Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

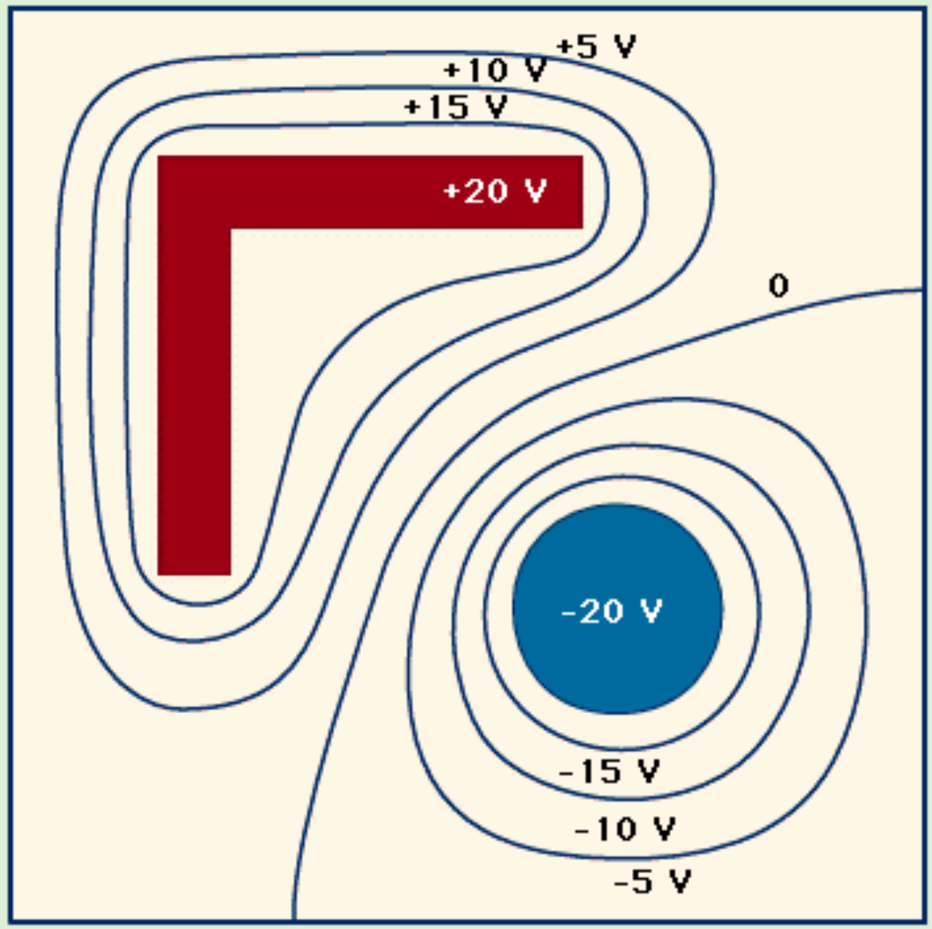
Electric Field and Potential

$$\overline{E} = -\nabla V$$

$$V_{BA} = V_B - V_A = -\int_A^B \overline{E} \bullet d\overline{\ell}$$

Reading Potential Maps

The distance between two equipotential surfaces, represented by the lines, indicates how rapidly the potential changes. The smallest distances correspond to the location of the greatest rate of change and therefore to the largest values of the electric field.

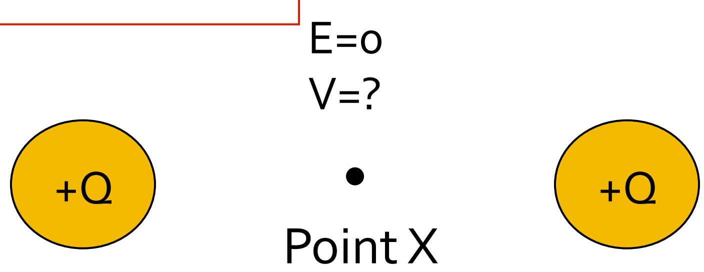


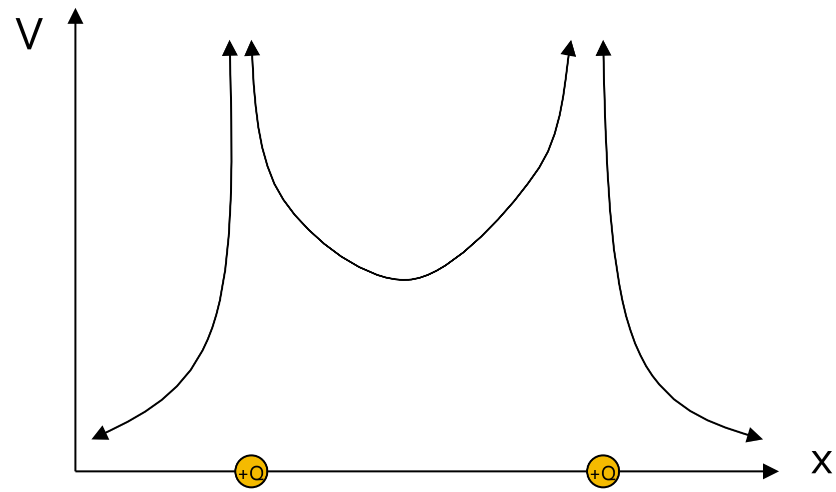
Concept Check

Two identical charge, +Q and +Q, are fixed in space. The electric potential (V) at the point X midway between the charges is:

A) Zero

B) Non-Zero





Potential of Two Charges

$\dot{Q}$ $\qquad\qquad$ $\dot{Q}$

$$V_1 = Q/4\pi\varepsilon_0 r_1 \qquad V_2 = Q/4\pi\varepsilon_0 r_2$$

$$V_{tot} = V_1 + V_2$$

Write coordinate-independent

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{|\vec{r}-\vec{r}_1|} + \frac{Q_2}{|\vec{r}-\vec{r}_2|}\right]$$

Say on x-axis

$$x_1 = 0 \quad , \quad x_2 = d$$

What is $V(d/2)$?

$$V = Q/4\pi\varepsilon_0 \left[\frac{1}{|d/2-0|} + \frac{1}{|d/2-d|}\right]$$

$$= Q/4\pi\varepsilon_0 \left(2/d + 2/d \right]$$

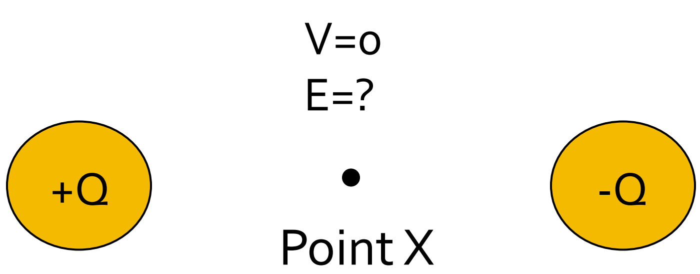
$$= Q/\pi\varepsilon_0 d$$

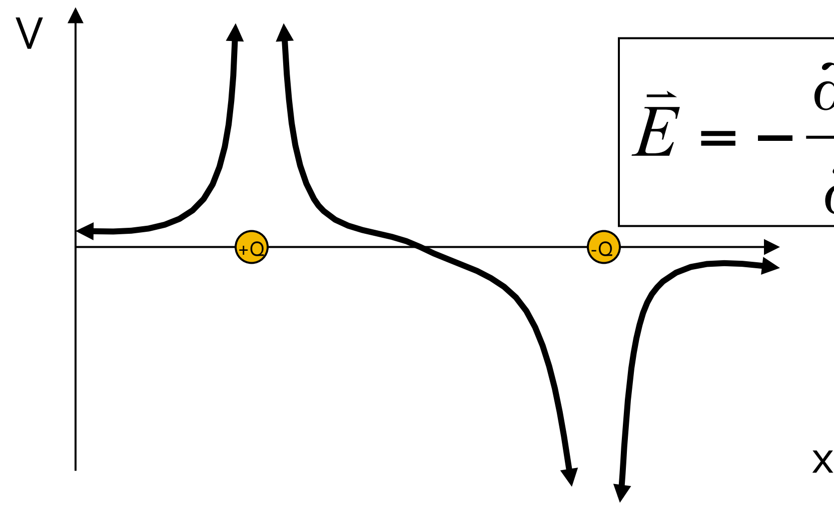
Concept Check

Two charges, +Q and -Q, are fixed in space. The electric field at the point X midway between the charges is:

A) Zero

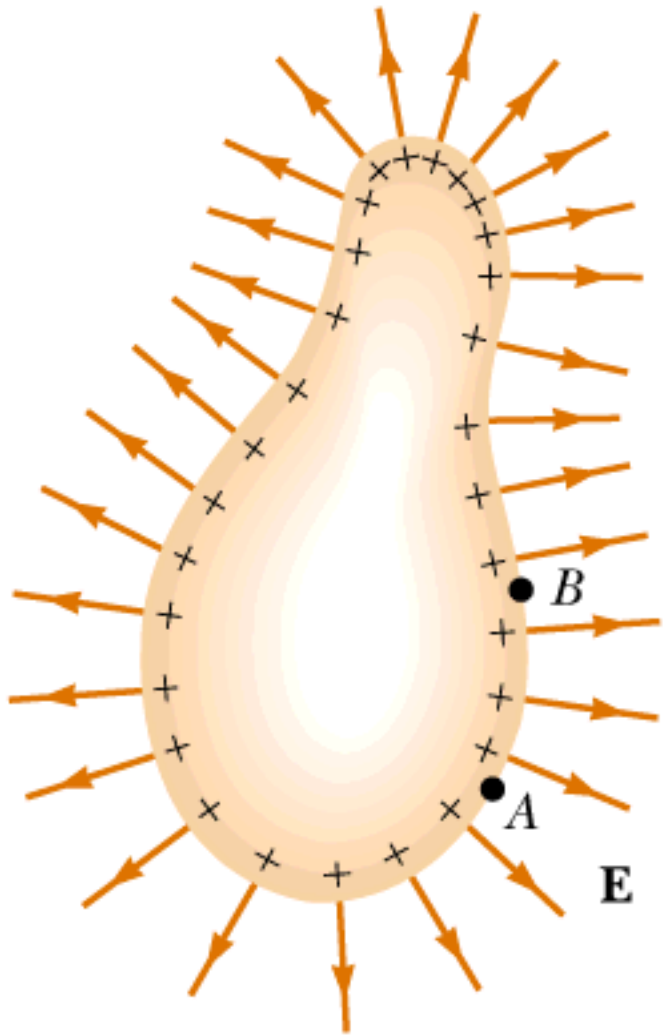
B) Non-Zero



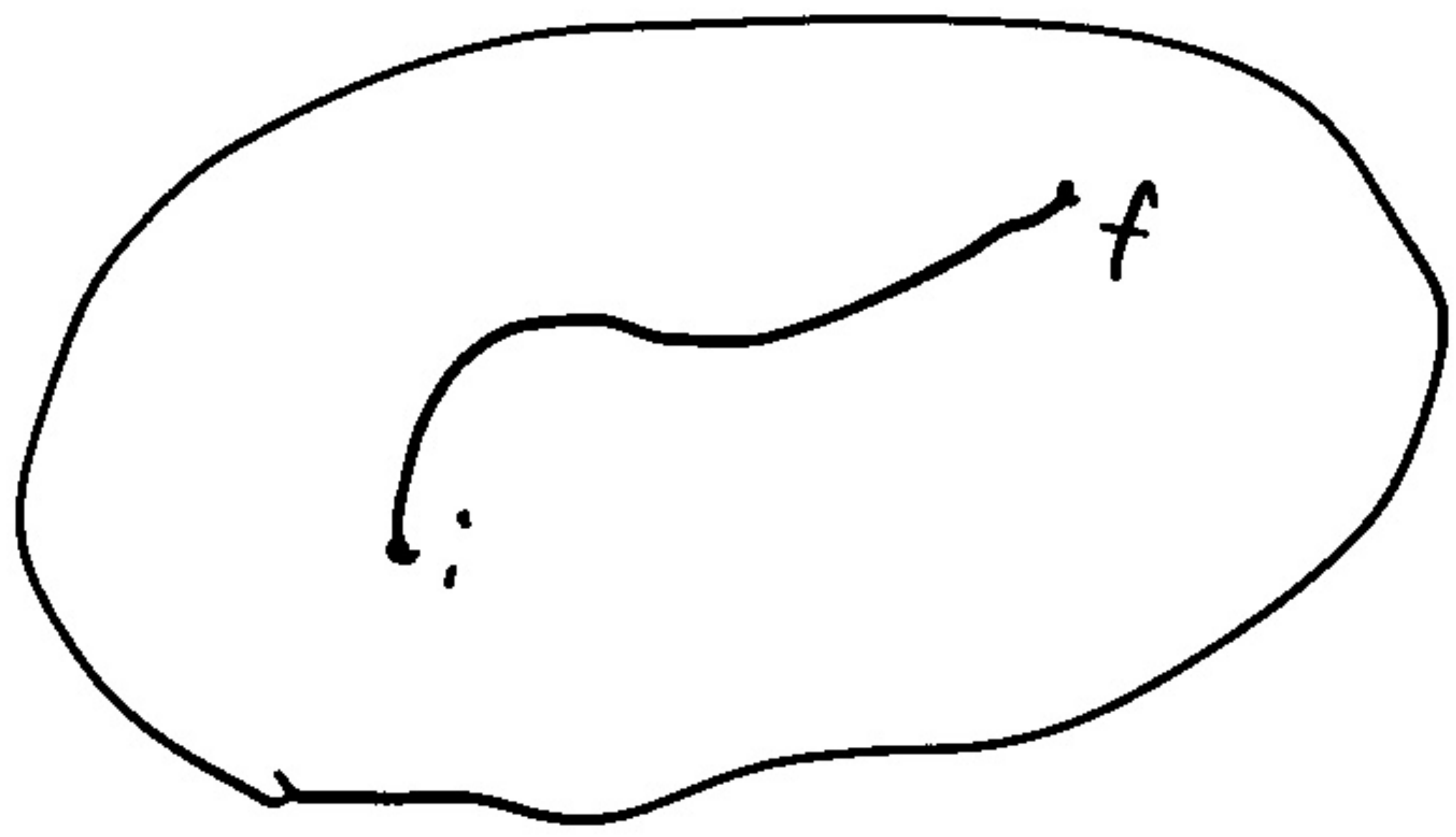


$$\vec{E} = -\frac{\partial V}{\partial x}\hat{x}$$

Electric Potential of A Conductor



Conductors



pick any path

$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$
$$= 0 \quad \text{since } \vec{E} = 0$$

so V = constant in conductor

- This means the surface of a conductor is an equipotential.

- $\vec{E} = -\nabla V$ must be perpendicular to equipotentials

- So $\vec{E}$ perpendicular to surface of conductor

Principle of superposition:

q_1 $\qquad$ q_2

$$V_{total} = V_1(\vec{r}) + V_2(\vec{r})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\vec{r}-\vec{r}_1|} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{|\vec{r}-\vec{r}_2|}$$

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2$$

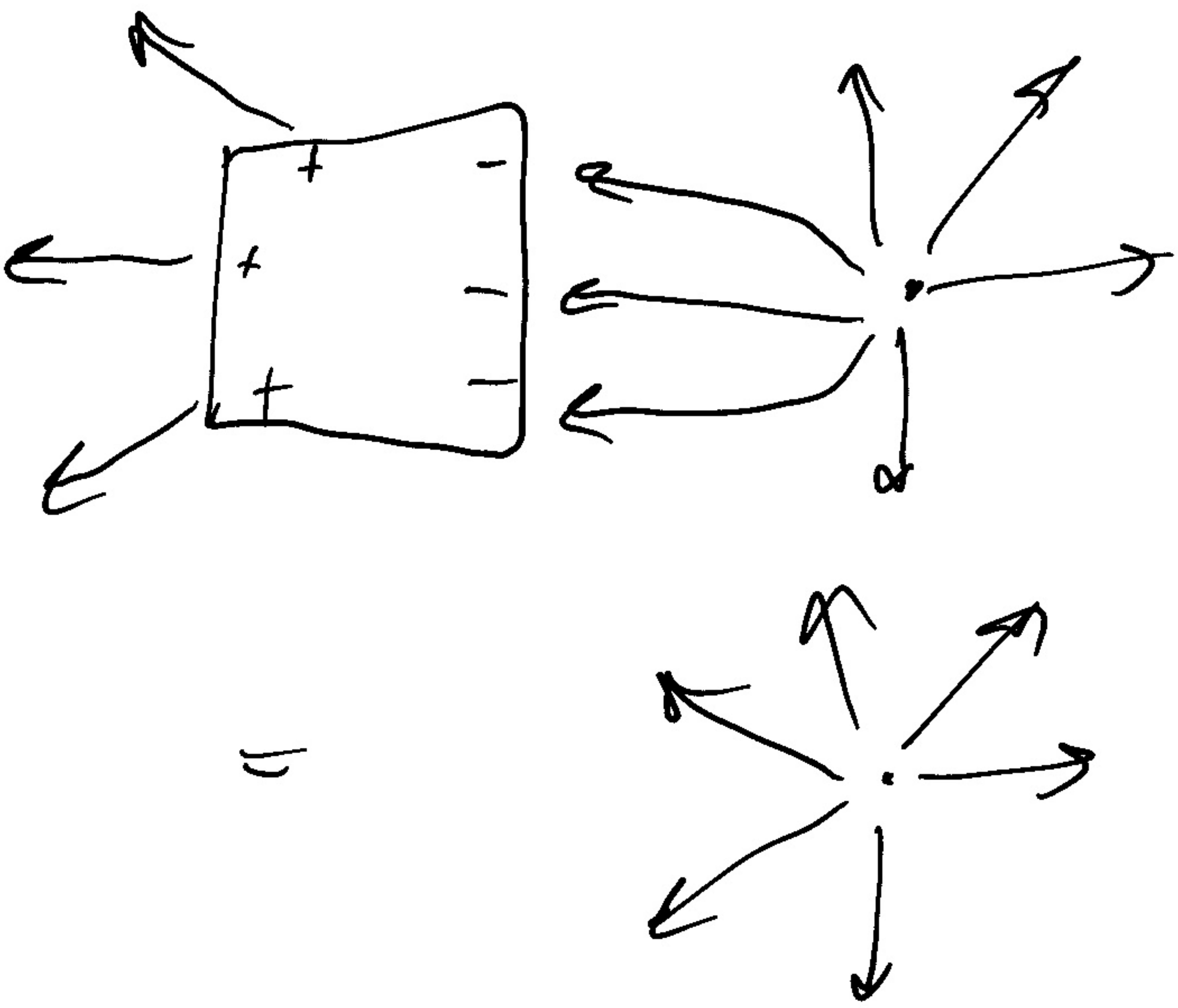
$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} + \frac{1}{4\pi\varepsilon_0}\frac{q_2(\vec{r}-\vec{r}_2)}{|\vec{r}-\vec{r}_2|^3}$$

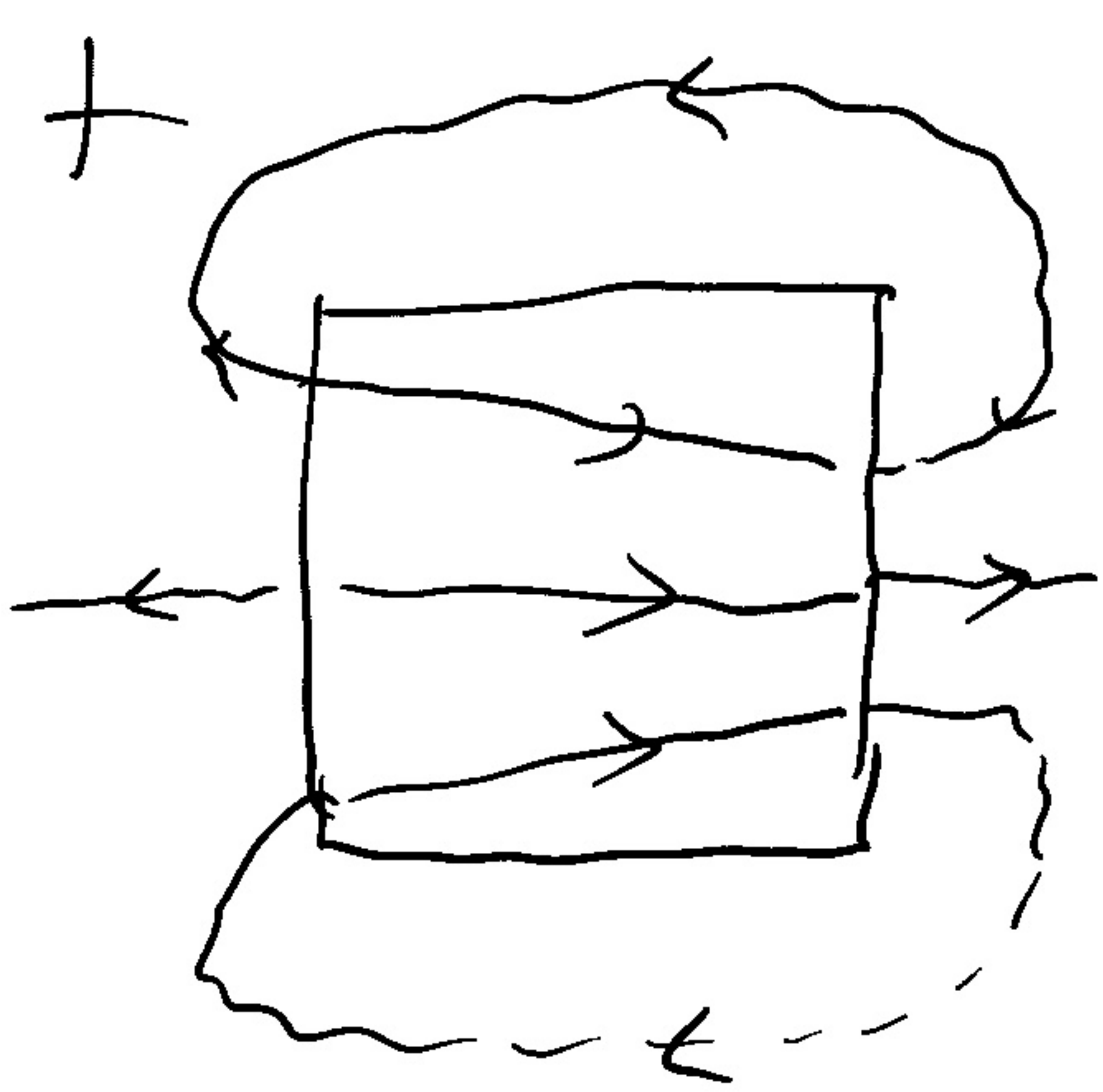
What about:

Field of uncharged conductor $= 0$. Field of $+q = q/4\pi\varepsilon_0 r^2 \,\hat{r}$

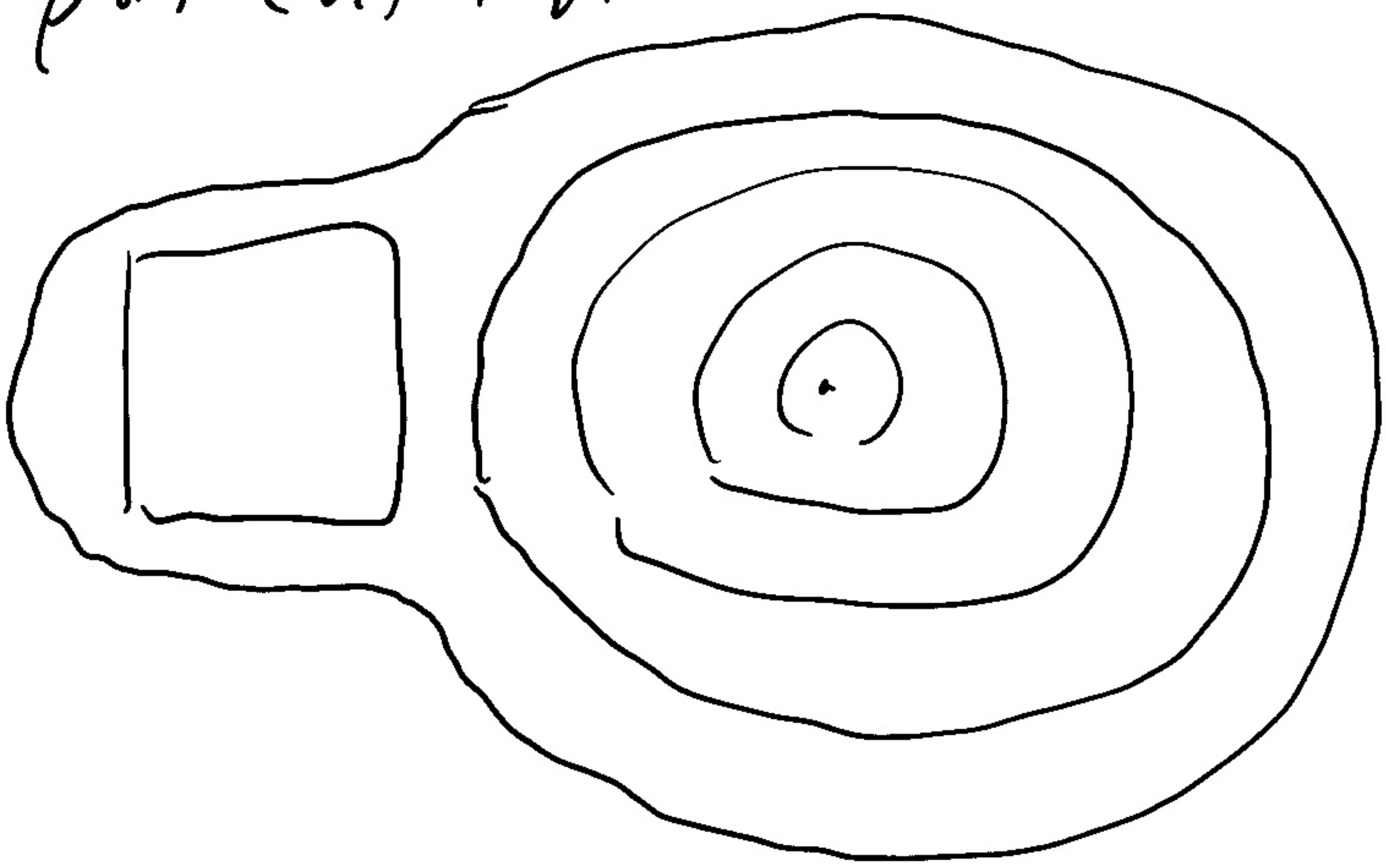
Field of point charge + conductor $\neq$ field of point charge + uncharged conductor.

Induced charge must be included





potential



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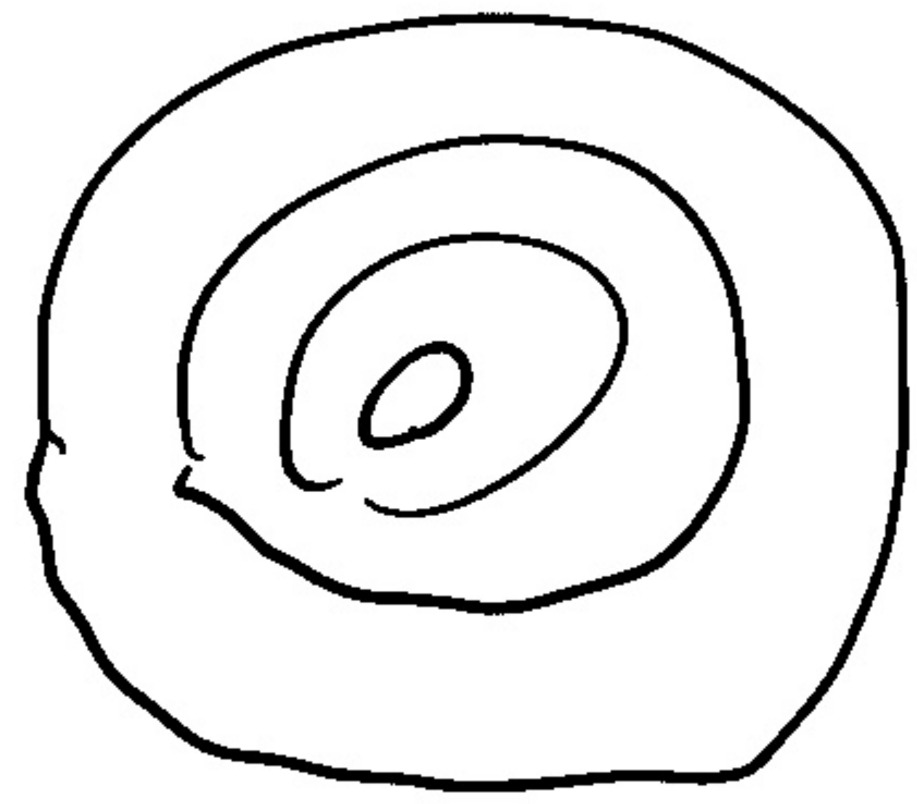

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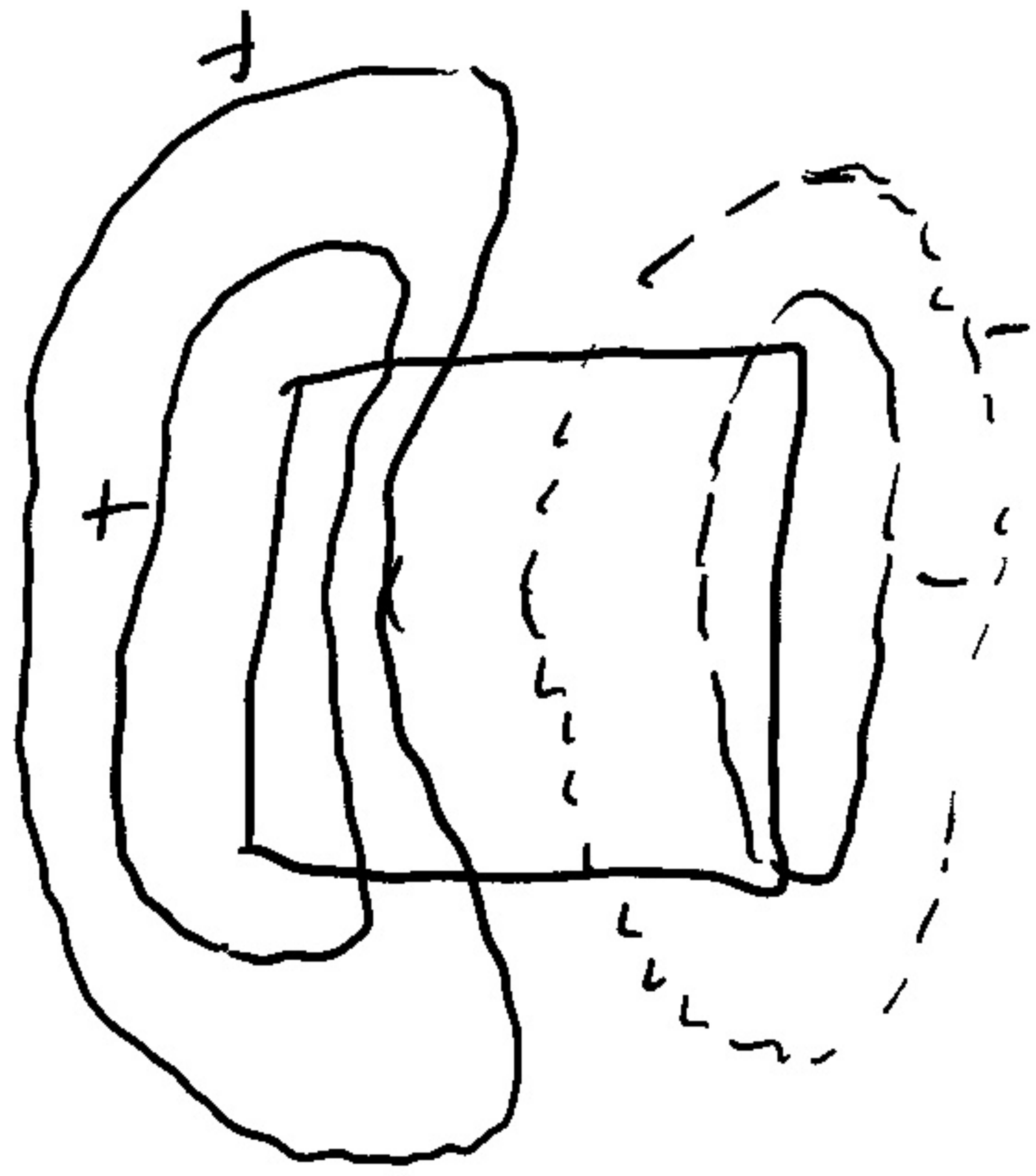


Concept Check

A positive and negative charge are held a distance R apart and are then released.

The two particles accelerate toward each other as a result of the Coulomb attraction.

As the particles approach each other, the total potential energy of the system…

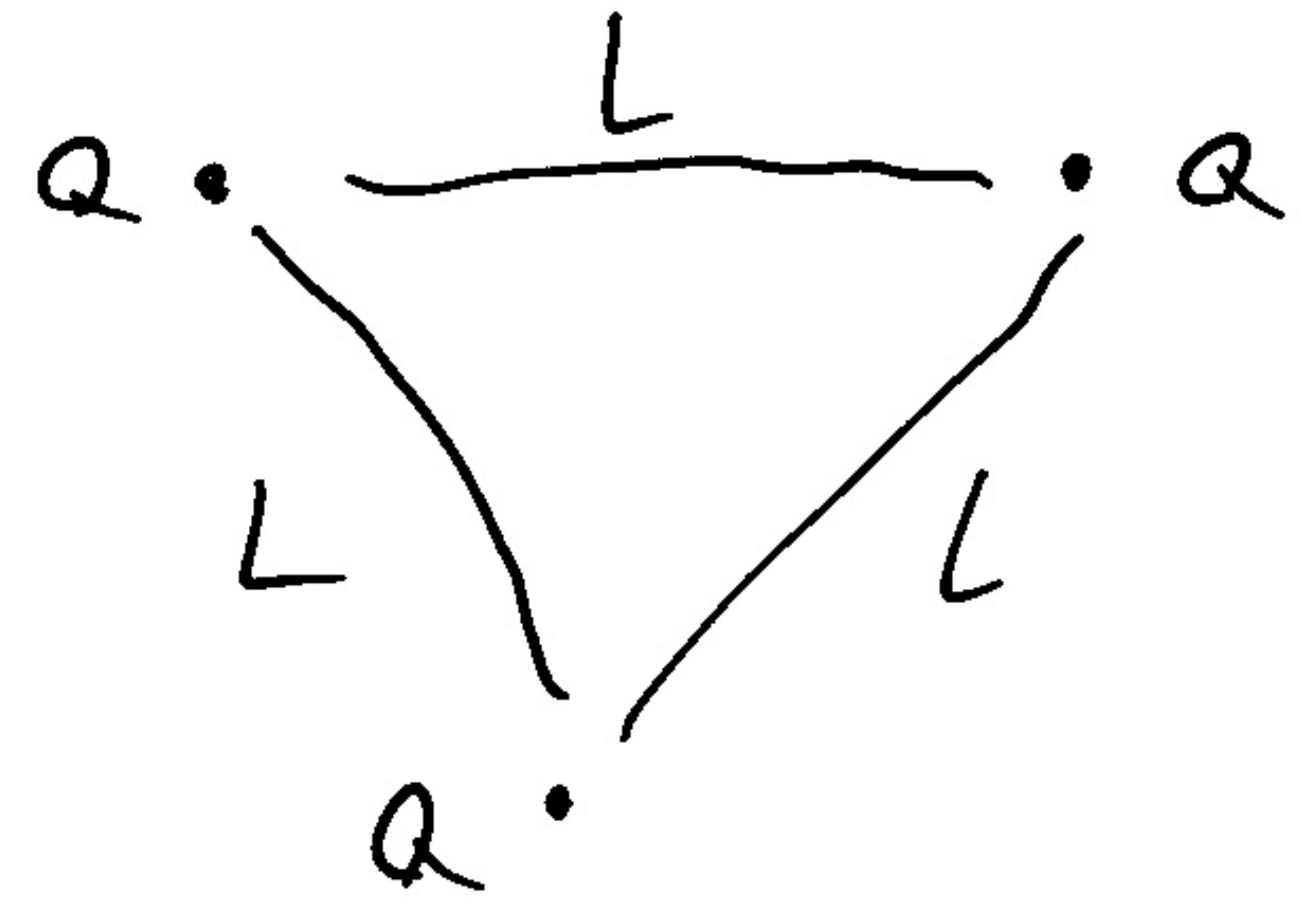
A: increases

B: decreases

C: stays the same

D: ??

Potential Energy of A System of Charges



- Bring first one from infinity - First one's free!

$$\Delta U_1 = 0$$

- Bring second one from infinity

V of 1st. charge $V_1 = Q_1 / 4\pi\varepsilon_0 |\vec{r} - \vec{r}_1|$

$$\Delta U_2 = Q_2 \Delta V$$
$$= Q_2 (V - V(\infty))$$
$$= Q_2 V_1$$
$$= Q_2 Q_1 / 4\pi\varepsilon_0 |\vec{r}_2 - \vec{r}_1|$$

- Bring third from infinity

V of 1st. and 2nd. charges

$$V_{12} = Q_1 / 4\pi\varepsilon_0 |\vec{r} - \vec{r}_1| + Q_2 / 4\pi\varepsilon_0 |\vec{r} - \vec{r}_2|$$

so $\Delta U_3 = Q_3 \left[\frac{Q_2}{4\pi\varepsilon_0 |\vec{r}_3 - \vec{r}_2|} + \frac{Q_1}{4\pi\varepsilon_0 |\vec{r}_3 - \vec{r}_1|} \right]$

$$U_{total} = \Delta U_1 + \Delta U_2 + \Delta U_3$$

$$U_{total} = \frac{1}{4\pi\varepsilon_0} \left[\frac{Q_1 Q_2}{|\vec{r}_2 - \vec{r}_1|} + \frac{Q_1 Q_3}{|\vec{r}_3 - \vec{r}_1|} + \frac{Q_2 Q_3}{|\vec{r}_3 - \vec{r}_2|} \right]$$

For this case

$$U = \frac{3 Q^2}{4\pi\varepsilon_0 L}$$

For more particles, need to count all pairs

For 4 we have 6 pairs
12, 13, 14, 23, 24, 34

- Change potential energy by changing distance
- or by changing charge!

Potential Energy of a System

- Potential energy of a system of particles:
  - Defined as the work needed to assemble a system, starting with all particles at infinity
  - To calculate, start with one particle, and bring in the rest one at a time