Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Announcements

- Final Date/Time Set
  - Tuesday May 10, 12:30-2:30, LR70 (this room)
- Midterm 1 one week from today
  - In class, closed book, no calculators, one 8.5x11 equation sheet (to be turned in with exam)
- Hardcopy homework (HW4) due tonight

Electrostatic Potential Energy

$$\Delta U = -q\int_{i}^{f} \vec{E} \cdot d\vec{r}$$

$$\vec{F}(\vec{r}) = -\nabla U(\vec{r}) = q\vec{\mathbf{E}}$$

Electric Field and Potential

$$\overline{E} = -\nabla V$$

$$V_{BA} = V_B - V_A = -\int_A^B \overline{E} \bullet d\overline{\ell}$$

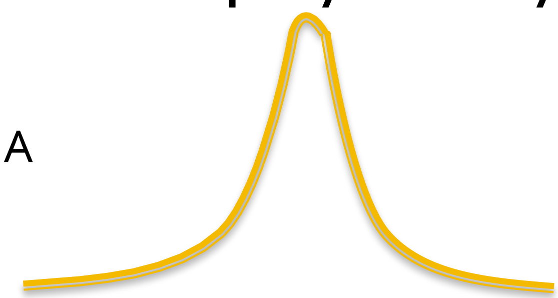
Units

- You have seen electric field primarily expressed in N/C (force per charge)
- However, electric potential is expressed in units of V (volts)
- This implies that electric field can be expressed in V/m (volts per meter)

Concept Check

- Which of these 1-d electric potential profiles is not physically plausible?

A



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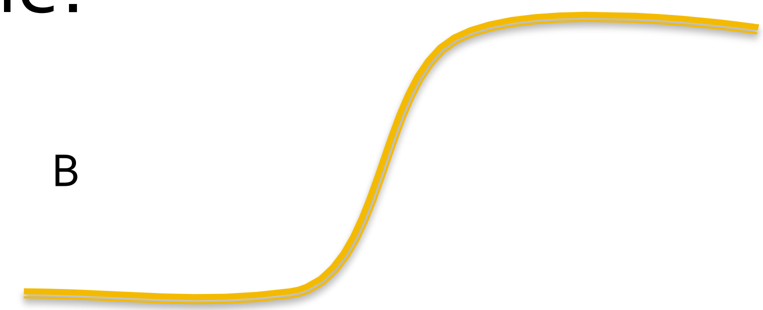

C

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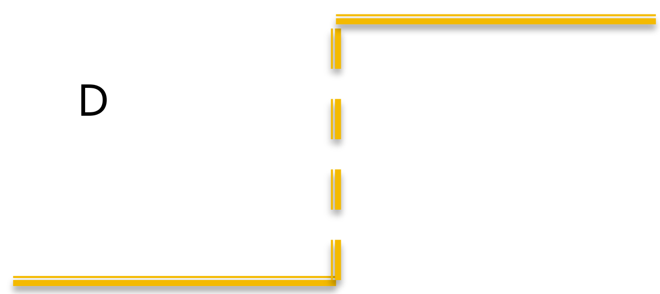


Important Point to Remember

- Electric Potential and Potential Energy are both continuous fields
  - Why?
  - Because any discontinuity would imply an infinite force and an infinite electric field

$\vec{E} = -\nabla V$ in cartesian

$$V = Q / 4\pi\varepsilon_0 r$$

$$= Q / 4\pi\varepsilon_0 |\vec{r} - \vec{r}_0|$$

w/ $\vec{r}_0$ position of Q

$$= \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{\sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}}$$

$$E_x = -\partial V / \partial x$$

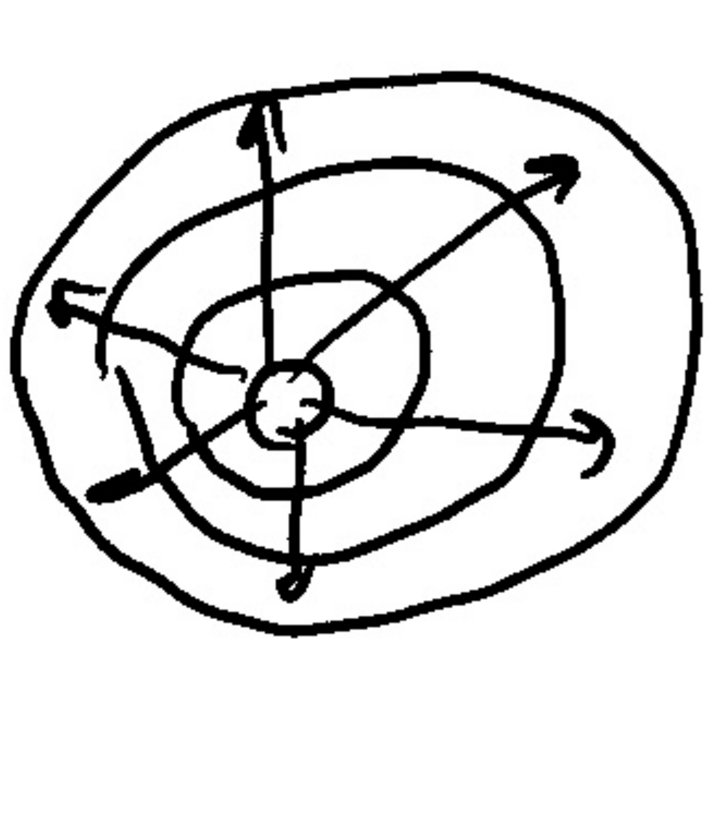
$$= Q/4\pi\varepsilon_0 \cdot \frac{\frac{1}{2} \cdot 2(x-x_0)}{((x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2)^{3/2}}$$

$$\vec{E} = [E_x, E_y, E_z]$$

$$= \frac{Q}{4\pi\varepsilon_0} \cdot \frac{(x-x_0)\hat{i} + (y-y_0)\hat{j} + (z-z_0)\hat{k}}{((x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2)^{3/2}}$$

$$= \frac{Q(\vec{r} - \vec{r}_0)}{4\pi\varepsilon_0 |\vec{r} - \vec{r}_0|^3}$$

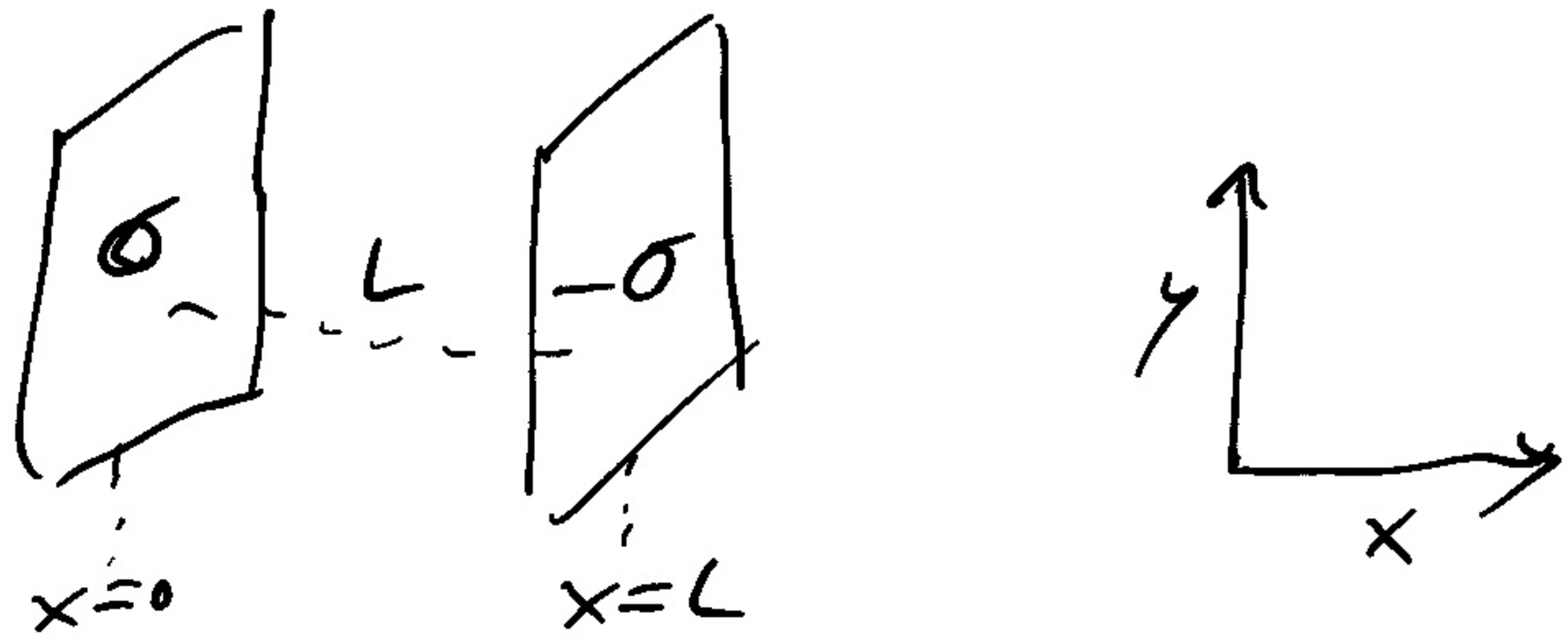
//



V = const. Equipotentials

$\vec{E} \perp$ (V = const.)

Potential Between Two Plates



From Gauss's Law we know:

$$\vec{E} = \sigma/\varepsilon_0 \, \hat{x} \quad \text{between plates}$$

start @ left plate

$U(L) - U(0)$ or $V(L) - V(0)$

$$= -q \int_0^L \vec{E} \cdot d\vec{x} \qquad = -\int_0^L \vec{E} \cdot d\vec{x}$$

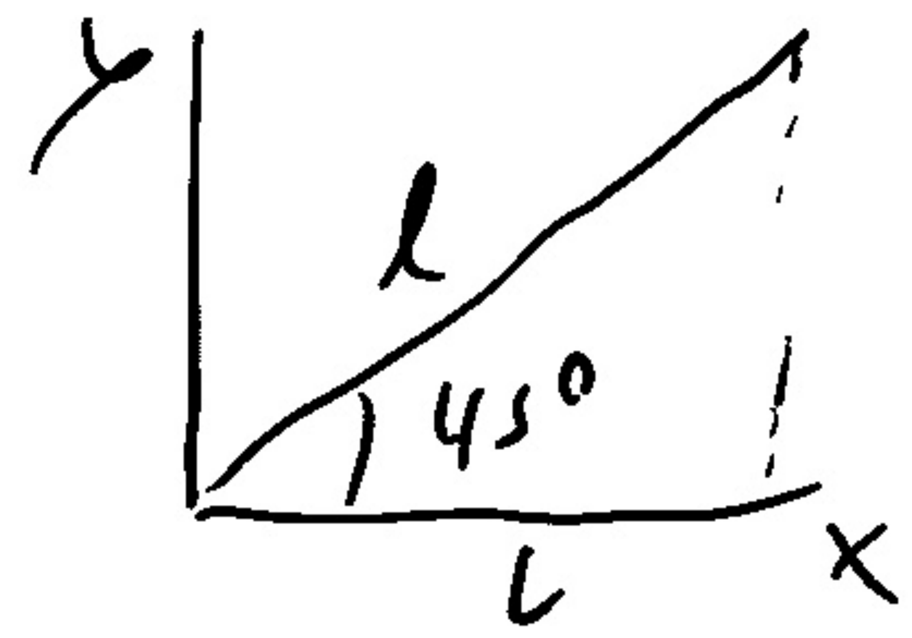
$$= -q \int_0^L \sigma/\varepsilon_0 \, dx \qquad = -\int_0^L \sigma/\varepsilon_0 \, dx$$

$$= -q\sigma L/\varepsilon_0 \qquad = -\sigma L/\varepsilon_0$$

$+q$ loses potential energy

$-q$ gains potential energy

How about a different path?



$$d\vec{l} = \hat{x}/\sqrt{2} + \hat{y}/\sqrt{2}$$

$$U_f - U_i = -q\int_i^f \vec{E}\cdot d\vec{l}$$

$$= -q\int_i^f \sigma/\varepsilon_0\,\hat{x}\cdot\left(\hat{x}/\sqrt{2} + \hat{y}/\sqrt{2}\right)dl$$

$$= -q\int_i^f \sigma/\varepsilon_0 \cdot 1/\sqrt{2}\cdot dl$$

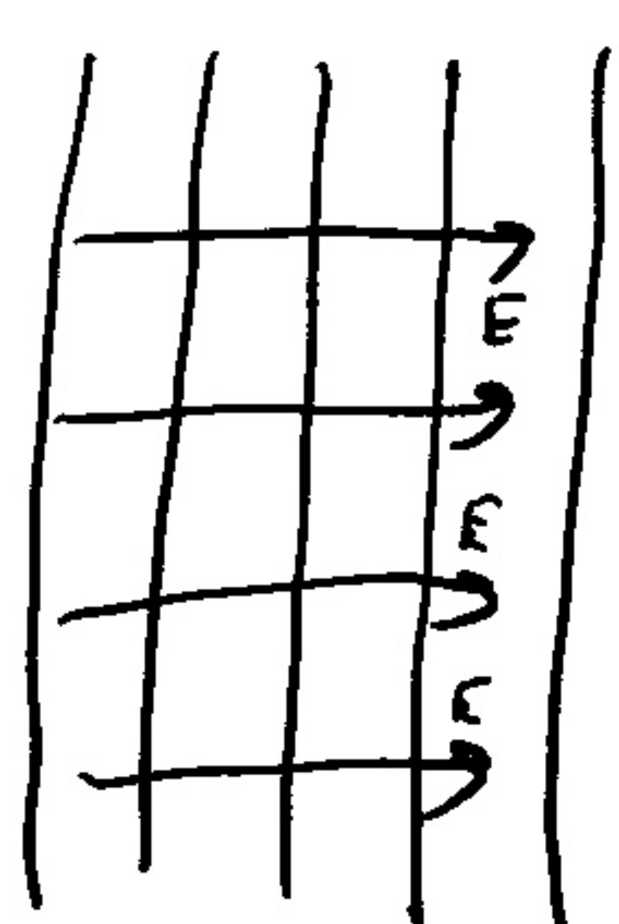
$$= -q\sigma/\varepsilon_0 \cdot 1/\sqrt{2}\cdot\int_i^f dl$$

$$= -q\sigma/\varepsilon_0 \cdot 1/\sqrt{2}\cdot l$$

$$L = l\cos 45^\circ$$
$$= l/\sqrt{2}$$

$$\Rightarrow l = \sqrt{2}\,L$$

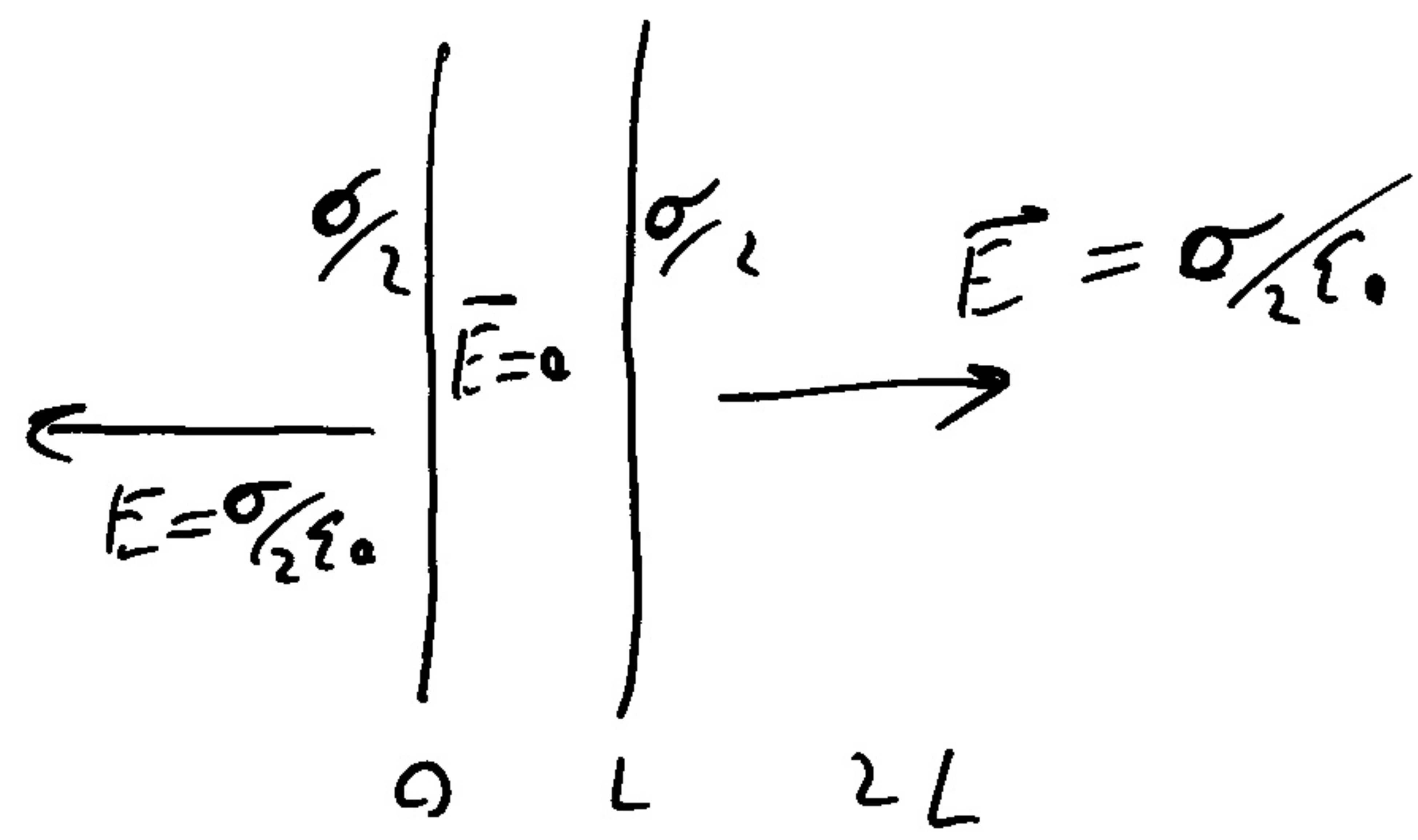
V = const.



$$\Rightarrow U_f - U_i = -q\sigma L/\varepsilon_0$$

- Same as before
- Since $\vec{E}$ normal to plates, the entire surface has a constant potential energy

- Potential near conducting sheet



say $V(2L) - V(0)$

$$= -\int_0^{2L} \vec{E}\cdot d\vec{x}$$

$$= -\int_0^{L} 0 \quad - \int_L^{2L} \sigma/2\varepsilon_0 \, dx$$

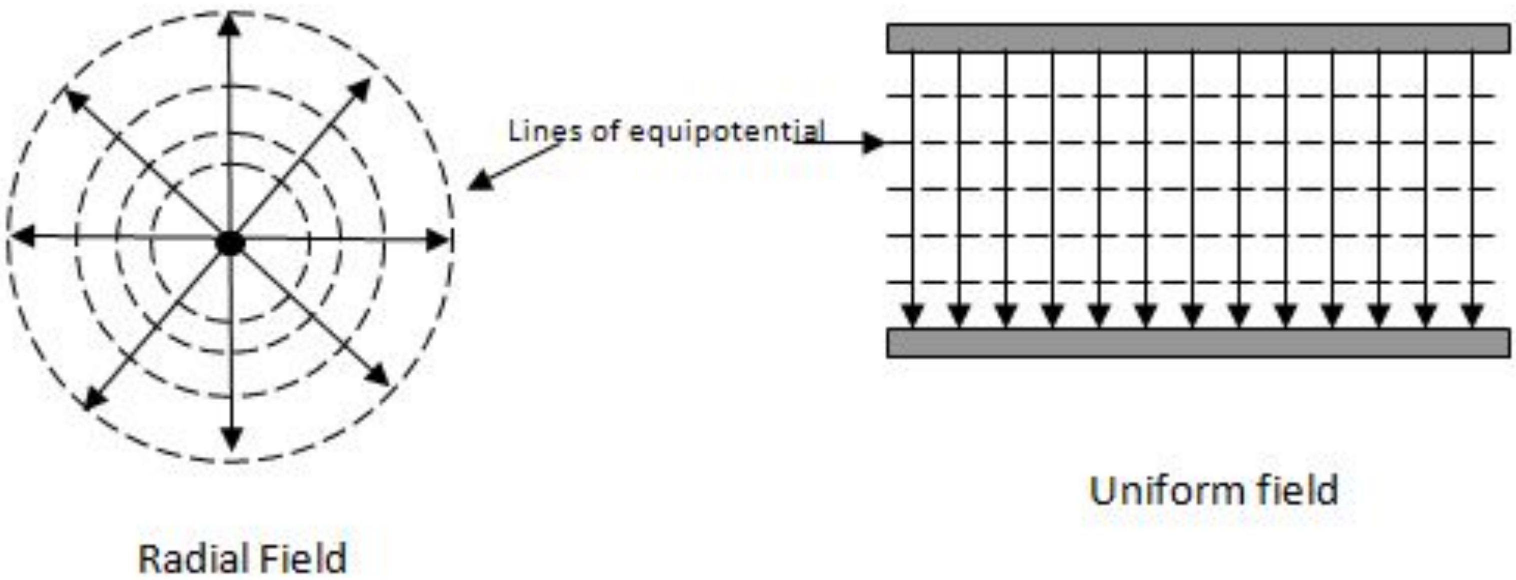
$$= -\sigma x/2\varepsilon_0 \Big|_L^{2L}$$

$$= \boxed{-\sigma L/2\varepsilon_0}$$

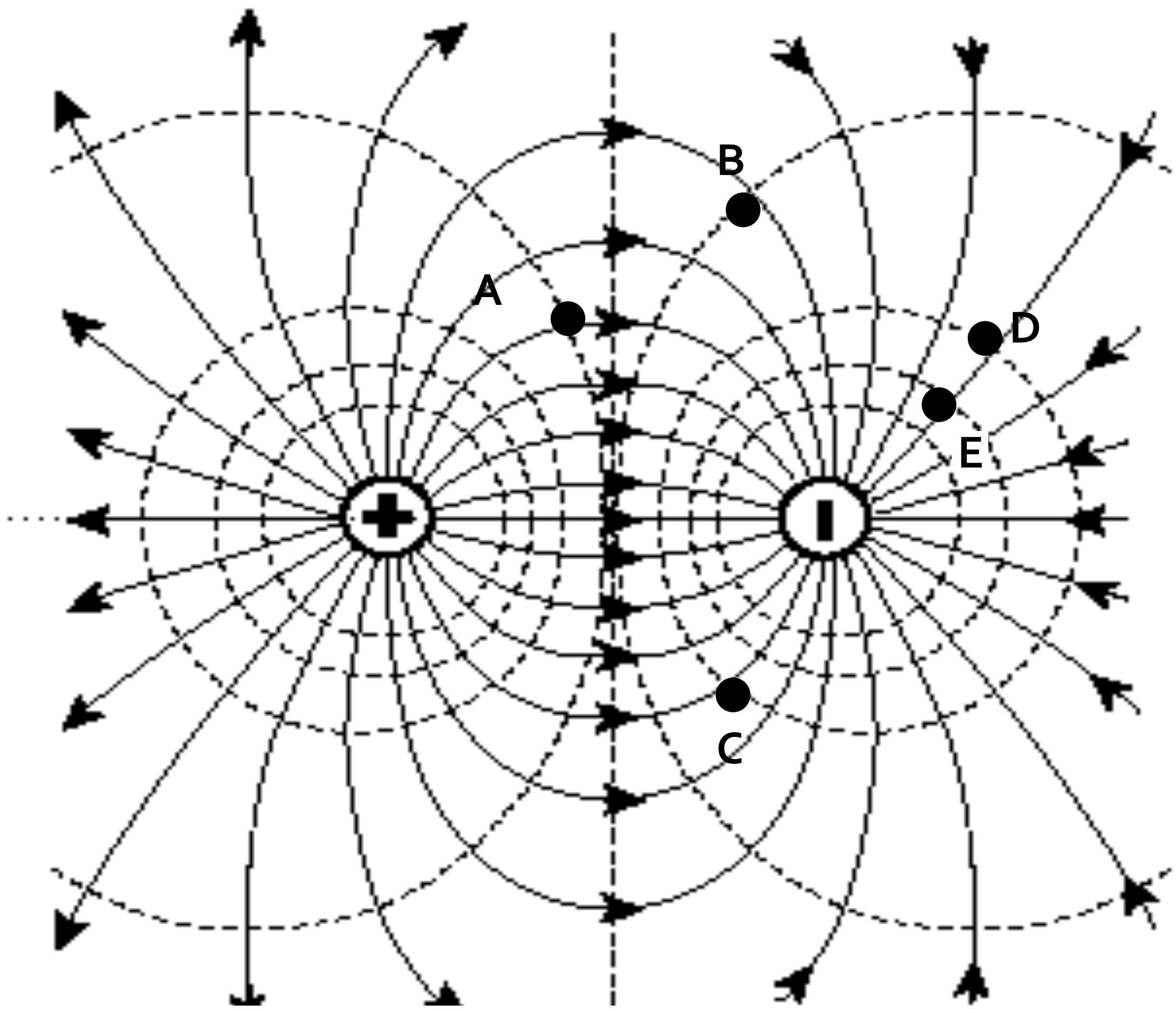
Electrical Topography

- Just as you can draw lines of constant elevation on a topographical map
- You can draw lines of constant electric potential on an electric potential map
- Electric field then points "downhill"
  - A positive charge wants to roll "downhill"
  - An negative charge wants to roll "uphill"!

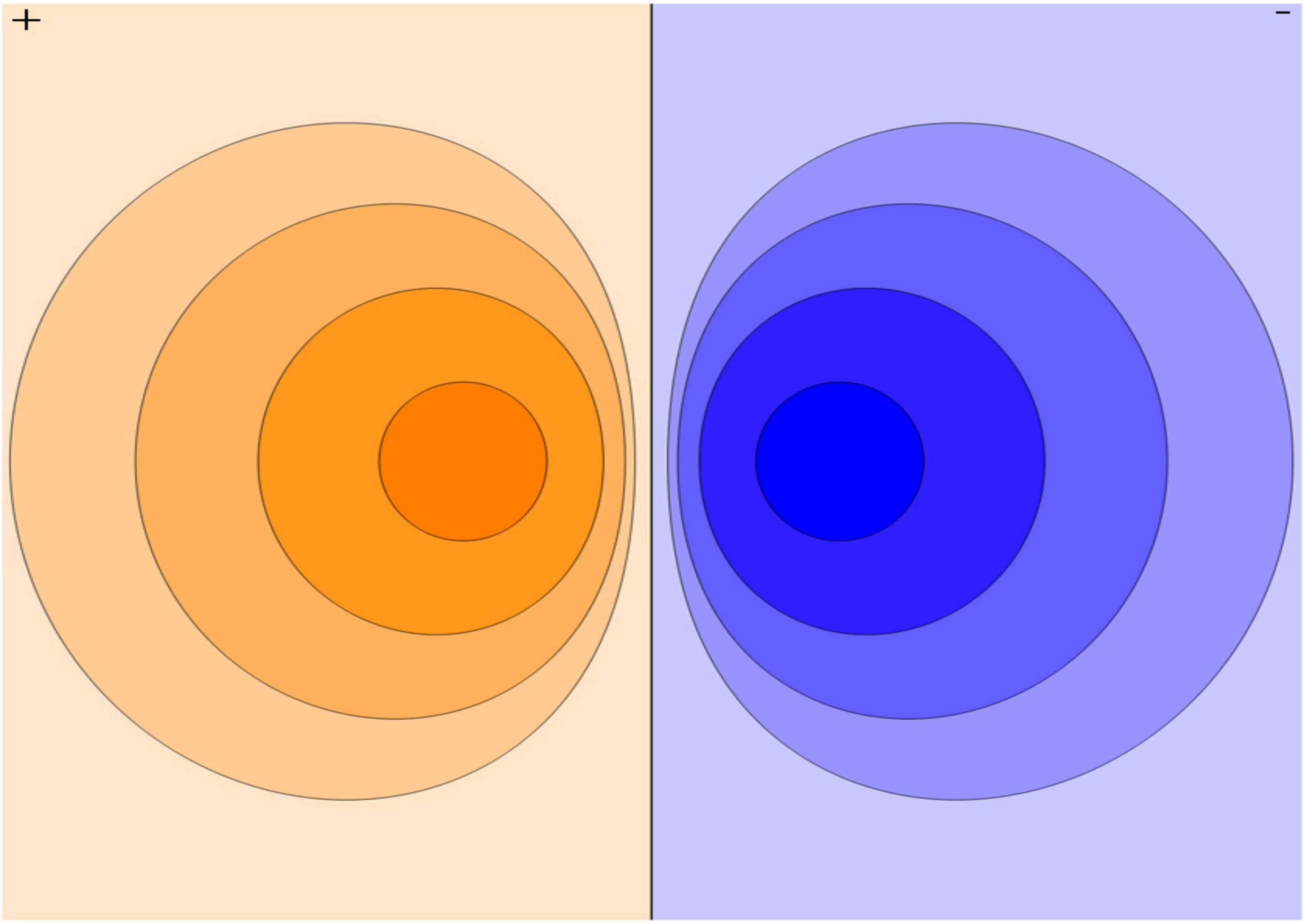
Potential Contours



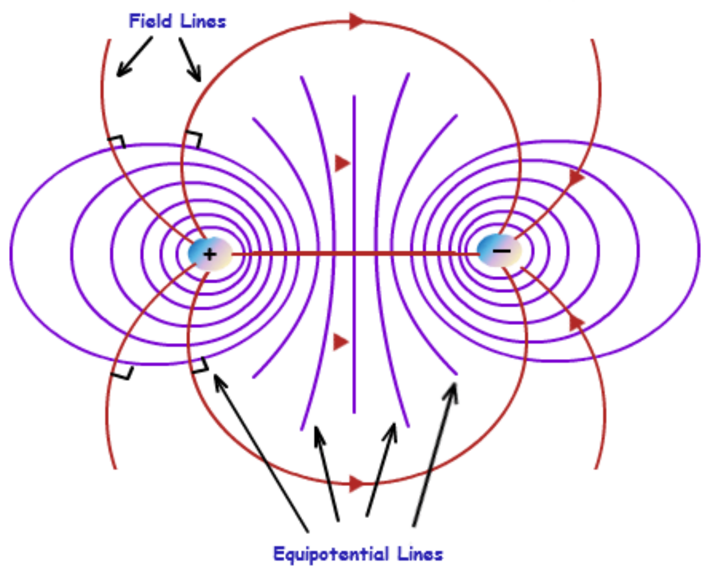
Potential Contours for Dipole

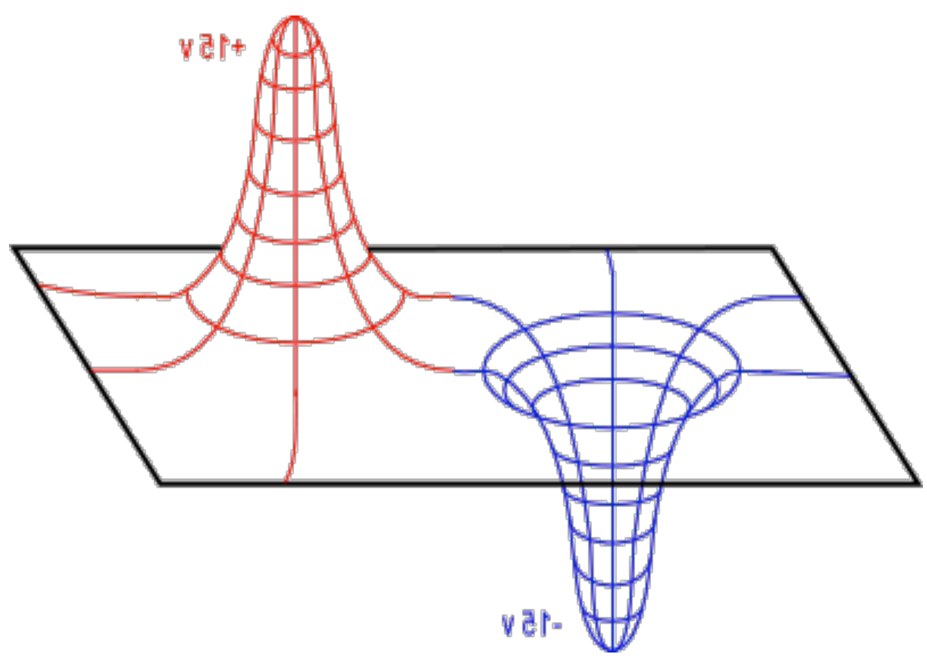


Potential Contours for Dipole



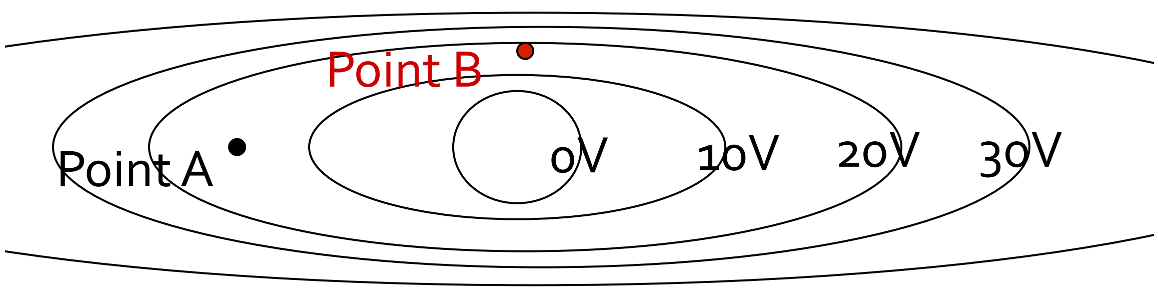
Potential Contours/Surfaces: Dipole





Concept Check

Drawn are a set of equipotential lines. Consider the electric field at points A and B. Which of the following statements is true?



A) $|E_A| > |E_B|$
B) $|E_A| < |E_B|$
C) $|E_A| = |E_B|$
D) Not enough information given.
E) None of the above

Reading Potential Maps

The distance between two equipotential surfaces, represented by the lines, indicates how rapidly the potential changes. The smallest distances correspond to the location of the greatest rate of change and therefore to the largest values of the electric field.