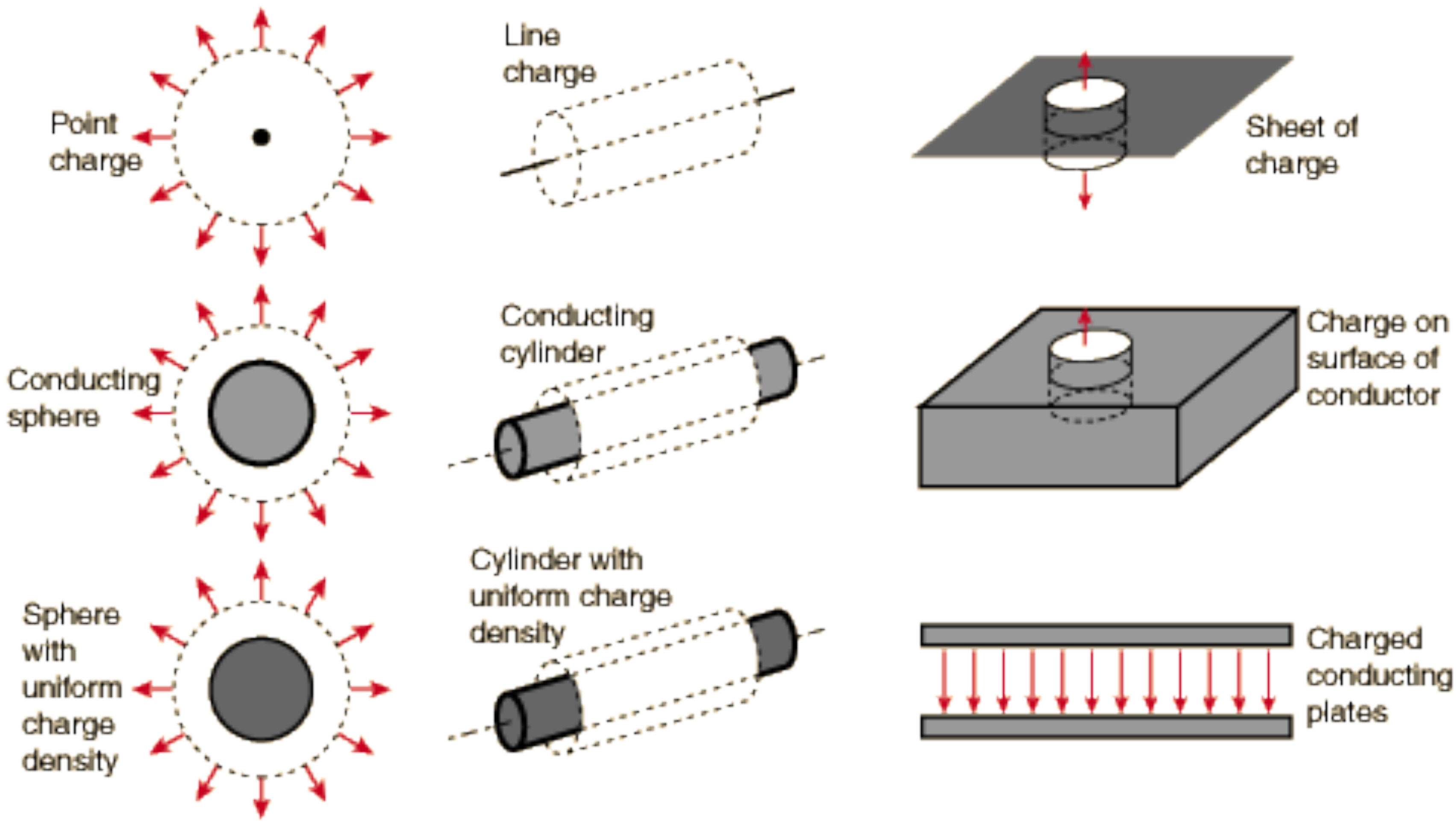
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

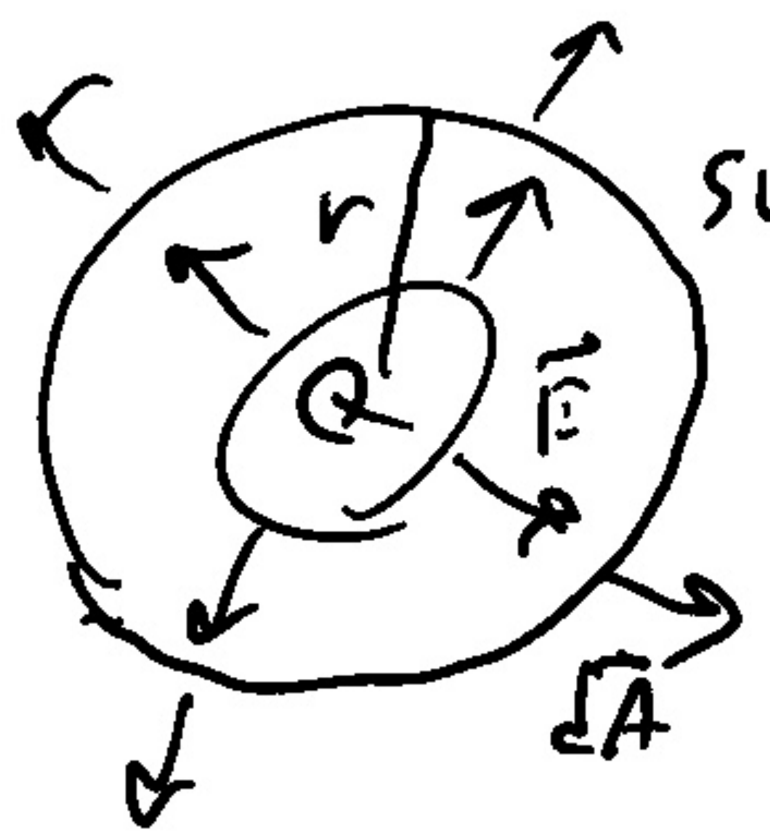
Announcements

- Hardcopy homework – available on “assignments” page – due Wed. 11:00 pm
  - Can turn in paper or electronic copy, but you *must* show all your work
- Sample Midterm Questions – now available on “notes” page
  - No sample questions on gravity – doesn’t mean no gravity questions on midterm!

Applying Gauss's Law



Spherically symmetric charge distribution:



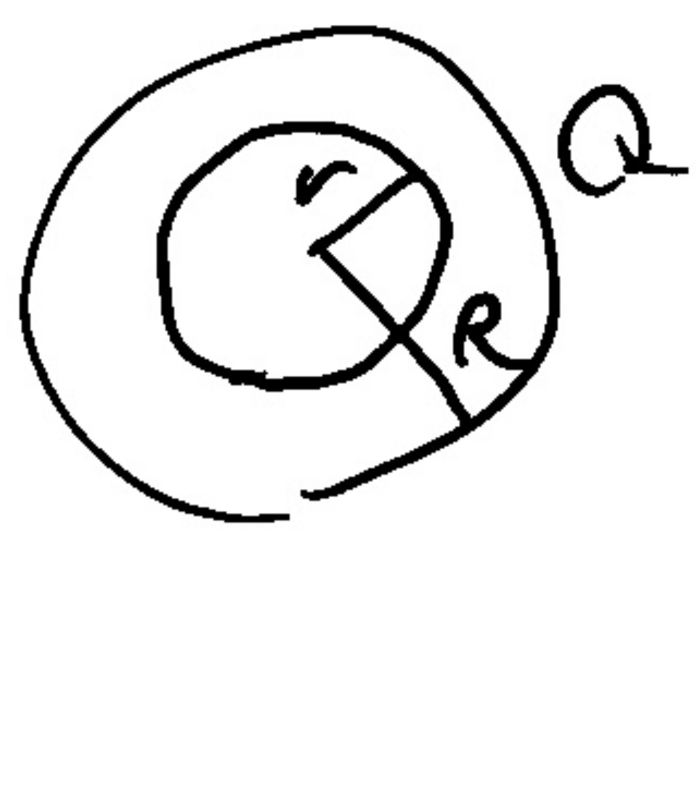
$$\oint \vec{E} \cdot d\vec{A} = Q/\varepsilon_0$$

$$= E \cdot 4\pi r^2$$

$$\Rightarrow E = Q/4\pi\varepsilon_0 r^2$$

- For point, or shell, or full sphere.
- Proves shell theorem

- What if you're inside?
- only enclosed charge matters
- Say evenly distributed charge

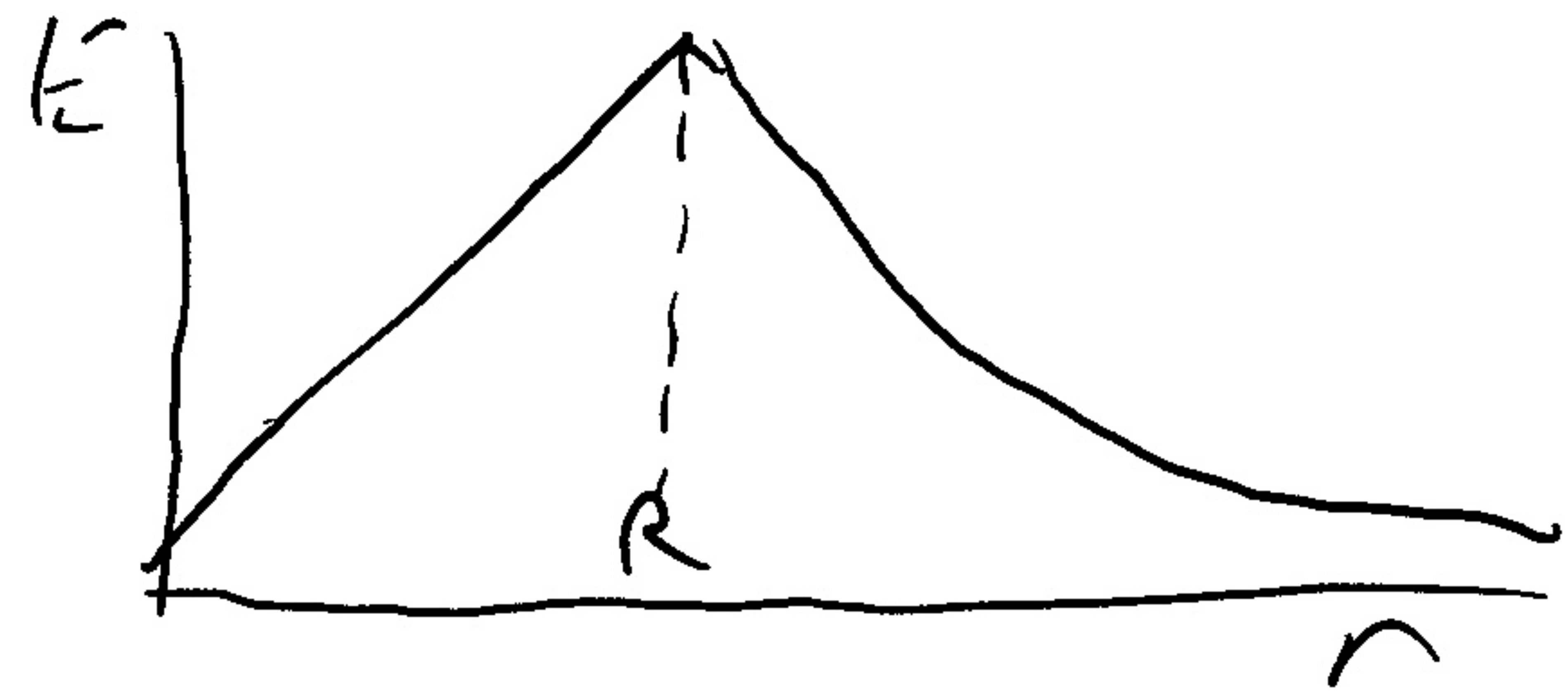


$$q_{enc} = \frac{4/3 \pi r^3}{4/3 \pi R^3} Q$$

$$= Q r^3/R^3$$

$$\oint \vec{E} \cdot d\vec{A} = 4\pi r^2 E = q_{enc}/\varepsilon_0$$

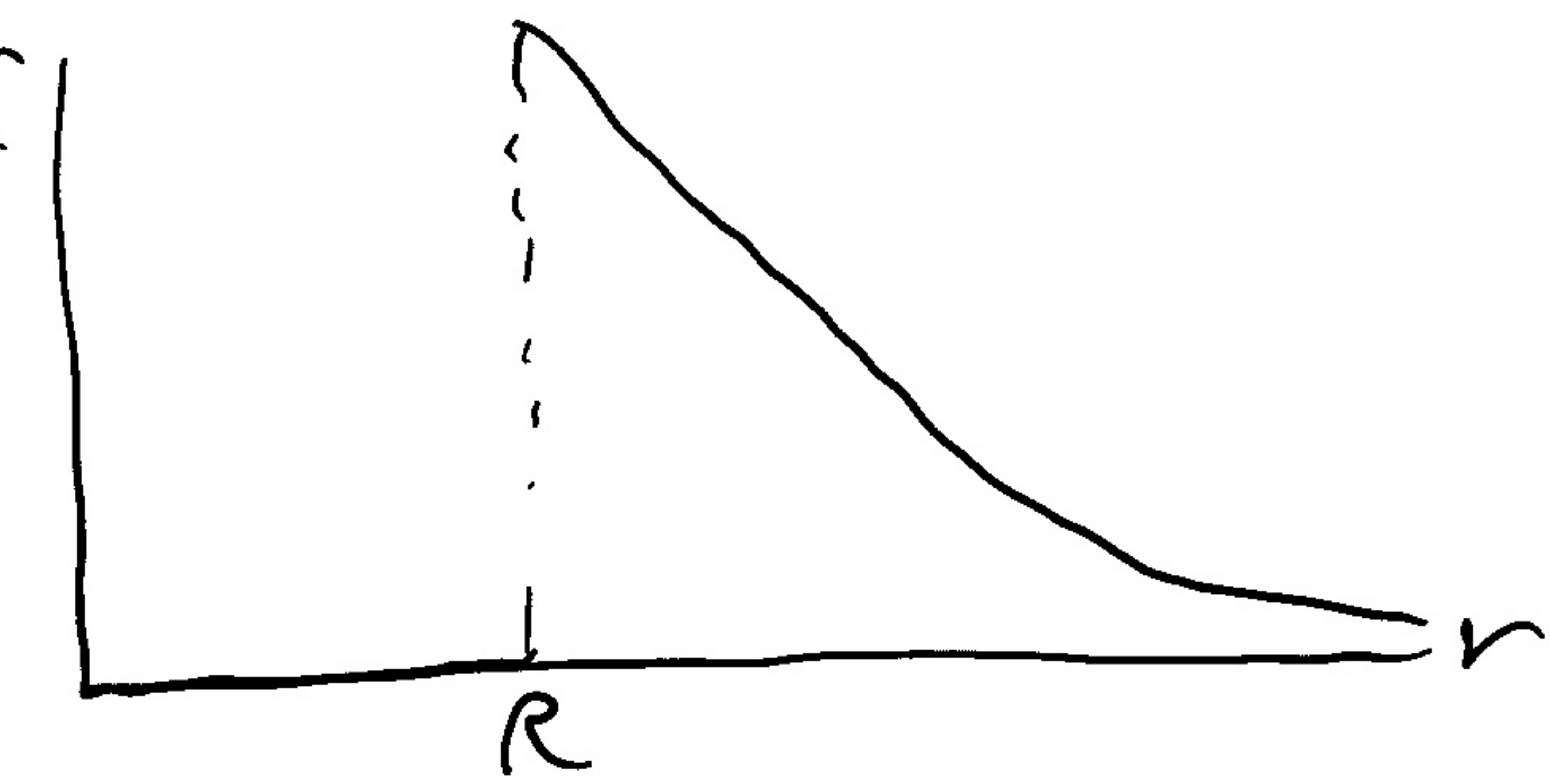
$$\Rightarrow E = Qr/4\pi\varepsilon_0 R^3$$

E (r) = 

- What if it's a conductor?

- Inside conductor, E = 0
- All charge @ surface
- Outside surface,

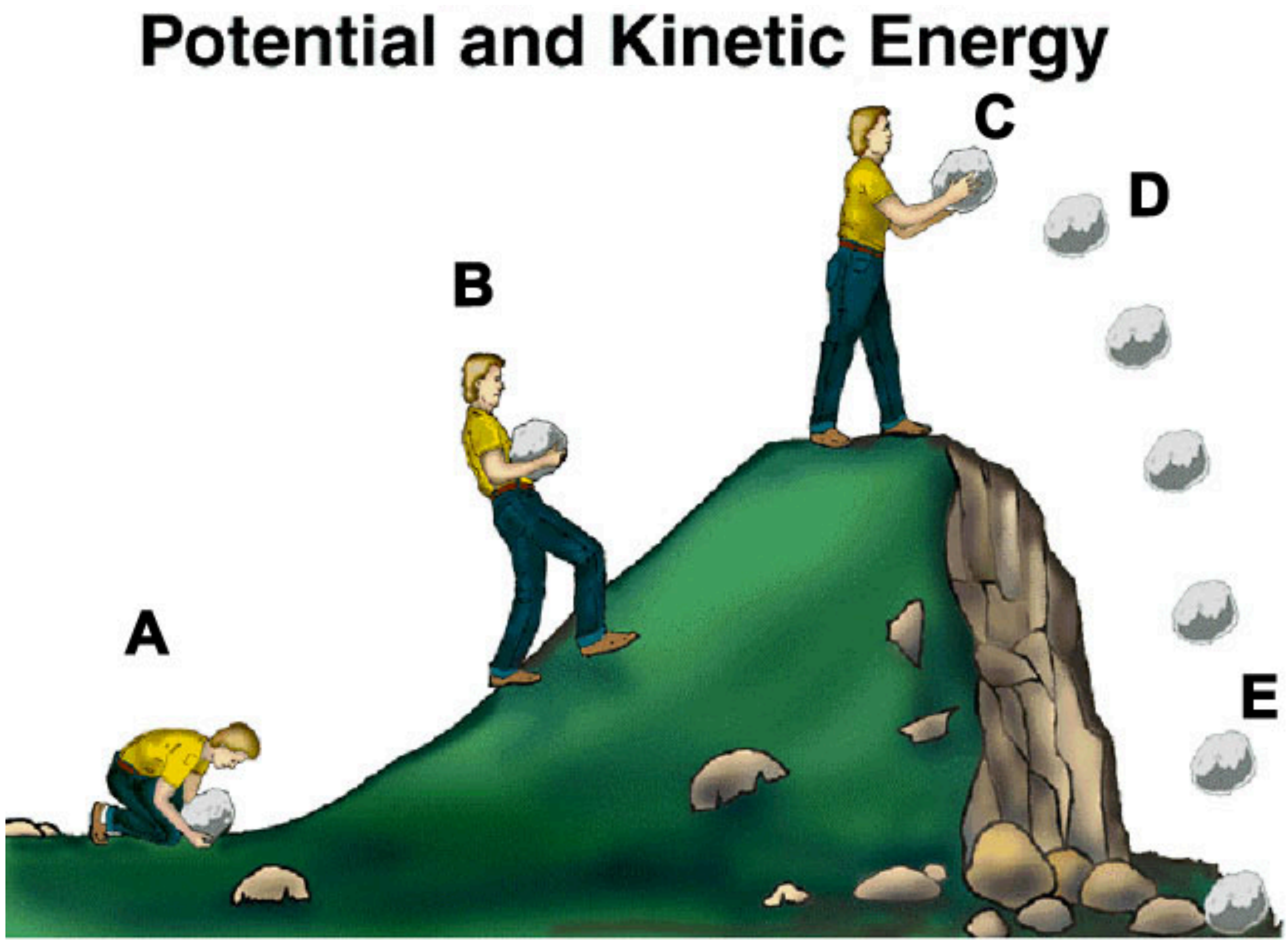
$$E = Q / 4\pi \varepsilon_0 r^2$$

E (r) = 

Gauss's Law & Shell Theorem

- Gauss's law proves the shell theorem
- Why?
  - Any spherically symmetric distribution with the same total charge produces the same electric flux through any surface outside all of the charge, so the field of a shell is the same as a point with the same charge
  - No charge outside of a surface produces net flux through it, so the resulting field is zero inside any shell of charge

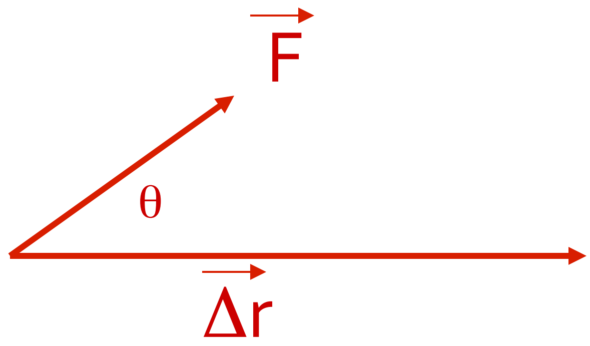
Potential Energy



Work

Work done by a (constant) force $\vec{F}$

$$W = \vec{F} \cdot \Delta\vec{r} = |\vec{F}||\Delta\vec{r}|\cos\theta$$

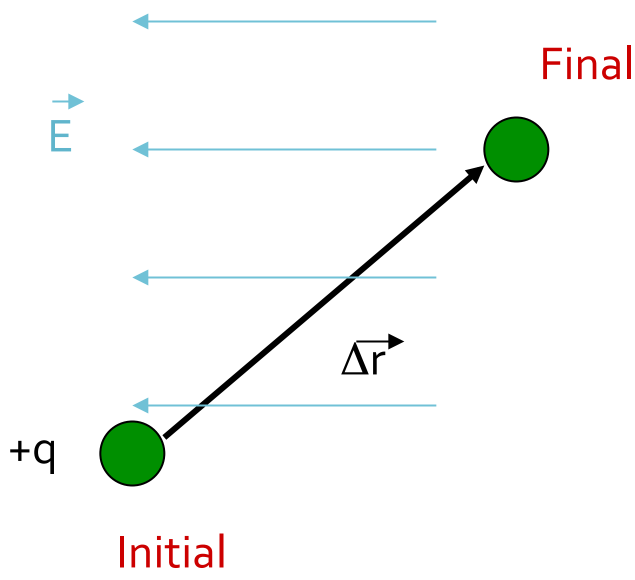


Work W is the negative of the change in potential energy

W = -ΔU

Electric Potential Energy

Definition of electrical potential energy.



ΔU_{elec} = change in U when moving +q from initial to final position.

$$\Delta U = U_f - U_i = +W_{ext} = -W_{field}$$

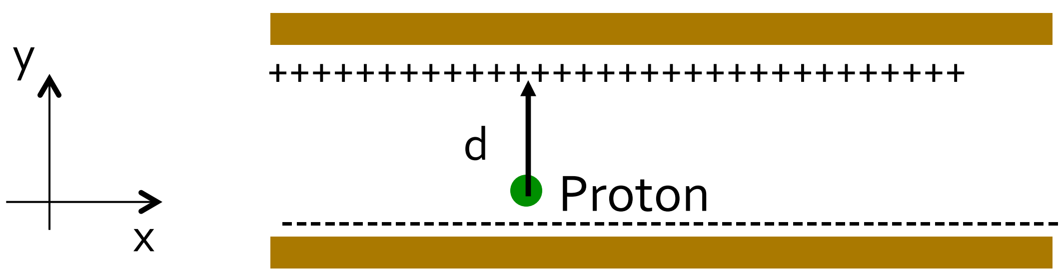
$$\Delta U = -W_{field} = -\vec{F}_{field} \cdot \Delta\vec{r} = -q\vec{E} \cdot \Delta\vec{r}$$

* In the case of constant E-field.

Electric Potential Energy

Two parallel conducting plates
(a capacitor) are charged as shown.
A proton is lifted by an external
agent (tweezers) a distance d as shown.
Ignore gravity in this problem.

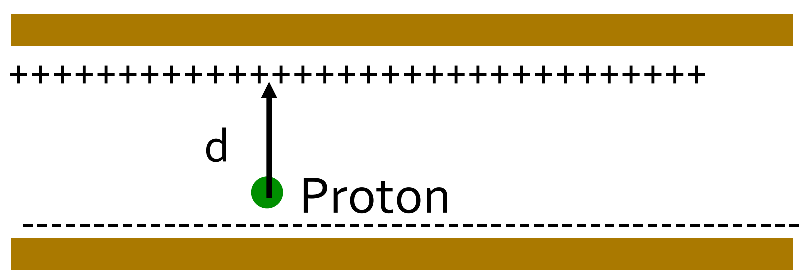
(There is a uniform electric field between the plates) $\vec{\mathbf{E}} = -E\,\hat{\mathbf{y}}$



Concept Check Part 1

What direction is the force on the proton due to the E-field in the capacitor?

A: up B: down C: zero

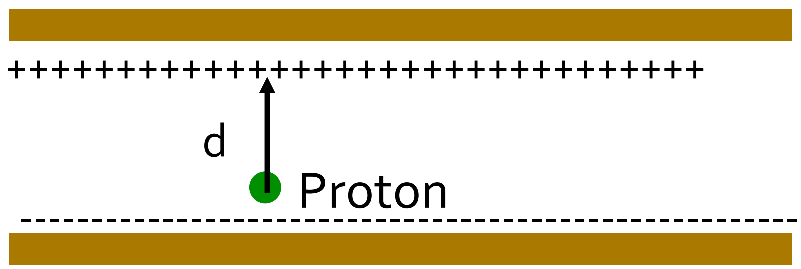


Concept Check Part 2

The sign of the work done by the E-field as the proton is moved upwards is…?

A: + B: - C: zero

D: Not enough info

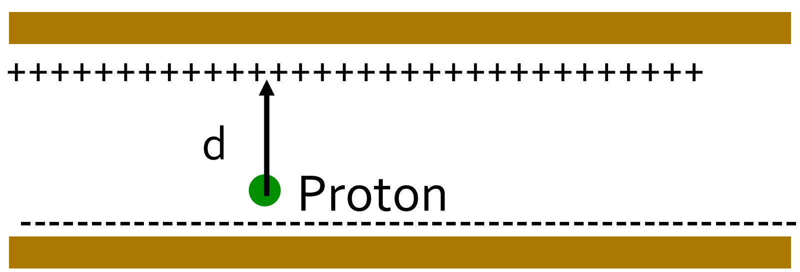


Concept Check Part 3

The sign of the work done by the external agent (the tweezers) is … ?

A: + B: - C: zero

D: Not enough info



Concept Check Part 4

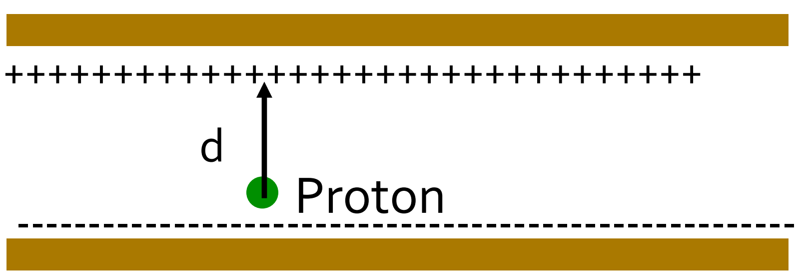
Change in potential energy is :

$$\Delta U = +W_{ext} = -W_{field}$$

If we define U(proton) = 0 at the bottom plate, then U(proton) near the top is...

A: + B: - C: zero

D: Not enough info



Electric Potential Energy

- Note that things are a little bit more complicated when you have positive and negative charges
- If you move two charges of opposite sign through a given electric field, one of them gains potential energy, while the other loses potential energy!

Force and Potential Energy

$$-\frac{dU}{dx} = F(x)$$

$$U(x) = -\int_{x_0}^{x} F(x)dx + U(x_0)$$

Vector Fun

What if the E-field is not constant?

$$\Delta U = -q\vec{E}\cdot\Delta\vec{r}$$

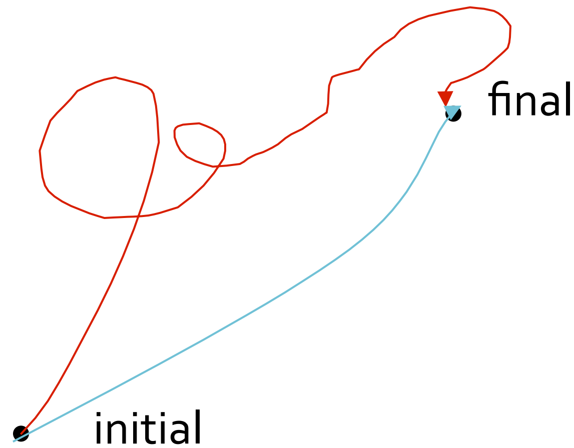
$$\Delta U = -q\int_{i}^{f}\vec{E}\cdot d\vec{r}$$

Integral over the path
from initial (i) position to final (f)
position.

Conservative Forces are Great!

Since Coulomb forces are conservative, it means that the change in potential energy is path independent.

$$\Delta U = -q\int_{i}^{f} \vec{E} \cdot d\vec{r}$$



The same was true for gravitational potential energy.

Electric Potential

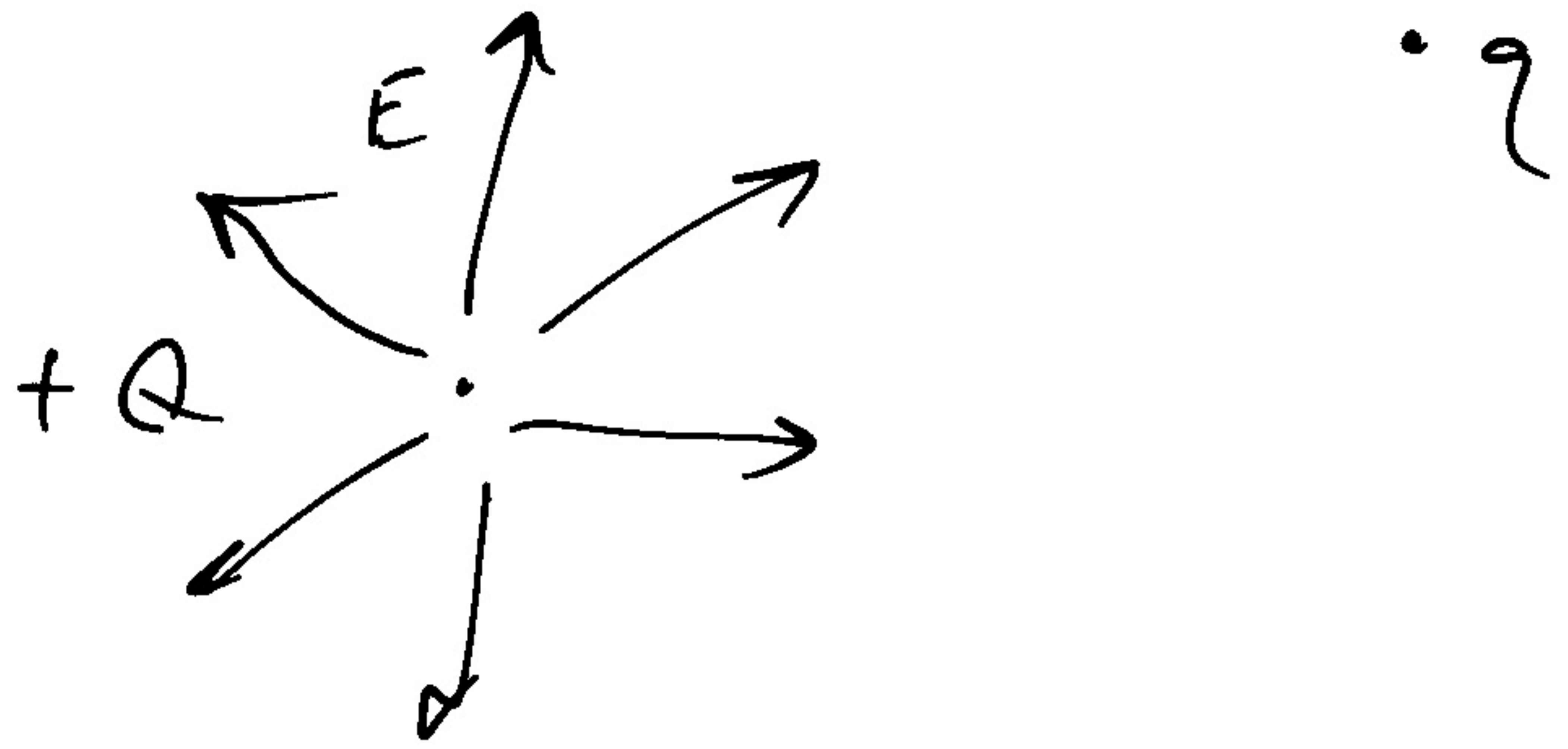
- Just as we defined a field $E = F/q$
- We define an electric potential $V = U/q$
- While the electric field is a vector field, the electric potential is a scalar field
- Electric potential $V \neq$
  Electric potential energy U !!

Electric Field and Potential

$$\overline{E} = -\nabla V$$

$$V_{BA} = V_B - V_A = -\int_A^B \overline{E} \bullet d\overline{\ell}$$

potential energy for test charge in field of another charge



- start at $r = \infty$ and radially integrate to $r = R$

$$\Delta U = U_f - U_i$$

$$= -q \int_i^f \vec{E} \cdot d\vec{\ell}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$$

$d\bar{\ell} = -\hat{r}\, dr$ since inward path

so $$\Delta U = -q \int_i^f \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r} \cdot -\hat{r}\, dr$$

$$= \frac{qQ}{4\pi\varepsilon_0} \int_i^f 1/r^2\, dr$$

$$= \frac{qQ}{4\pi\varepsilon_0} \left[-1/r_i - (-1/r_f) \right]$$

$r_i = \infty$
$r_f = R$

so $U_f - U_i = \frac{qQ}{4\pi\varepsilon_0 R}$

$V = U/q \Rightarrow V_f - V_i = Q/4\pi\varepsilon_0 R$

If we set $U(\infty) = 0$
then in general

$$U(r) = qQ/4\pi\varepsilon_0 r$$
or $V(r) = Q/4\pi\varepsilon_0 r$

Note $F = -\nabla U(r)$
$= -\partial U/\partial r \, \hat{r}$

$= - \; -qQ/4\pi\varepsilon_0 r^2 \, \hat{r}$

$= qQ/4\pi\varepsilon_0 r^2 \, \hat{r}$

$= q\vec{E}$

Recovers Coulomb's Law

Could also solve $\vec{E} = -\nabla V$
to get same answer w/o q included