Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Review: Five Most Important Things

1. Know what gravitational and electrostatic fields and potentials are and how to compute them directly from charge/mass distributions

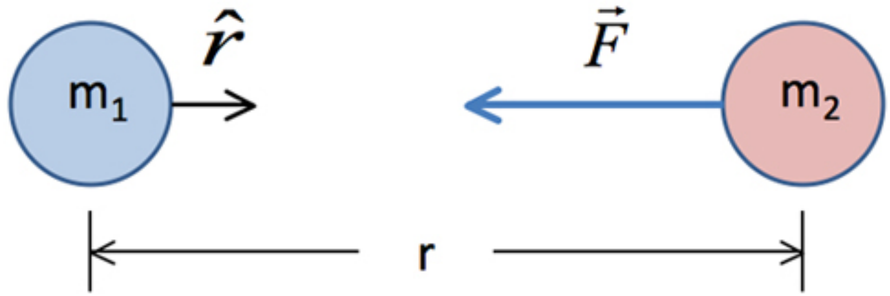
2. Know how to use Gauss's law to compute the fields from symmetric charge/mass distributions

3. Know how to go from field to potential and back

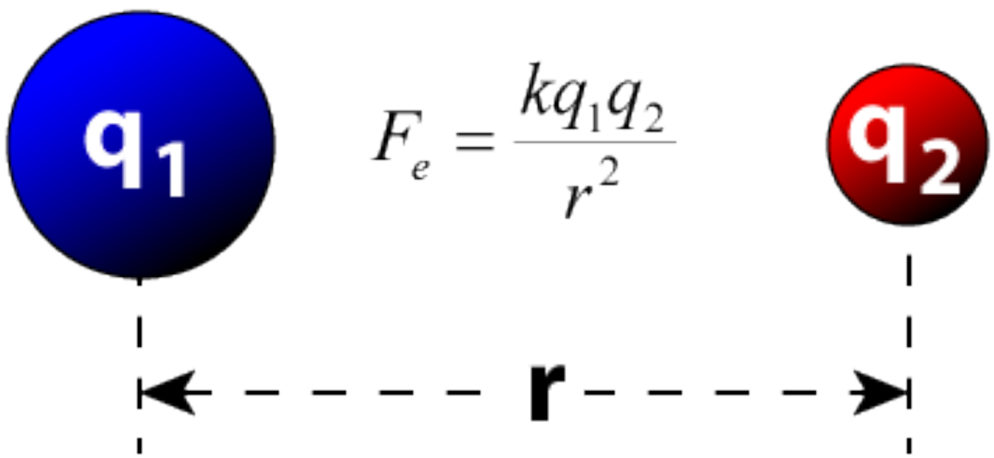
4. Know how to use conservation of energy

5. Know how to deal with conductors

Newton's and Coulomb's Laws



$$\vec{F} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

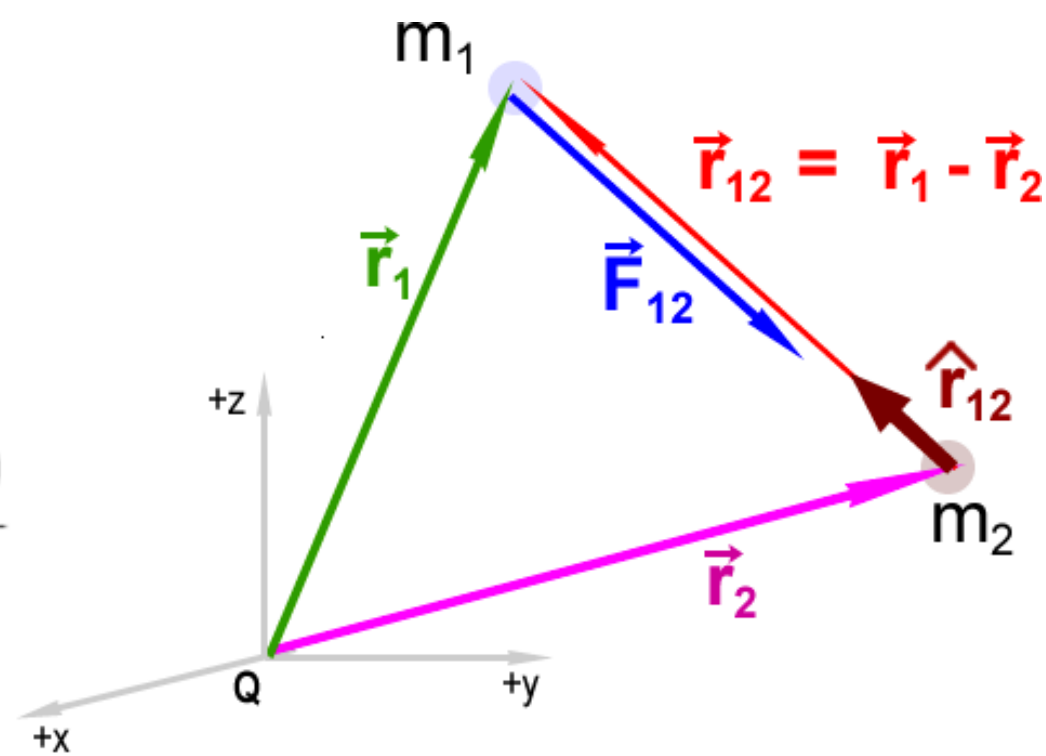


$$F_e = \frac{kq_1q_2}{r^2}$$

Explicit Vector Form

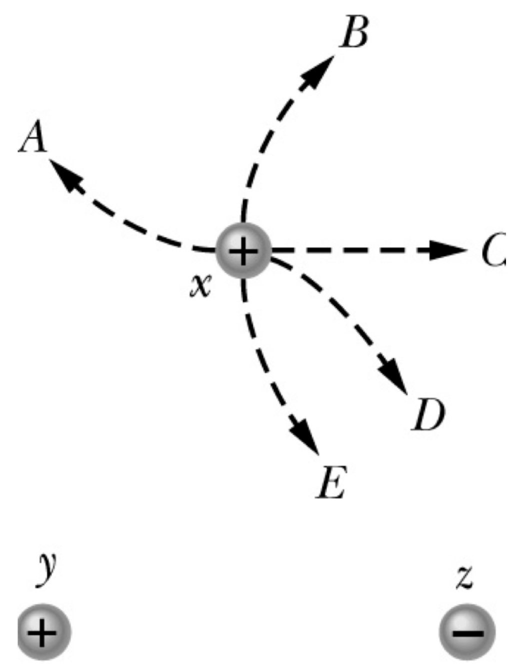
$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{|\mathbf{r}_{12}|^2}\,\hat{\mathbf{r}}_{12}$$

$$F_{12} = \frac{G m_1 m_2 (\mathbf{r_2} - \mathbf{r_1})}{|\mathbf{r_2} - \mathbf{r_1}|^3}$$

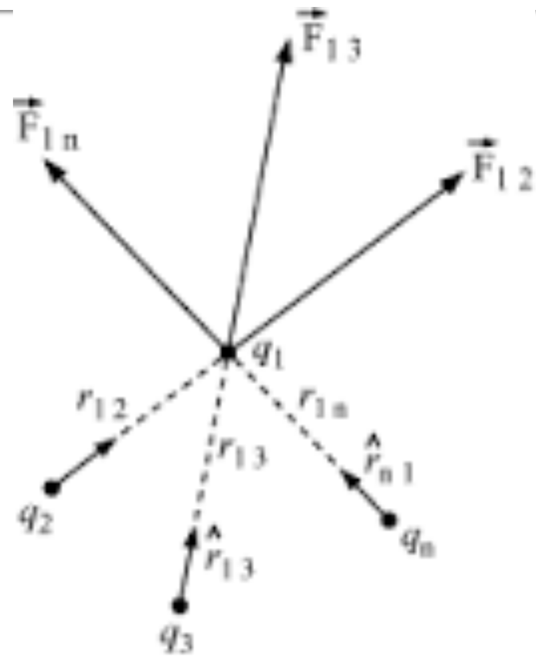


Concept Check

Q25) The figure shows three small spheres that have charges of equal magnitudes and rest on a frictionless surface. Spheres y and z are fixed in place and are equally distance from sphere x. If sphere x is released from rest, which of the five paths shown will it take?



Multiple Sources



$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \ldots + \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathbf{r}}_{13} + \ldots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathbf{r}}_{1n}\right]$$

$$= \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{1i}^2}\hat{\mathbf{r}}_{1i}$$

Gravitational and Electric Fields

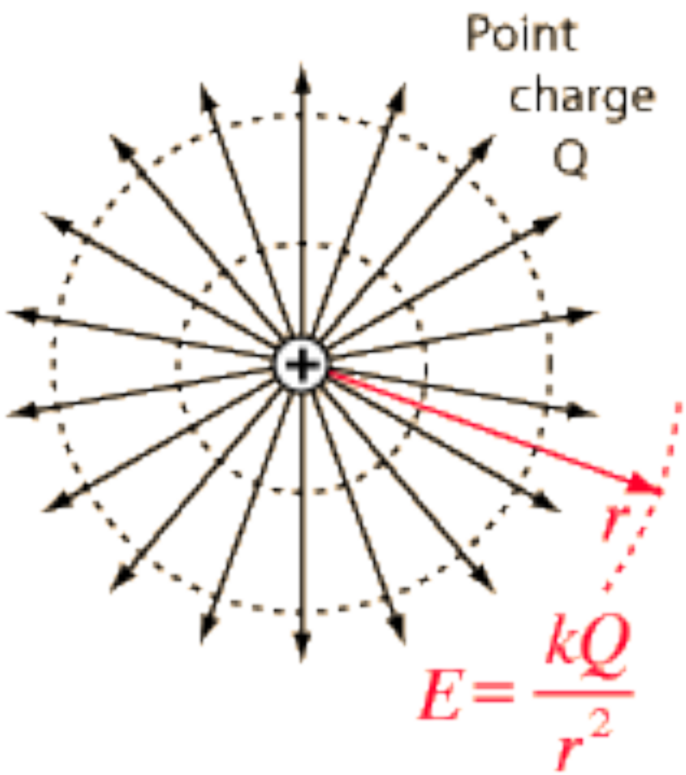
Electric field in N/C or volts/m.

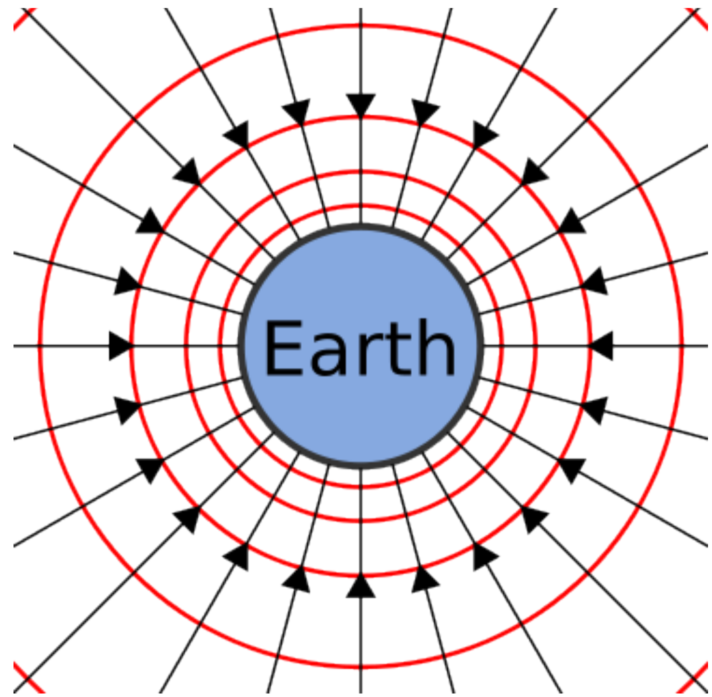
$$\vec{E} = \frac{\vec{F}}{q}$$

electric force in Newtons

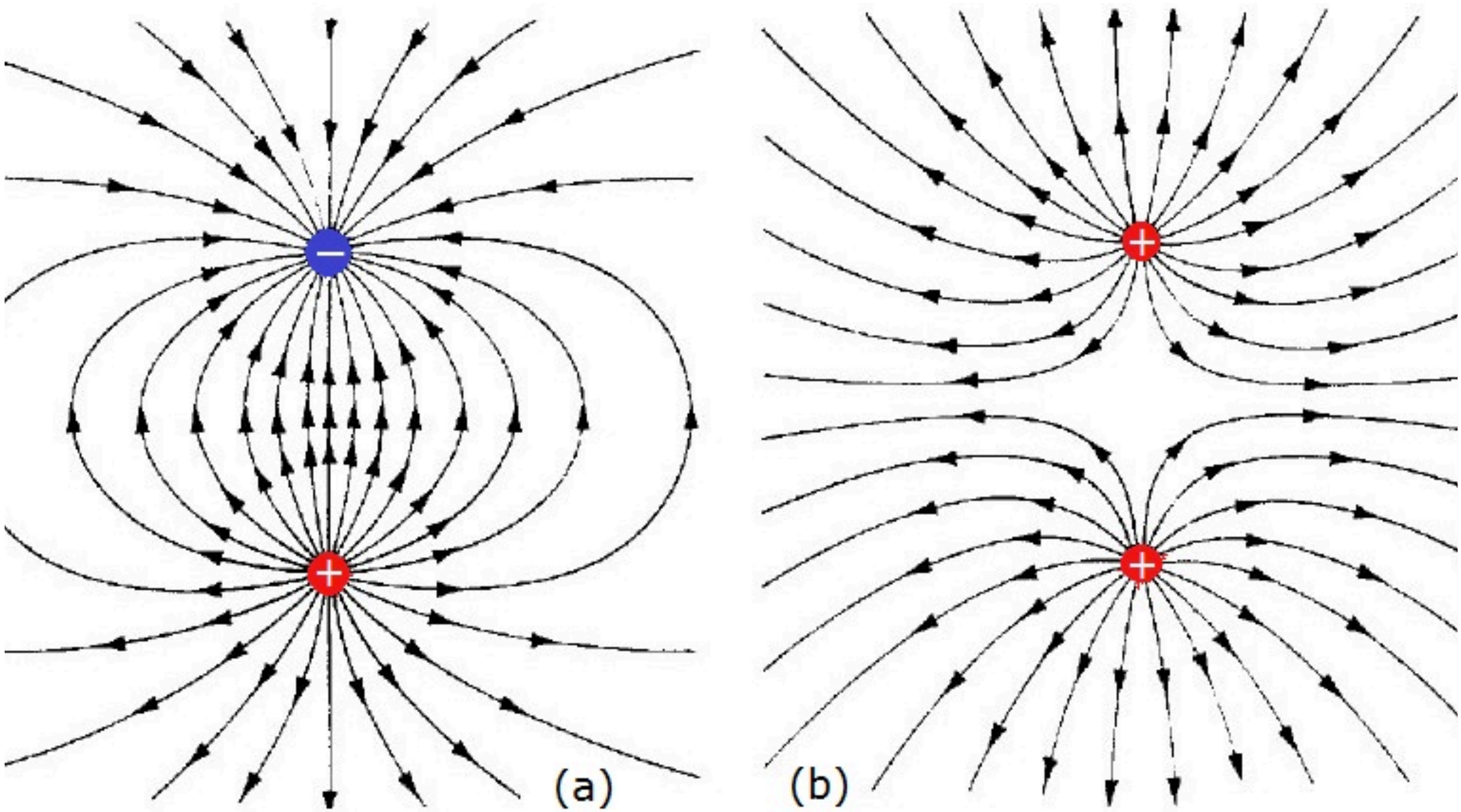
charge in Coulombs

$$\vec{g} = \frac{\vec{F}_g}{m} = -G\frac{mM}{mr^2}\hat{r} = -\frac{GM}{r^2}\hat{r}$$





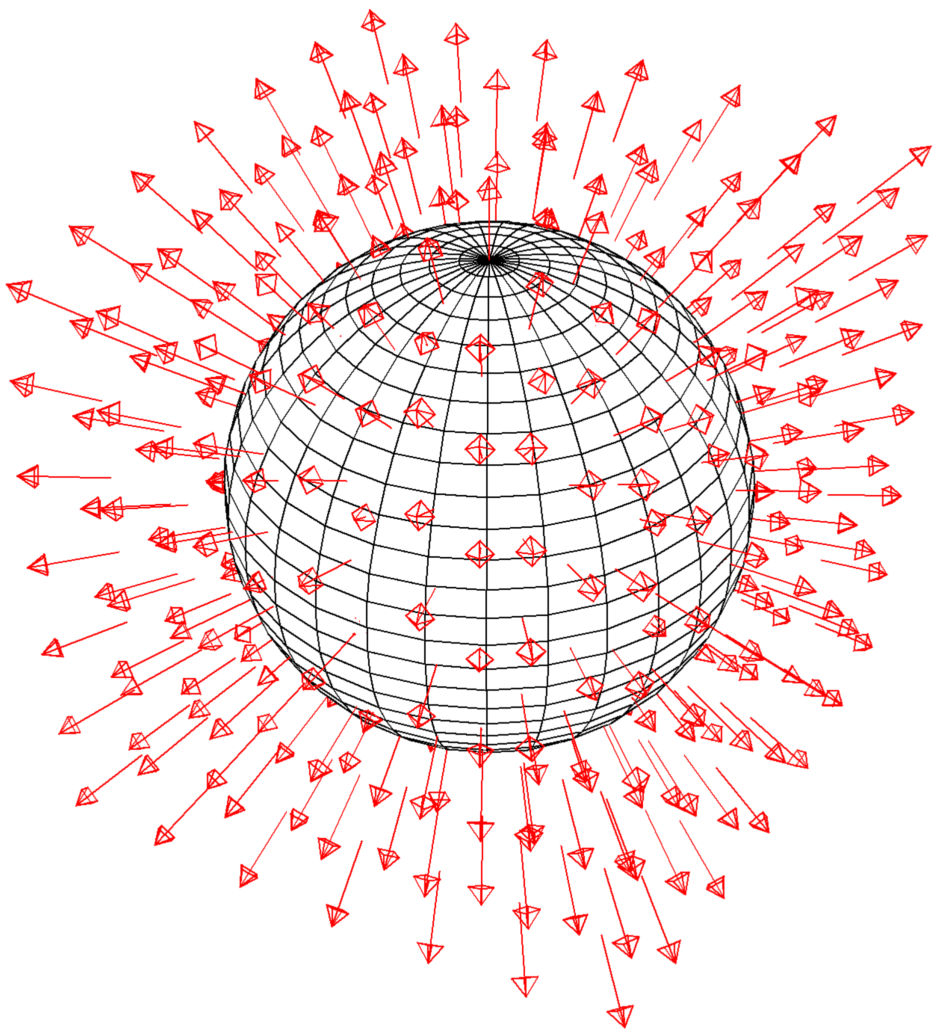
Field Lines



Electric Field Lines

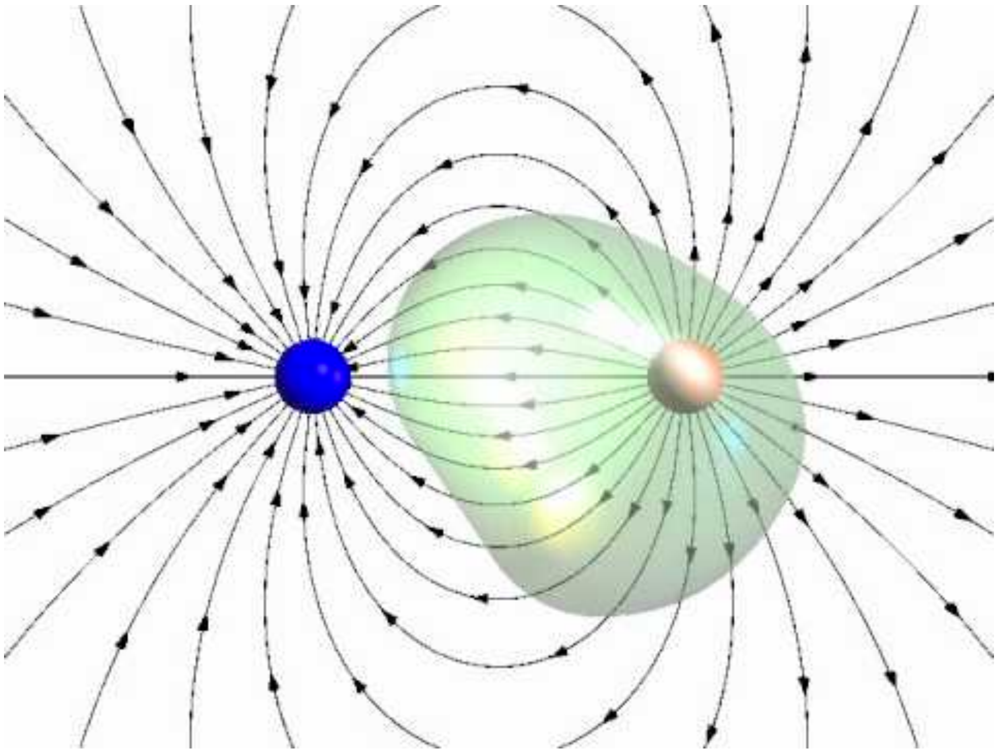
- Point in the direction tangent to the electric field [the direction of the force on a positive test charge]
- Start and end only at charges
  - Out from +Q, into -Q
  - The more charge Q, the more field lines!
- Are more closely spaced where fields are stronger

Field Lines Through Surface



The number of field lines through a closed surface does not depend on the shape of the surface!

Electric Flux and Gauss's Law



$$\Phi_e = \oint \mathbf{E} \bullet d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

$\mathbf{E}$ = Electric Field

$\mathbf{A}$ = Area Element

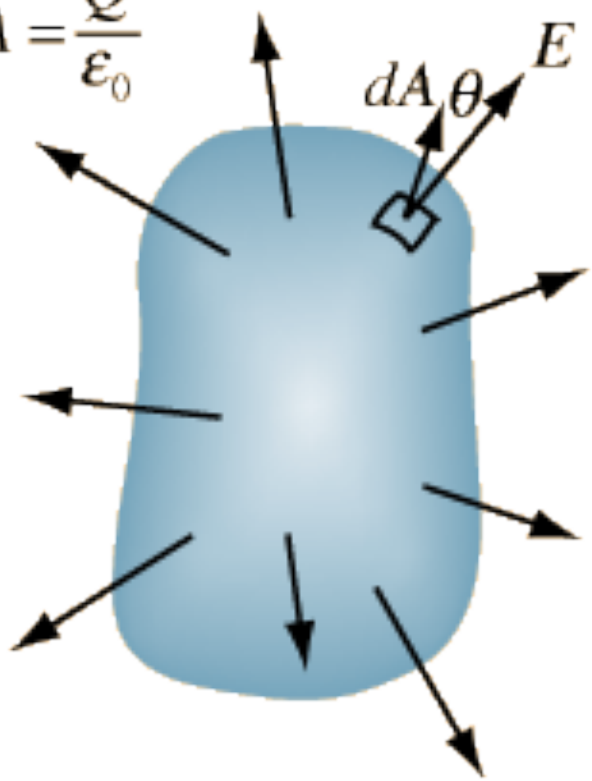
Q = Enclosed Charge

Φ_e = Electric Flux

ε_0 = Permittivity

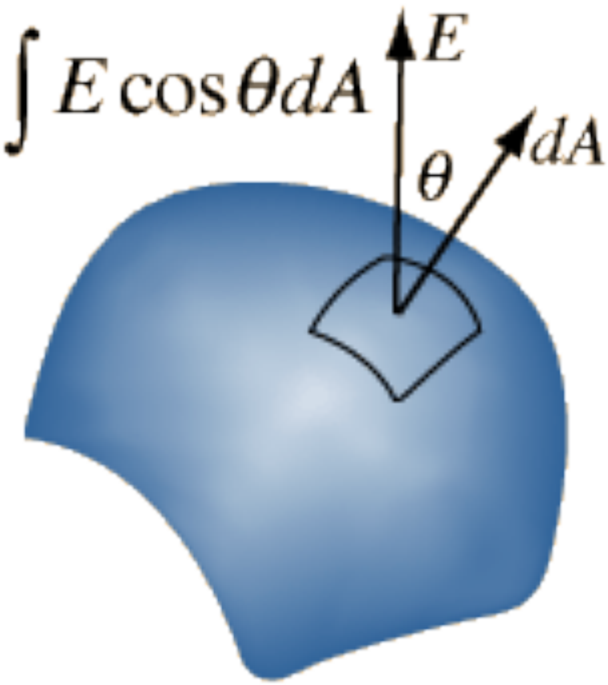
Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$



Electric flux:

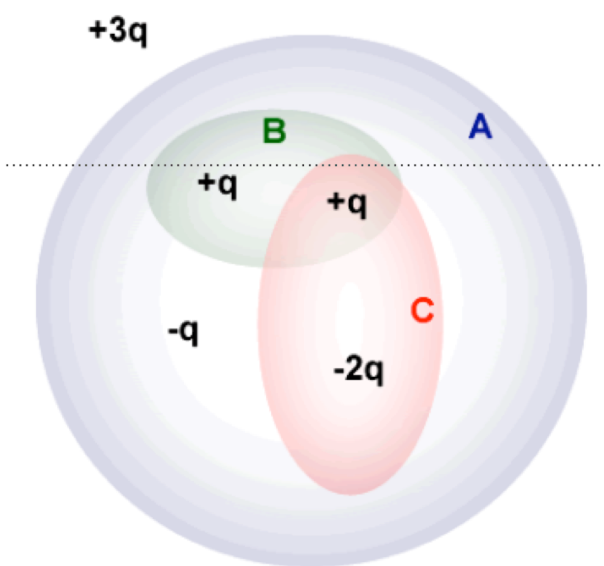
$$\Phi = \int E \cos\theta \, dA$$



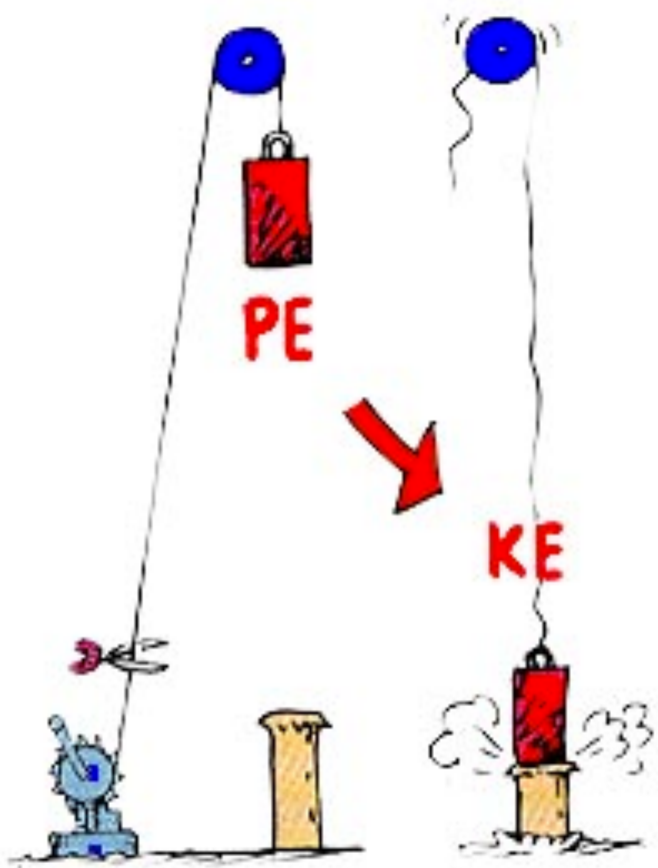
Concept Check

Q9) Rank the closed surfaces shown in increasing order of electric flux.

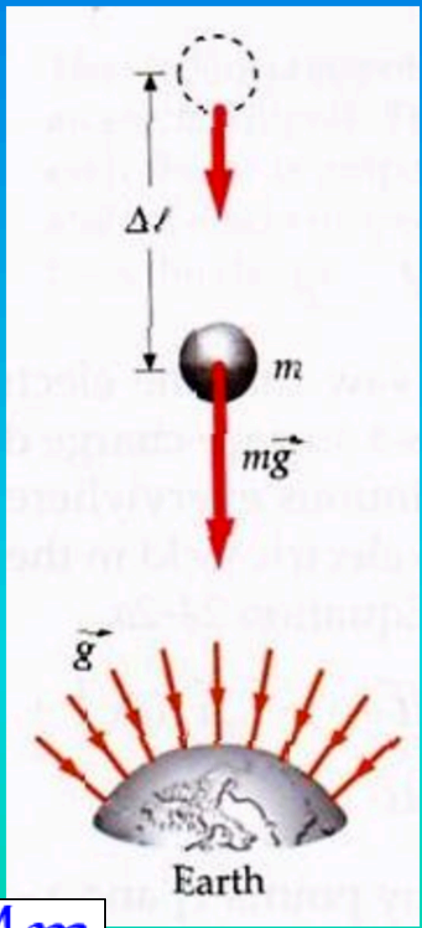
1) A < B < C
2) C < B < A
3) B < C < A
4) A = C < B

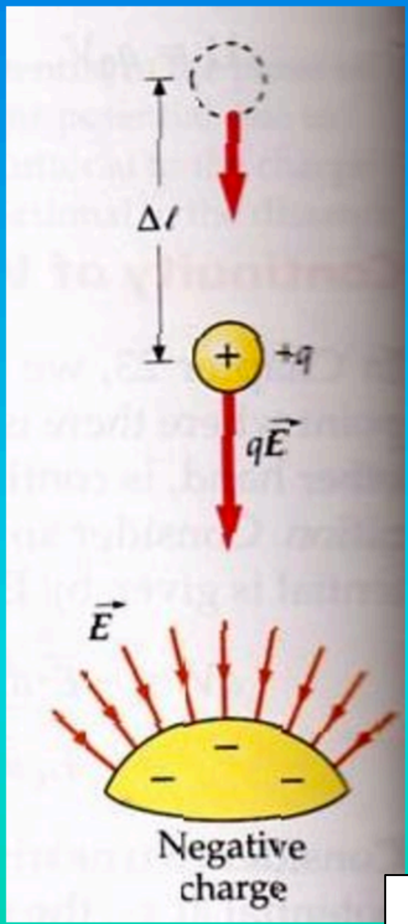


Potential Energy



Potential Energy





$$U = \frac{-GMm}{r}$$

$$U = k\frac{q_1 q_2}{r}$$

Potential Energy

$$W = \vec{F} \cdot \Delta\vec{r} = |\vec{F}||\Delta\vec{r}|\cos\theta$$

$$\Delta U = U_f - U_i = +W_{ext} = -W_{field}$$

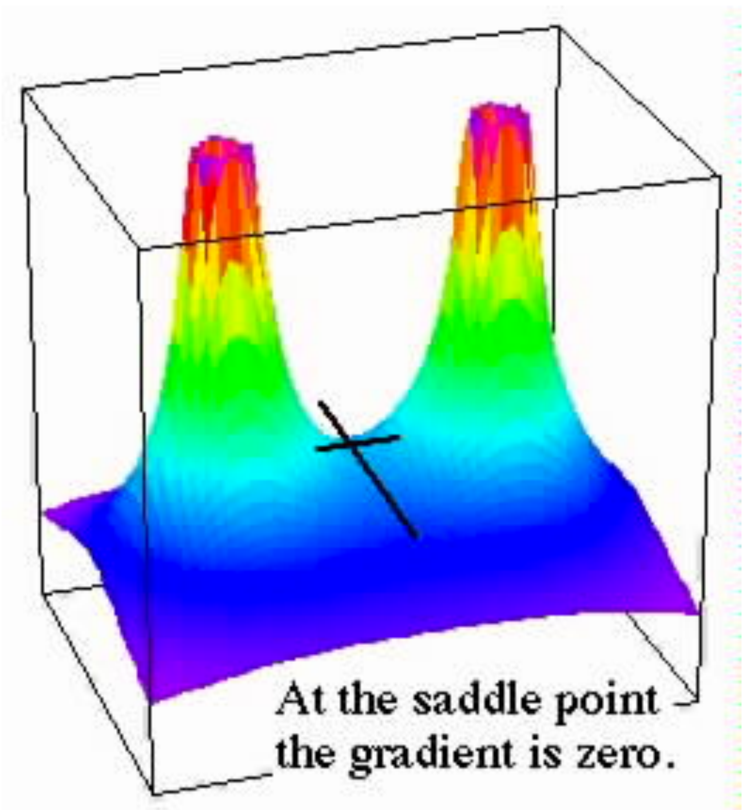
$$\Delta U = -W_{field} = -\vec{F}_{field} \cdot \Delta\vec{r} = -q\vec{E} \cdot \Delta\vec{r}$$

More Generally $\Delta U = -q\int_i^f \vec{E} \cdot d\vec{r}$

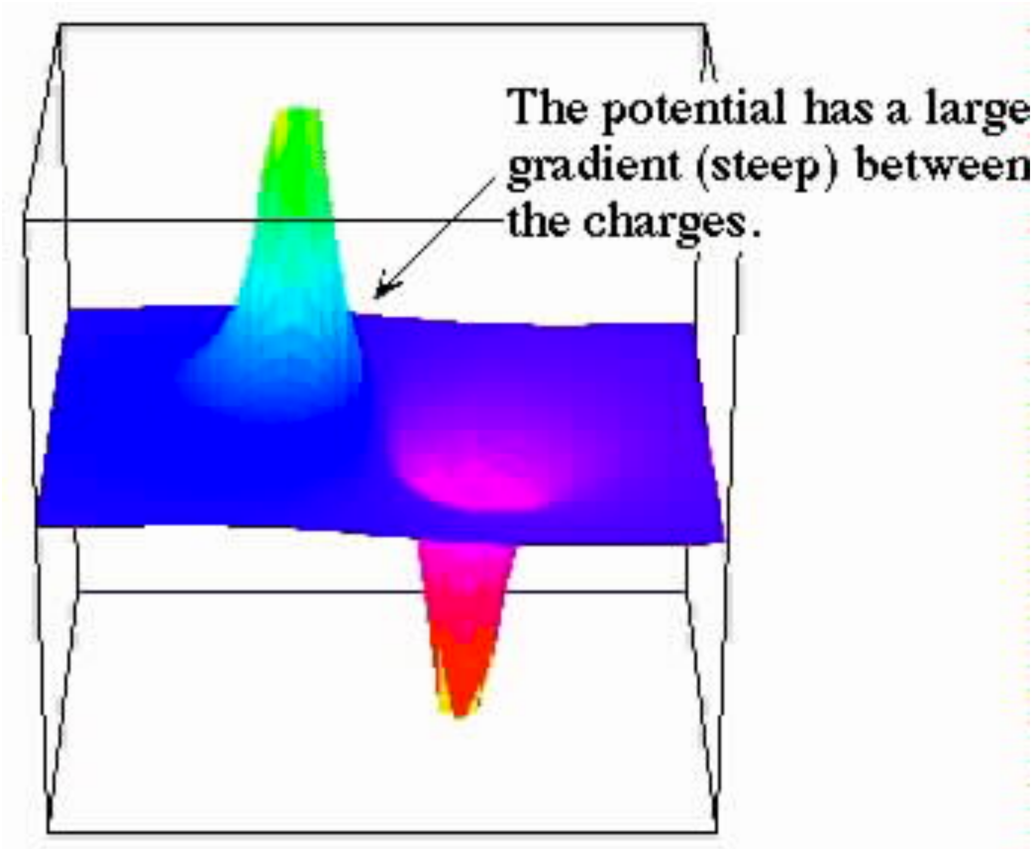
Electric Potential V = U/q

Electric Potential

+Q, +Q



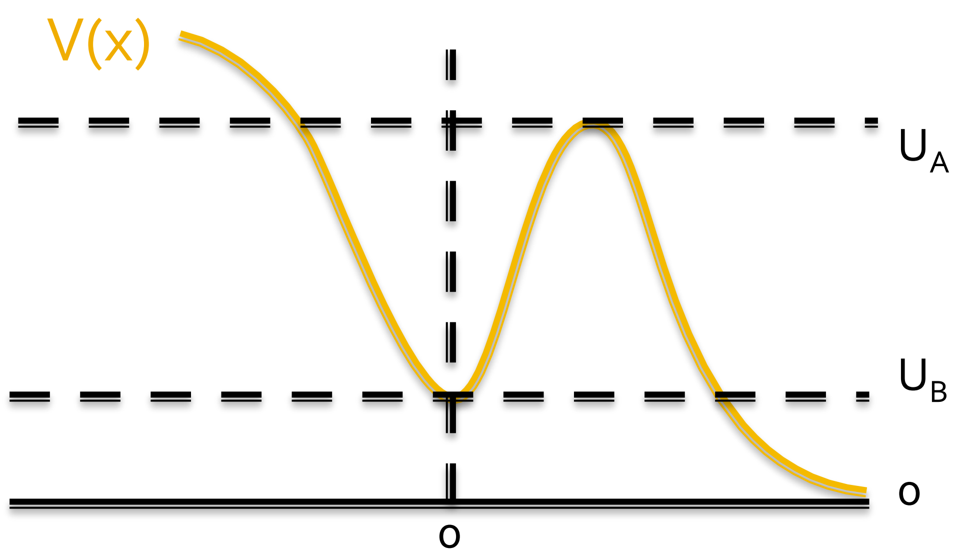
+Q, -Q



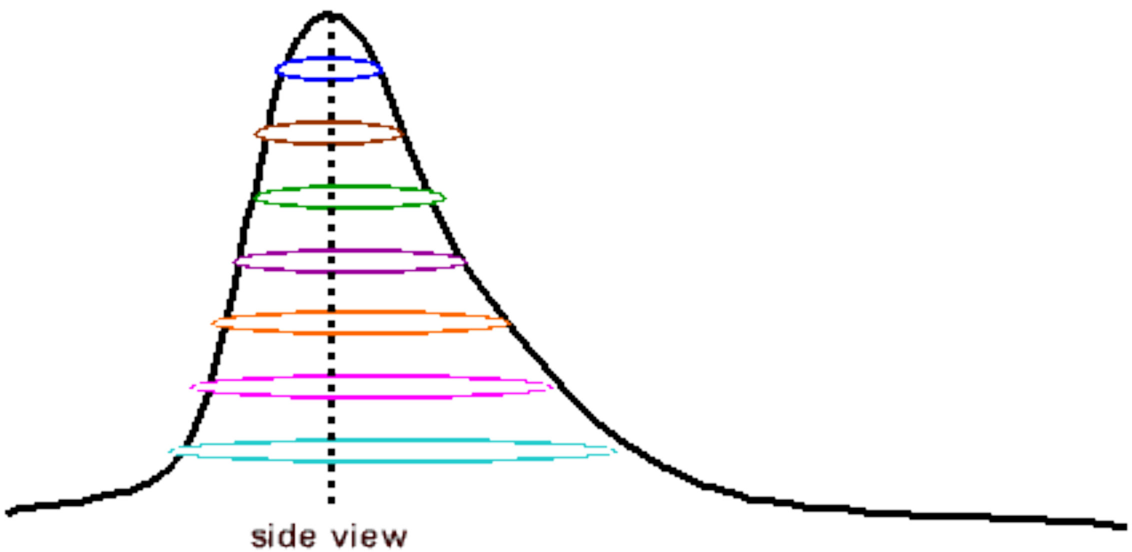
Concept Check

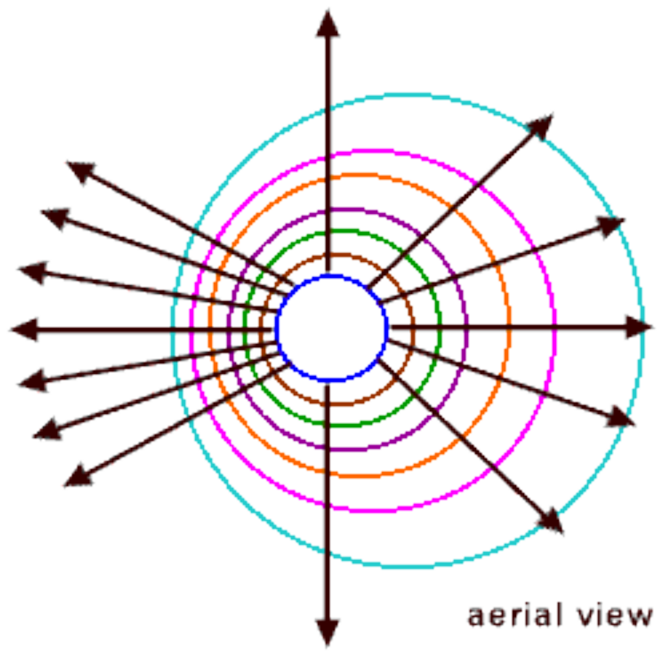
- Given this electric potential, how much kinetic energy would an electron (charge = –e) starting at x = 0 need to escape to x = +∞?

1. $KE = e(U_A - U_B)$
2. $KE = e(U_B)$
3. $KE = e(U_A)$
4. $KE = 0$



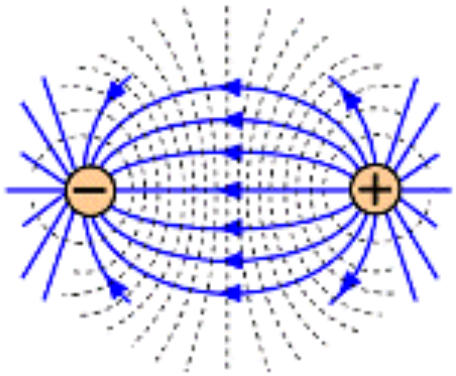
Potential And Field

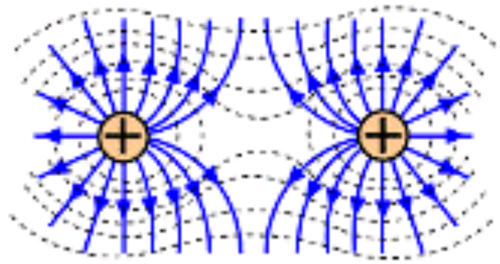


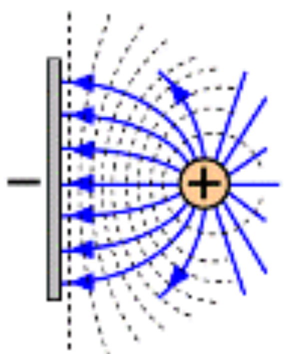


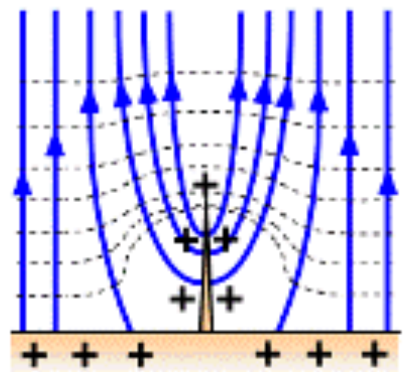
Field always points downhill!

Potential and Field



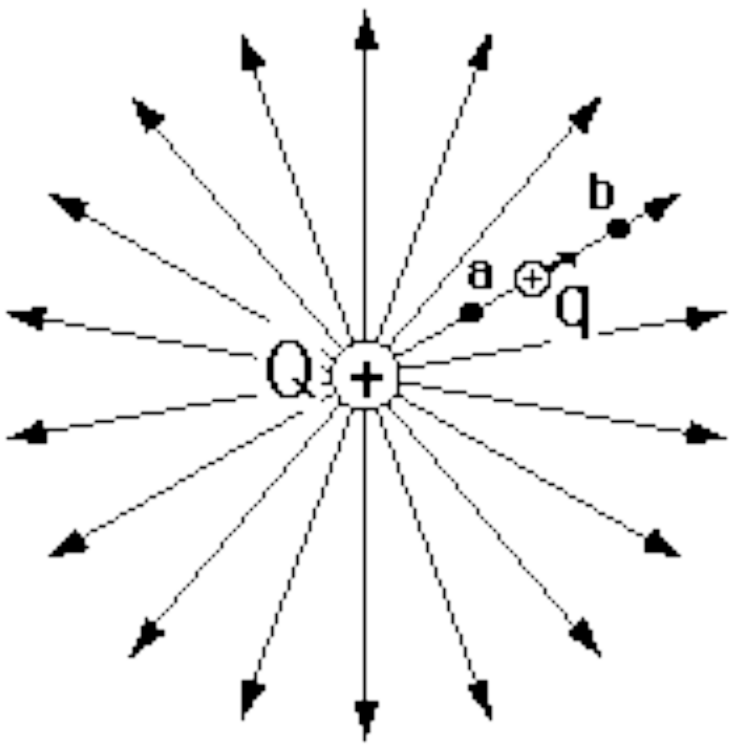






$$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right)$$

Potential and Field



Generally

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

For Spherical Symmetry

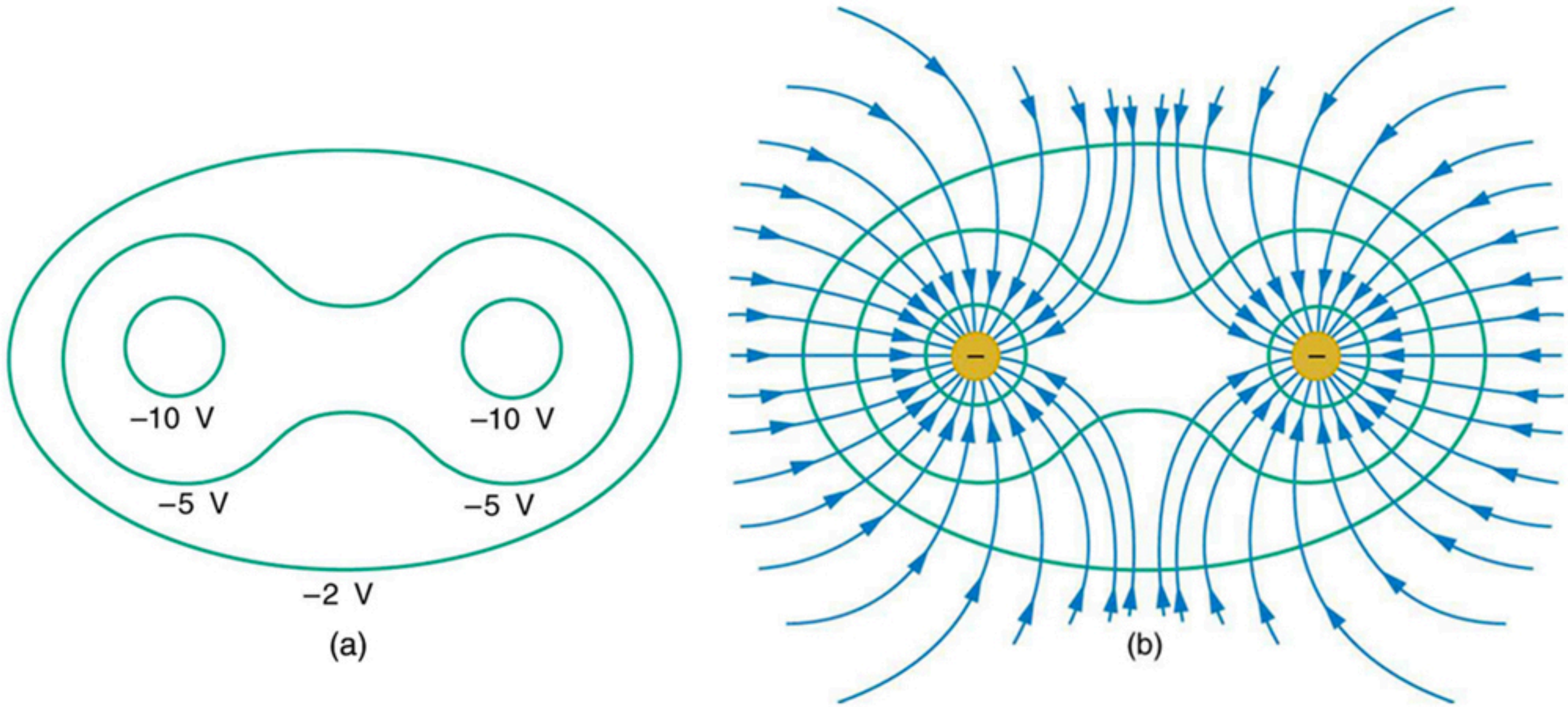
$$= -\int_{r_A}^{r_B} \frac{kQ}{r^2}\, dr$$

$$= \left. \frac{kQ}{r} \right|_{r_A}^{r_B}$$

$$= kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

Electric Potentials and Force/Energy

Force is along field lines, so perpendicular to equipotentials



Positive charges want to "roll downhill" (gaining kinetic energy)
Negative charges want to "roll uphill" (also gaining kinetic energy)

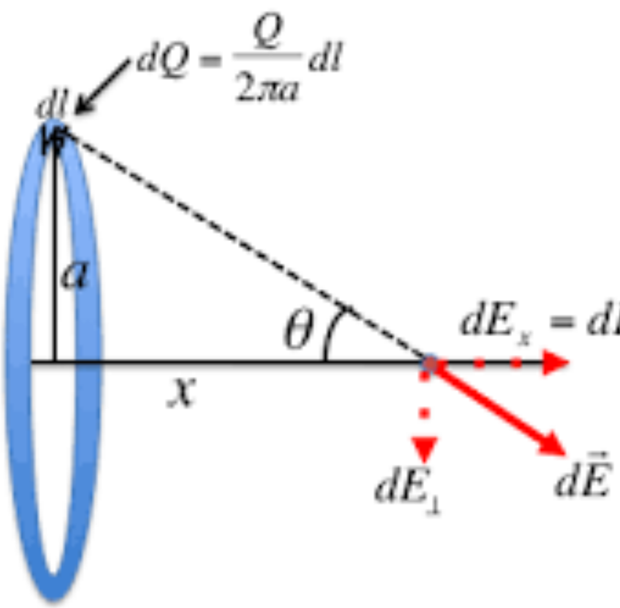
Field and Potential of a Ring

Electric Potential of Charged Ring [tt22]

Total charge on ring of radius a: $Q = \int dq$

$$V(x) = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{\sqrt{x^2+a^2}} = \frac{1}{4\pi\epsilon_0}\frac{1}{\sqrt{x^2+a^2}}\int dq$$

$$\Rightarrow V(x) = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$ (at points on axis of ring)

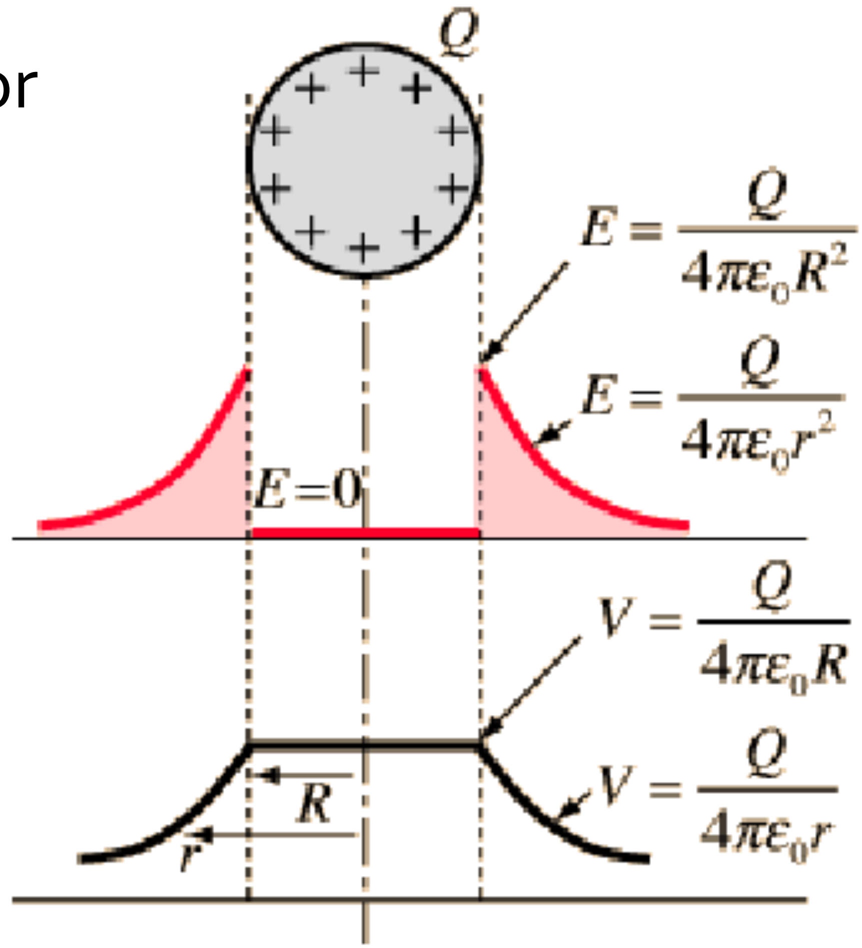


$$dE_x = dE\cos\theta = dE\frac{x}{\left(x^2+a^2\right)^{1/2}}$$

$$E_x = \frac{1}{4\pi\epsilon_0}\frac{qx}{[a^2+x^2]^{3/2}}$$

Spherical Geometry

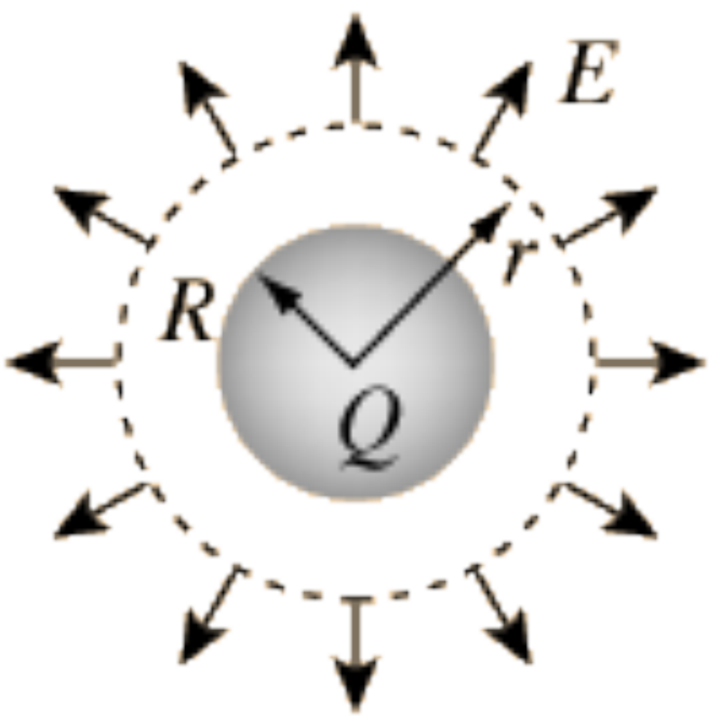
Conductor



Spherical Geometry

Uniform Charge Density

$$\Phi = EA = E4\pi r^2 = \frac{Q}{\varepsilon_0}$$



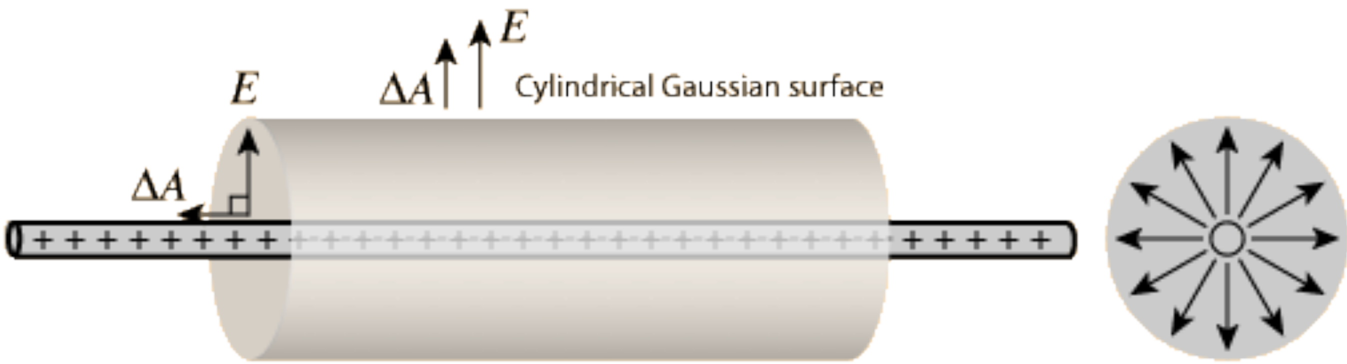
For $r > R$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

For $r < R$

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

Cylindrical Geometry

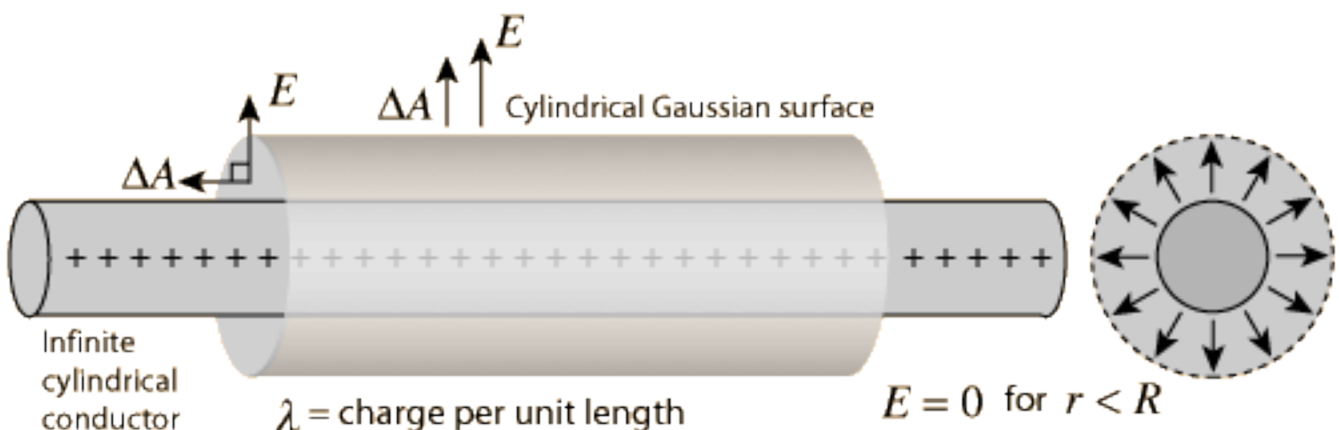


λ = charge per unit length

$$\Phi = E2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi r\varepsilon_0}$$

This expression is a good approximation for the field close to a long line of charge.



λ = charge per unit length

$E = 0$ for $r < R$ as it is inside any conductor at equilibrium.

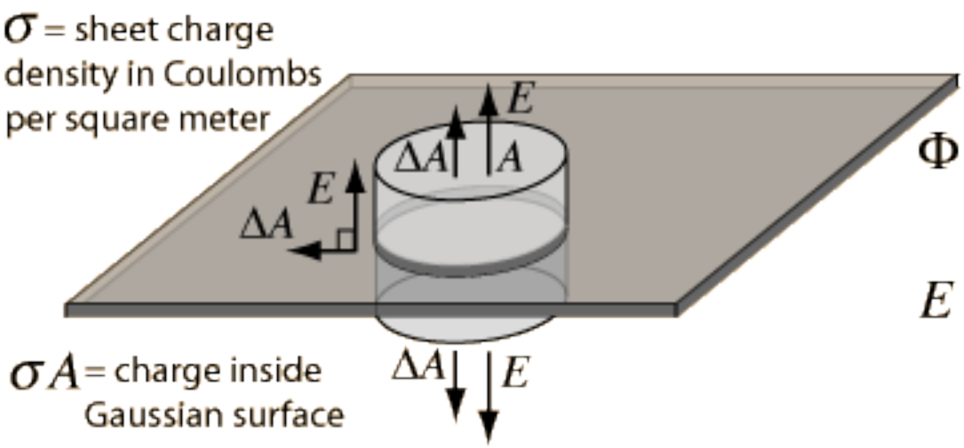
$$\Phi = E2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

For $r \geq R$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

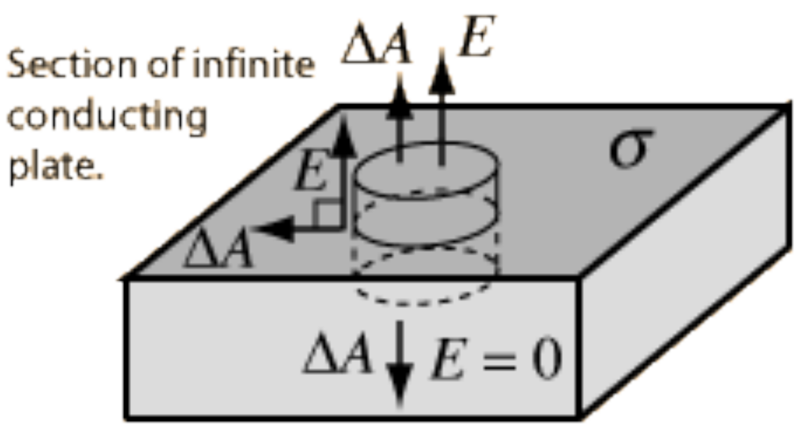
This expression is a good approximation for the field close to a long conducting cylinder.

Planar Geometry



$$\Phi = E2A = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$



σ = charge per unit area

$$\Phi = EA = \frac{\sigma A}{\varepsilon_0}$$

σA on the surface is the charge enclosed in the Gaussian surface.

$$E = \frac{\sigma}{\varepsilon_0}$$

Concept Check

Given two adjacent conductors with finite thickness but infinite lateral extent, each with a total area charge density σ on it, how will their charge density be distributed?

A. On right side of right conductor, left side of left conductor
B. On left side of right conductor, right side of left conductor
C. On right side of both conductors
D. On left side of both conductors

Conductors

- Always have zero electric fields inside them (at least in steady state, which is usually quickly reached)
  - Any net charge goes to the exterior
  - Any imposed field is canceled out in the interior