Homework #4 is a hard-copy long-form assignment
  - It is available in PDF from the “assignments” page on the main course web page

Gauss's Law

Gauss's Law

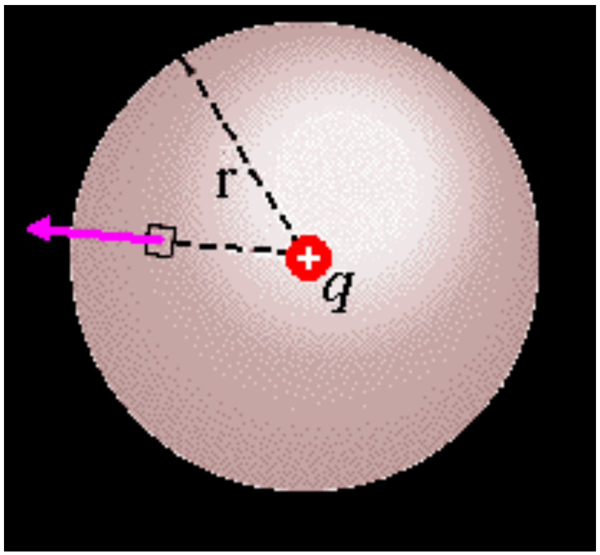
$$\Phi_E = \oint \vec{E} \bullet d\vec{a} = \frac{Q_{inside}}{\varepsilon_0}$$



Circle = integral over a closed surface!
For a closed surface, $\vec{da}$ is always outward.

The electric flux thru any **closed surface** S is a constant ($1/\varepsilon_0$) times the **net charge enclosed** by S.

Gauss's Law for Point Charge



$$\oint_{surf} \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\varepsilon_0}$$

$$\oint_{surf} |\vec{E}|\,|d\vec{a}| = \frac{+q}{\varepsilon_0}$$

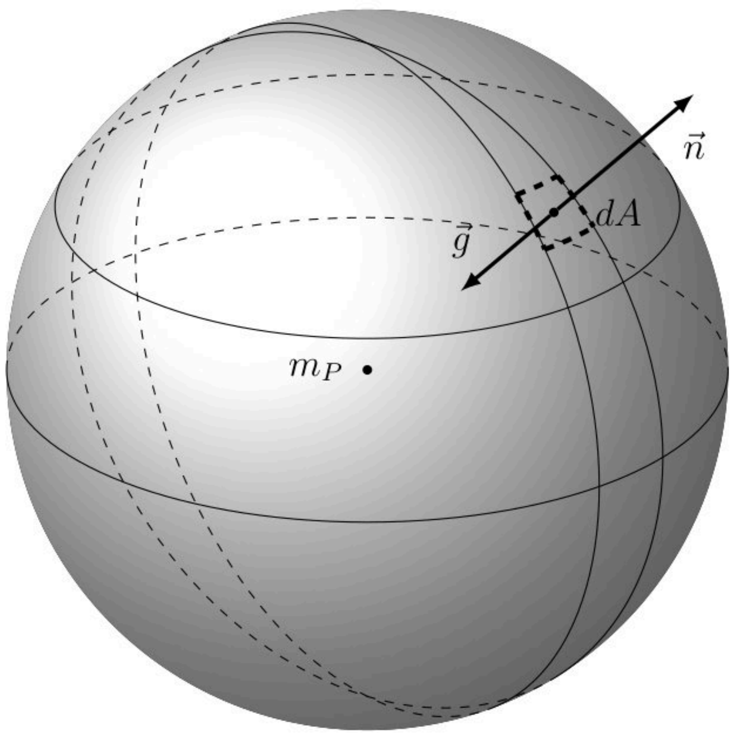
$$|\vec{E}| \oint_{surf} |d\vec{a}| = \frac{+q}{\varepsilon_0}$$

$$|\vec{E}|(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$|\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Gauss's Law of Gravity

$$\int_S \vec{g} \cdot d\vec{A} = 4\pi G M_{enc}$$



$|\mathbf{g}|A = |\mathbf{g}|\, 4\pi r^2 = -4\pi\, G\, M$

$|\mathbf{g}| = -GM/r^2$

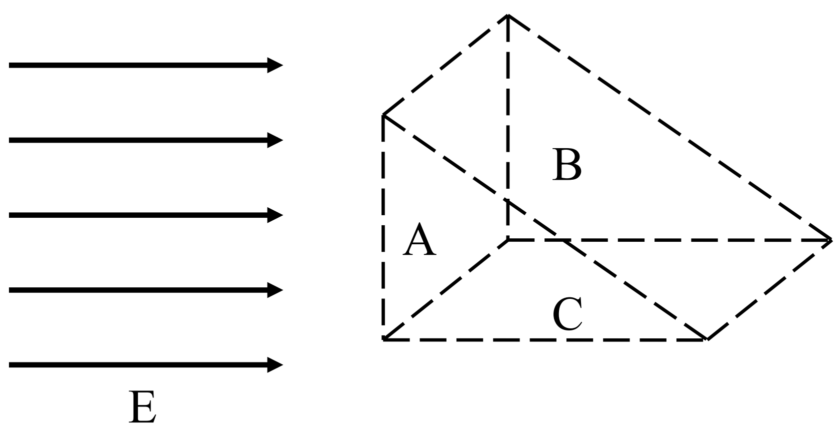
Gauss's Law: A Warning

- You can pick as complicated a surface as you want (though I don't recommend it) to evaluate: $$\oint_{surf} \vec{E} \cdot d\vec{a} = \frac{Q_{inside}}{\varepsilon_0}$$

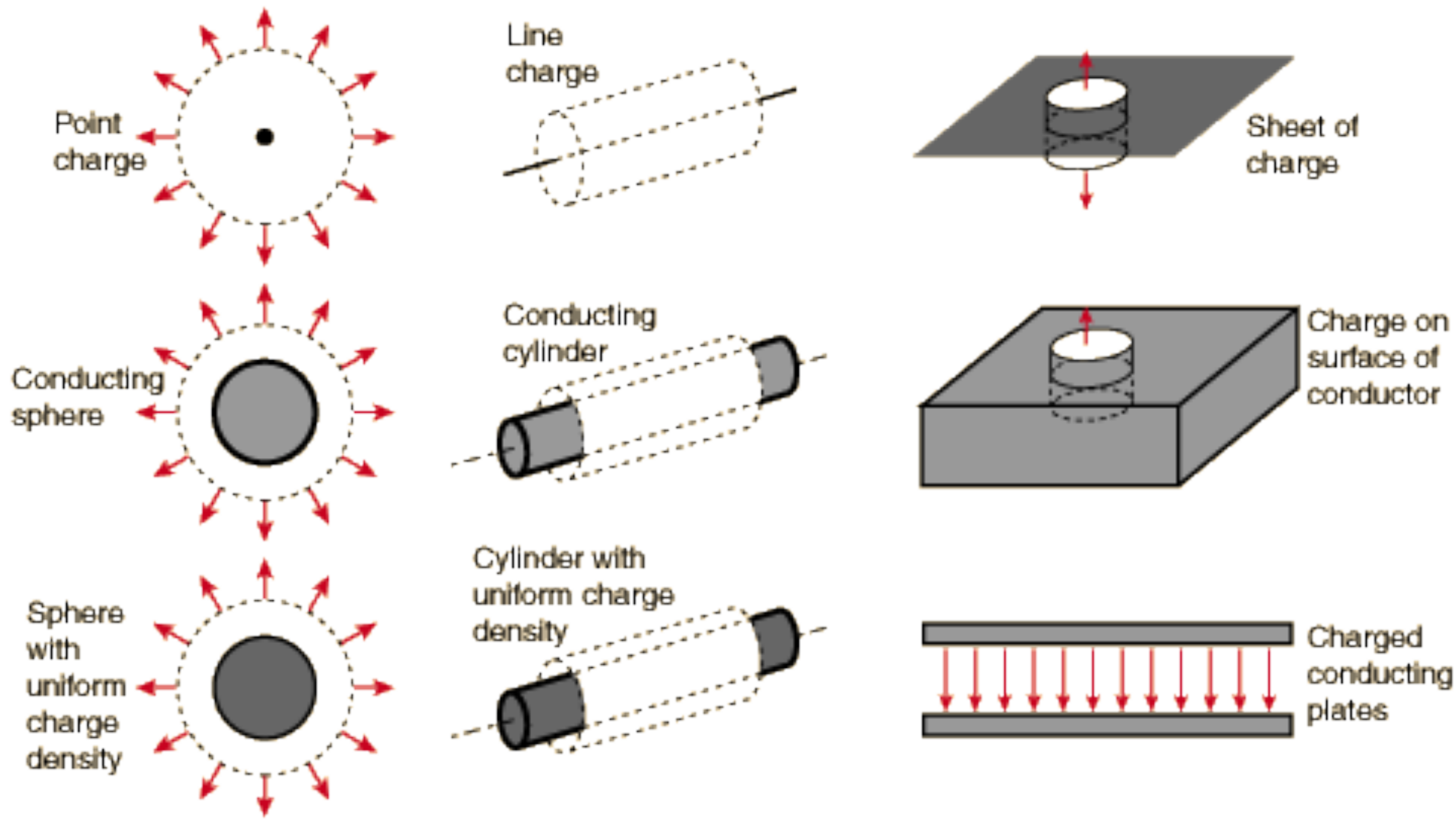
- But, you have to follow two rules:
  - The surface has to be closed
  - The surface normal has to be outward from the enclosed region

Gauss's Law: Another Warning

- Just because there is no charge enclosed does not mean there is no electric field or flux
- It only means there is no **net** electric flux through the surface, like this example we did last time



Applying Gauss's Law

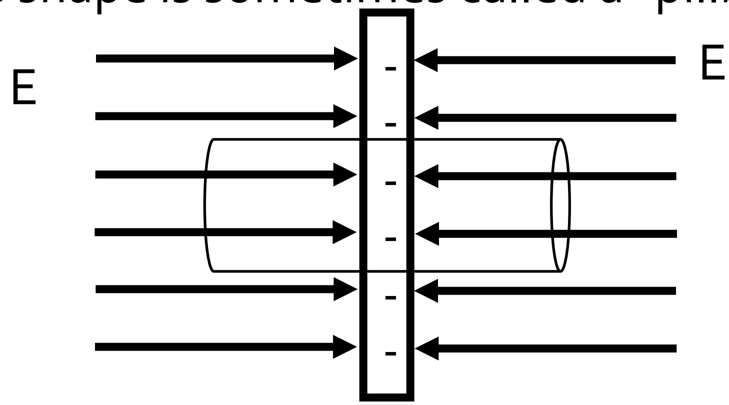


Uniform sheet of charge

A uniform, infinite plane of - charge creates a uniform E-field ⊥ to the plane.

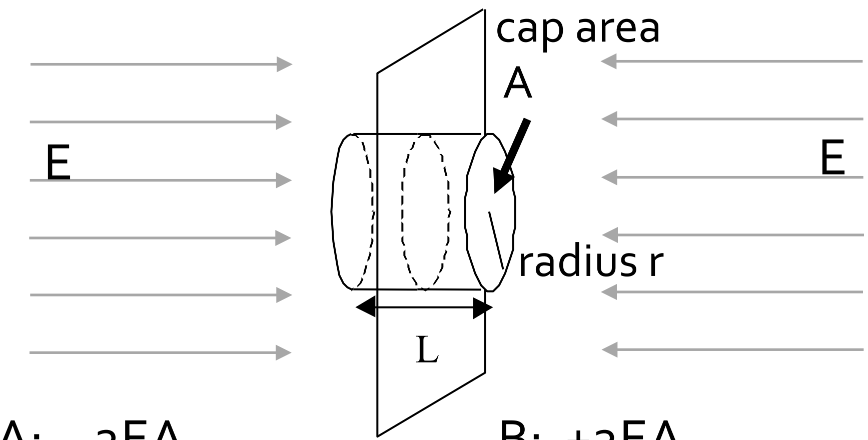
An imaginary Gaussian surface in the shape of a right cylinder is shown.

(This shape is sometimes called a "pillbox")



Concept Check

The electric flux through the pillbox surface is…



A: - 2EA

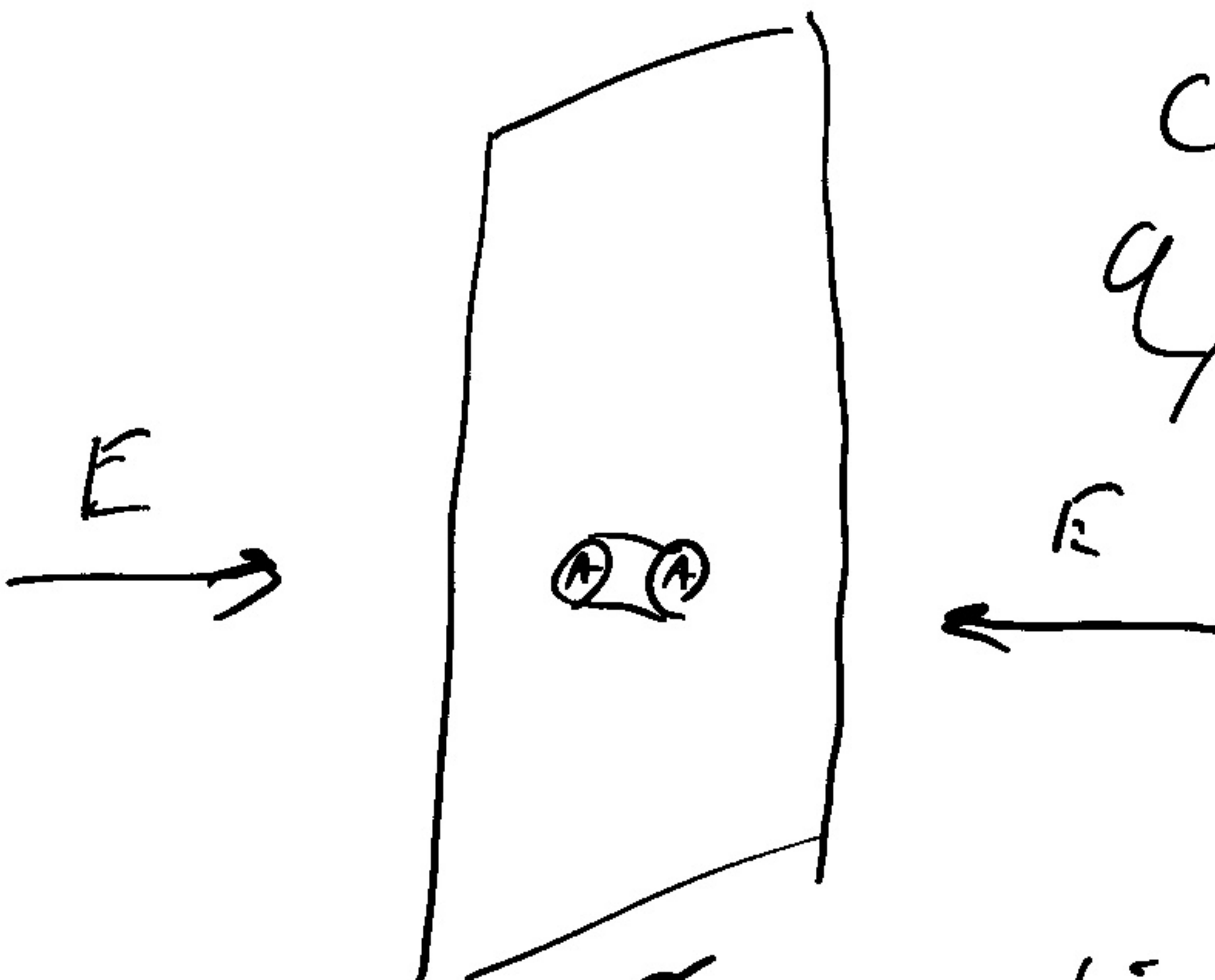
B: +2EA

C: -EA

D: $L\pi r^2$ E

E: EA

Sheet of charge



Charge density

$q/A = \sigma$

σ negative for E towards

$q_{enc} = A \cdot \sigma$

flux through left cap

$\vec{E} \rightarrow \quad \leftarrow d\vec{A}$

$$\varphi_1 = \int \vec{E} \cdot d\vec{A} = -EA$$

flux through right cap

$d\vec{A} \rightarrow \quad \leftarrow \vec{E}$

$$\varphi_2 = -EA$$

flux through sides $= 0$

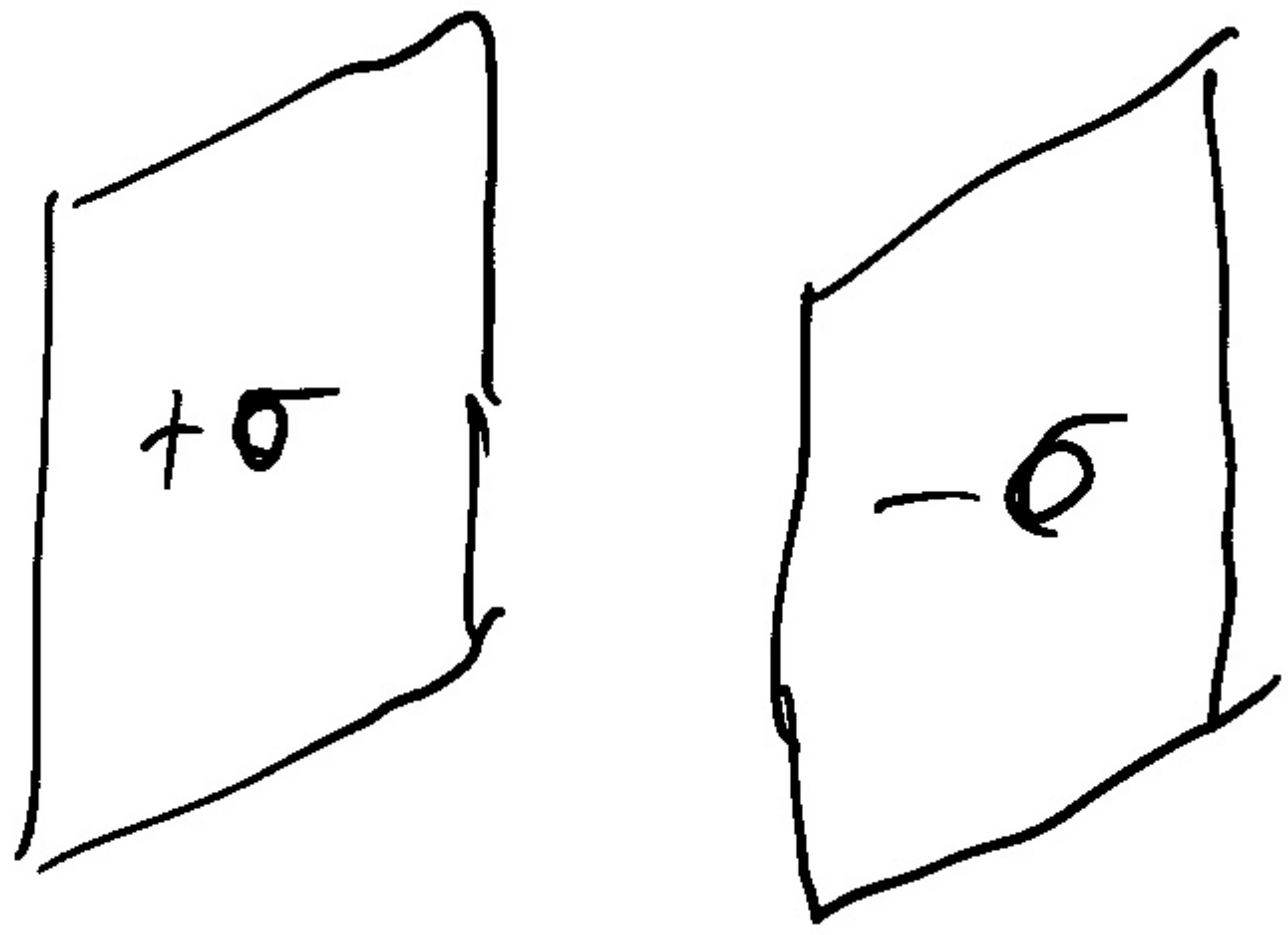
since $\vec{E} \cdot d\vec{A} = 0$

so $A\sigma/\varepsilon_0 = 2EA$

or $\boxed{E = \sigma/2\varepsilon_0}$

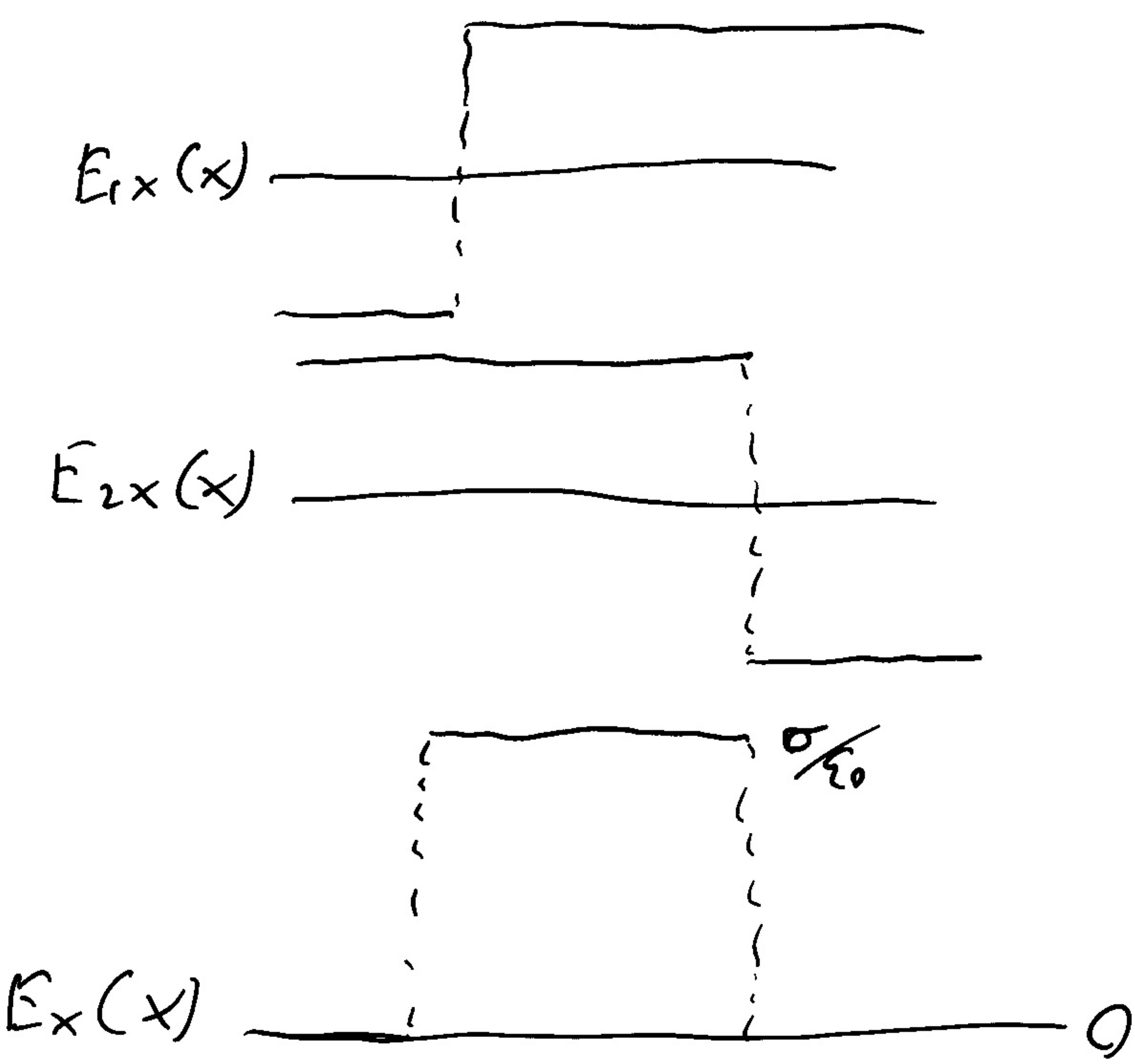
Good approx. close to any sheet.

What about two sheets



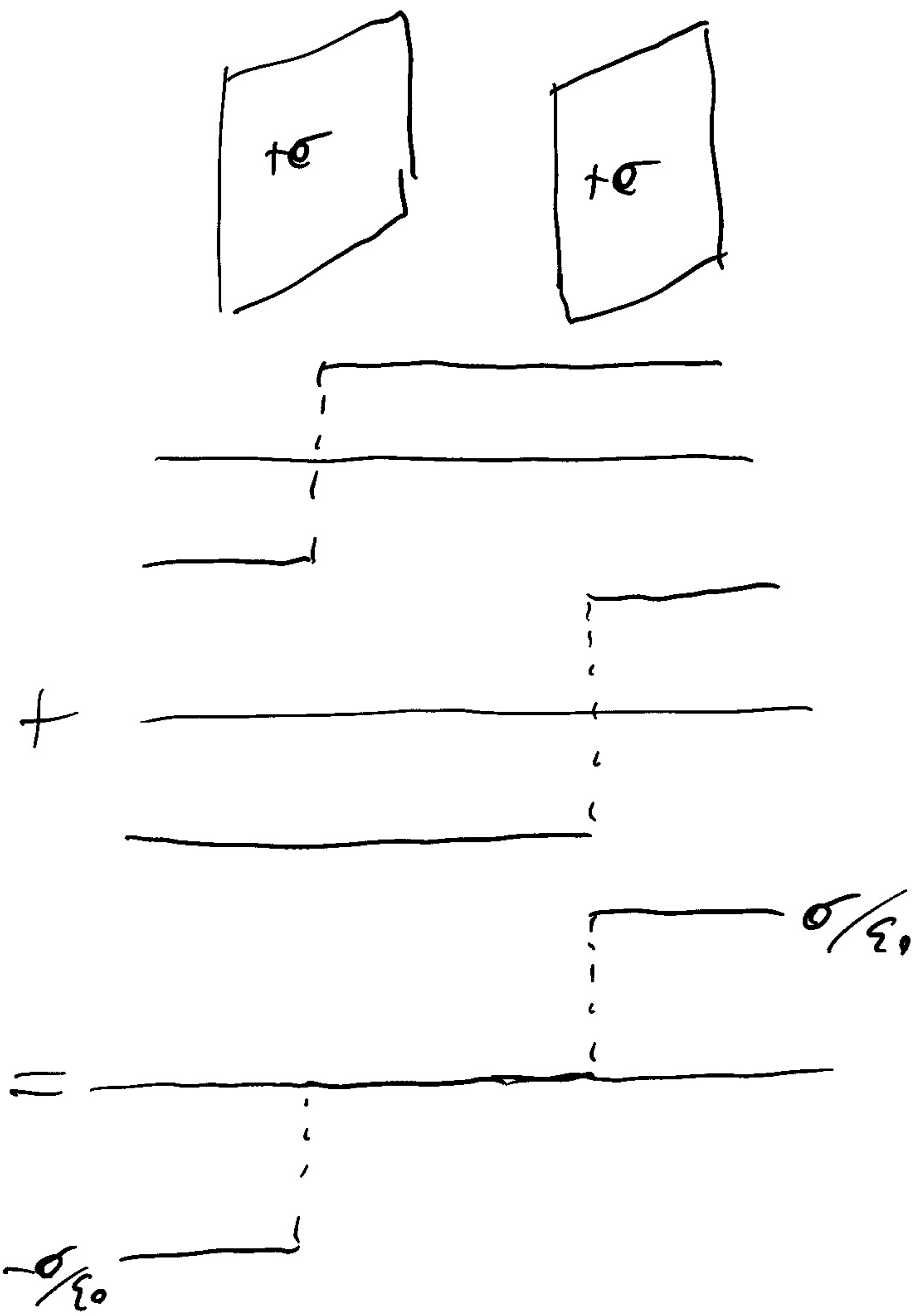
$E_1 = \sigma/2\varepsilon_0$ out from left sheet

$E_2 = -\sigma/2\varepsilon_0$ in toward right sheet



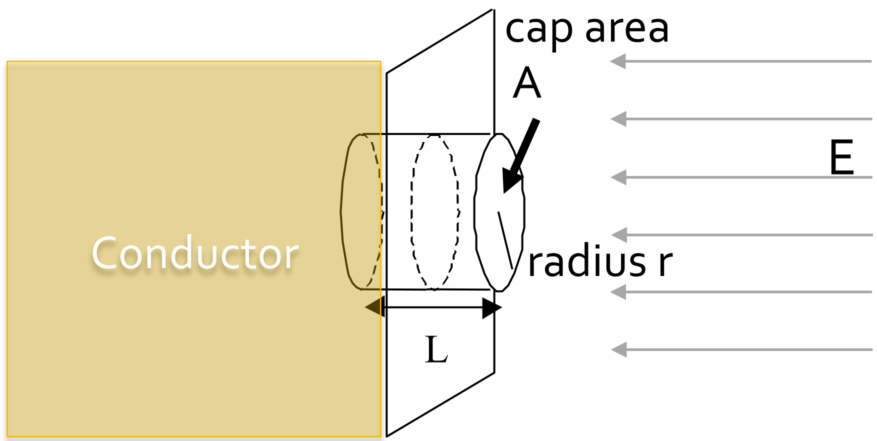
This is a capacitor

Like charged sheets:



Concept Check

What if the sheet of charge is the surface of a conductor?



A: -2EA

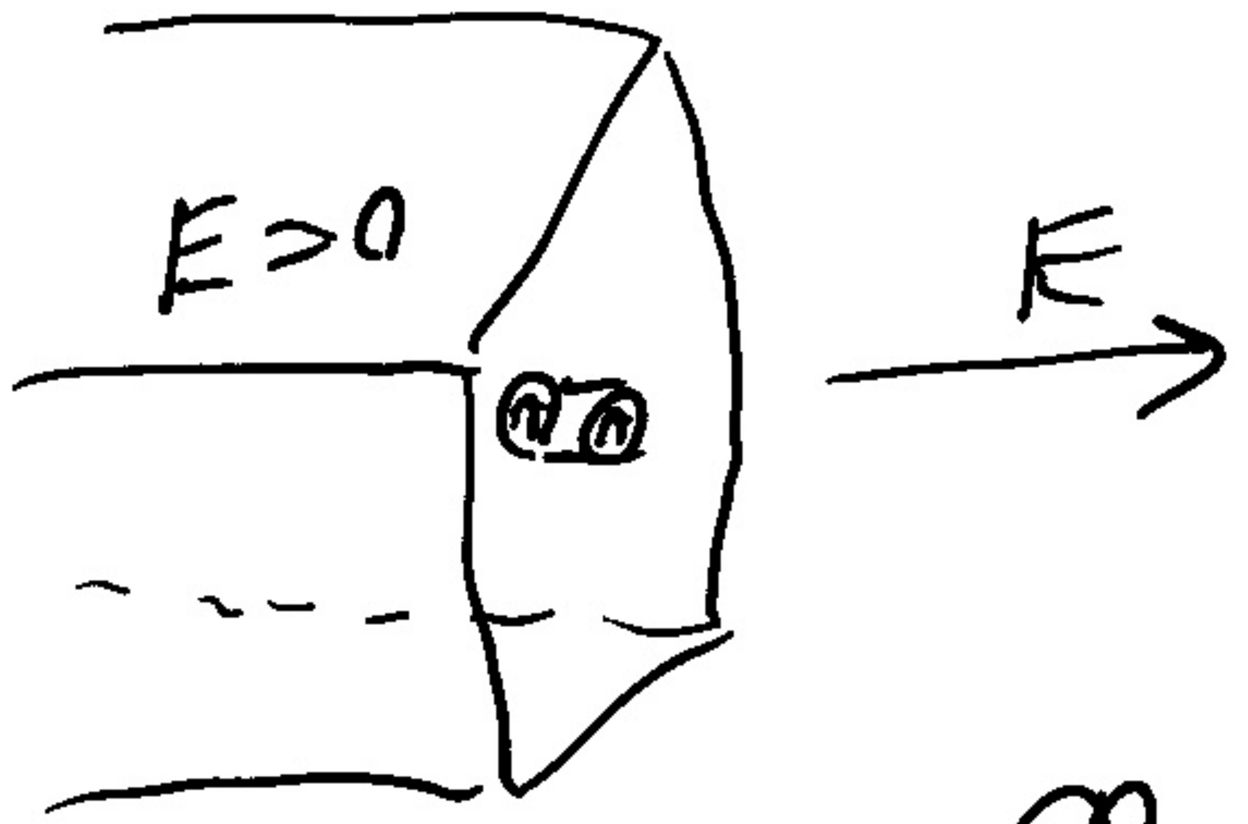
B: +2EA

C: -EA

D: $L\pi r^2 E$

E: EA

Now a conductor:



Φ_1 now zero, since $E=0$

$$\Phi_2 = EA$$

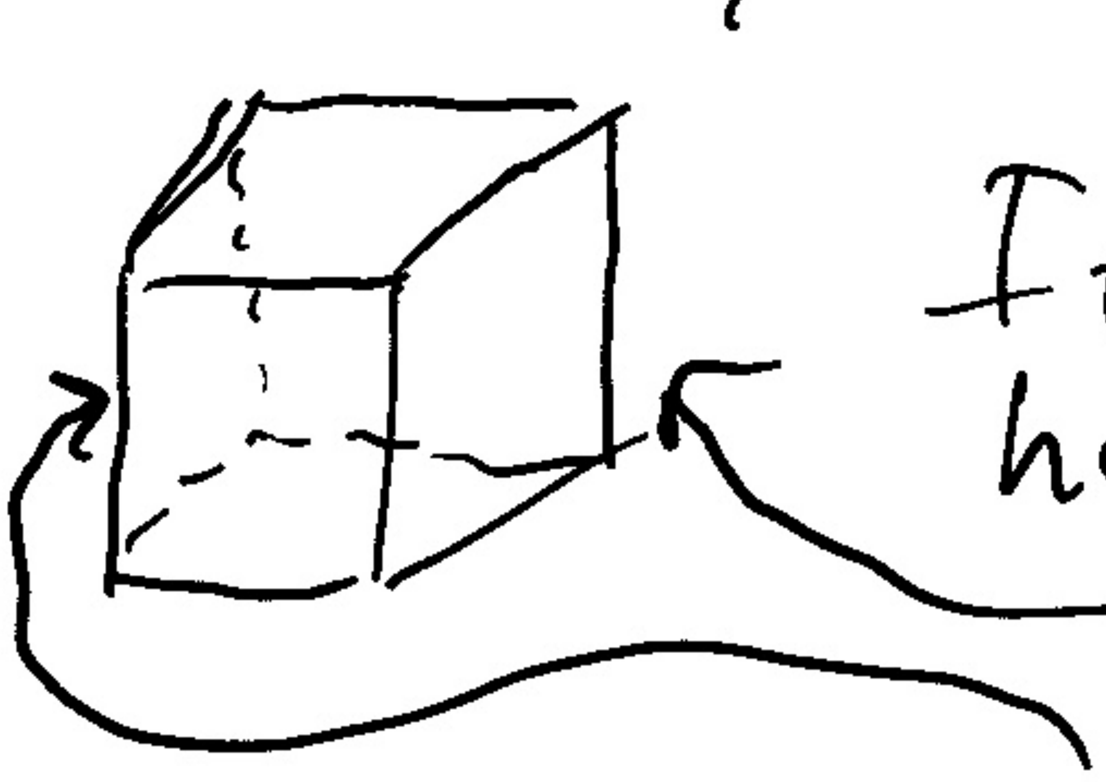
so $\sigma A/\varepsilon_0 = EA$

$$\boxed{E = \sigma/\varepsilon_0}$$

Twice that for isolated sheet of charge! WTF!

All the field lines forced to go out one side.

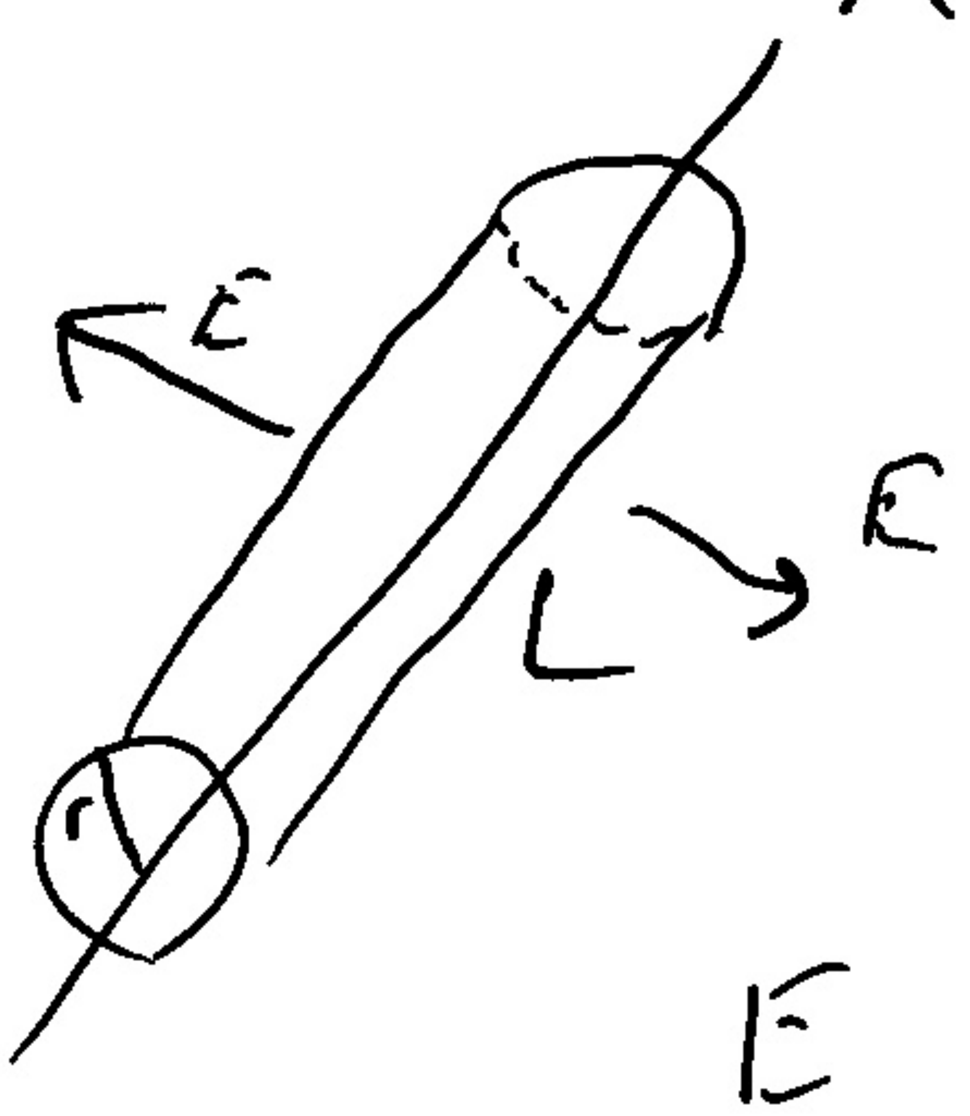
Also, the other side matters!



If you put charge here, you also have to put it here, to make $E=0$ inside

Field of a wire

$\lambda = q/L$



E radial so flux through end caps is zero

$$\Phi = \oint \vec{E} \cdot d\vec{A} = E \cdot A$$
$$= E \cdot 2\pi r L$$

$$q_{enc} = \lambda \cdot L$$

So $2\pi r L E = \lambda L / \varepsilon_0$

or $$\boxed{E = \lambda / (2\pi \varepsilon_0 r)}$$

Good approx. close to any wire.