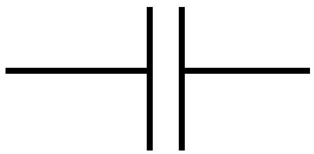
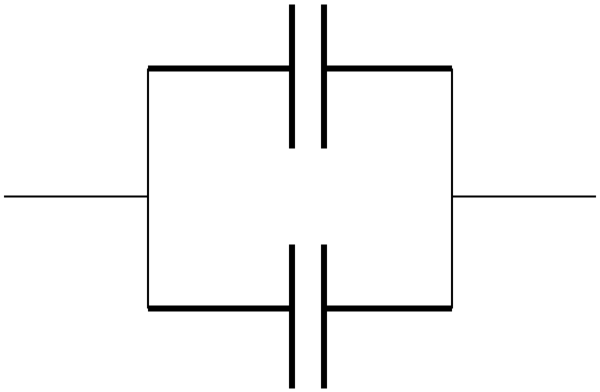
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Capacitors In Parallel



Circuit symbol for a capacitor

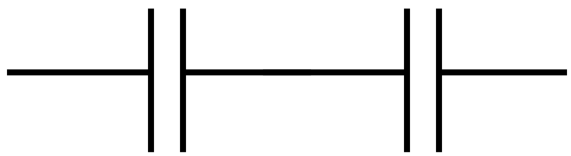


Two capacitors in parallel

$$C_{tot} = C_1 + C_2$$

Think about them like one giant capacitor with the area of 1 and 2 together.

Capacitors in Series

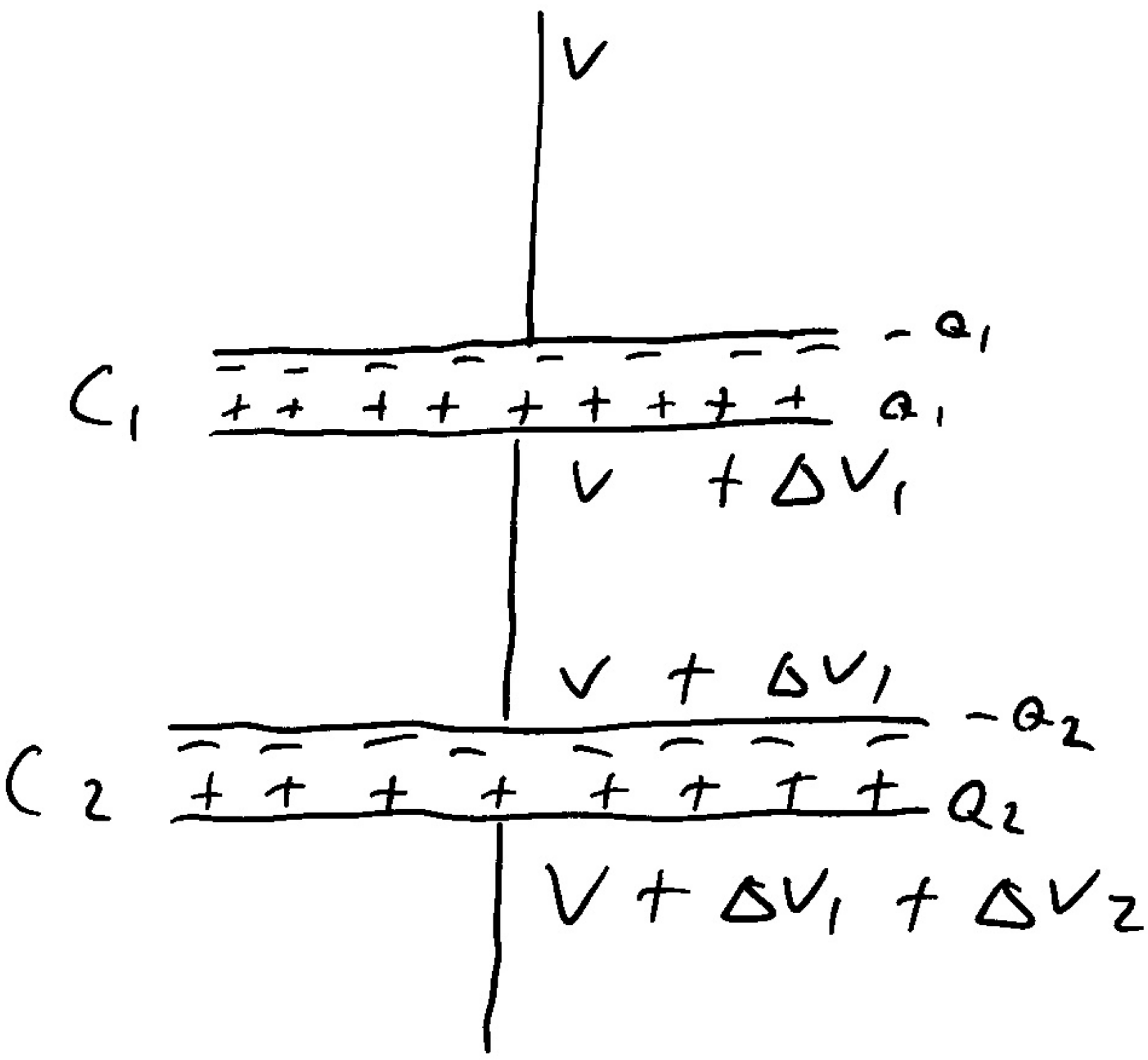


Two capacitors in series

$$C_{tot} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

One can easily extend these rules to more than two capacitors and more complex combinations.

Capacitors in Series



$C_1 = Q_1 / \Delta V_1$

$C_2 = Q_2 / \Delta V_2$

- But $Q_1 + -Q_2 = 0$ since the region between capacitors is an uncharged conductor before voltage is applied

So $Q_1 = Q_2 = Q$

$C_{tot} = Q / \Delta V_{tot} = Q / (\Delta V_1 + \Delta V_2)$

$$= Q / \left(Q_1/C_1 + Q_2/C_2 \right)$$

$$= 1 / \left(1/C_1 + 1/C_2 \right)$$

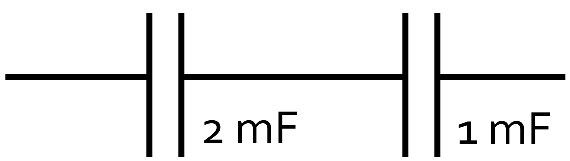
$$= \frac{C_1 C_2}{C_1 + C_2}$$

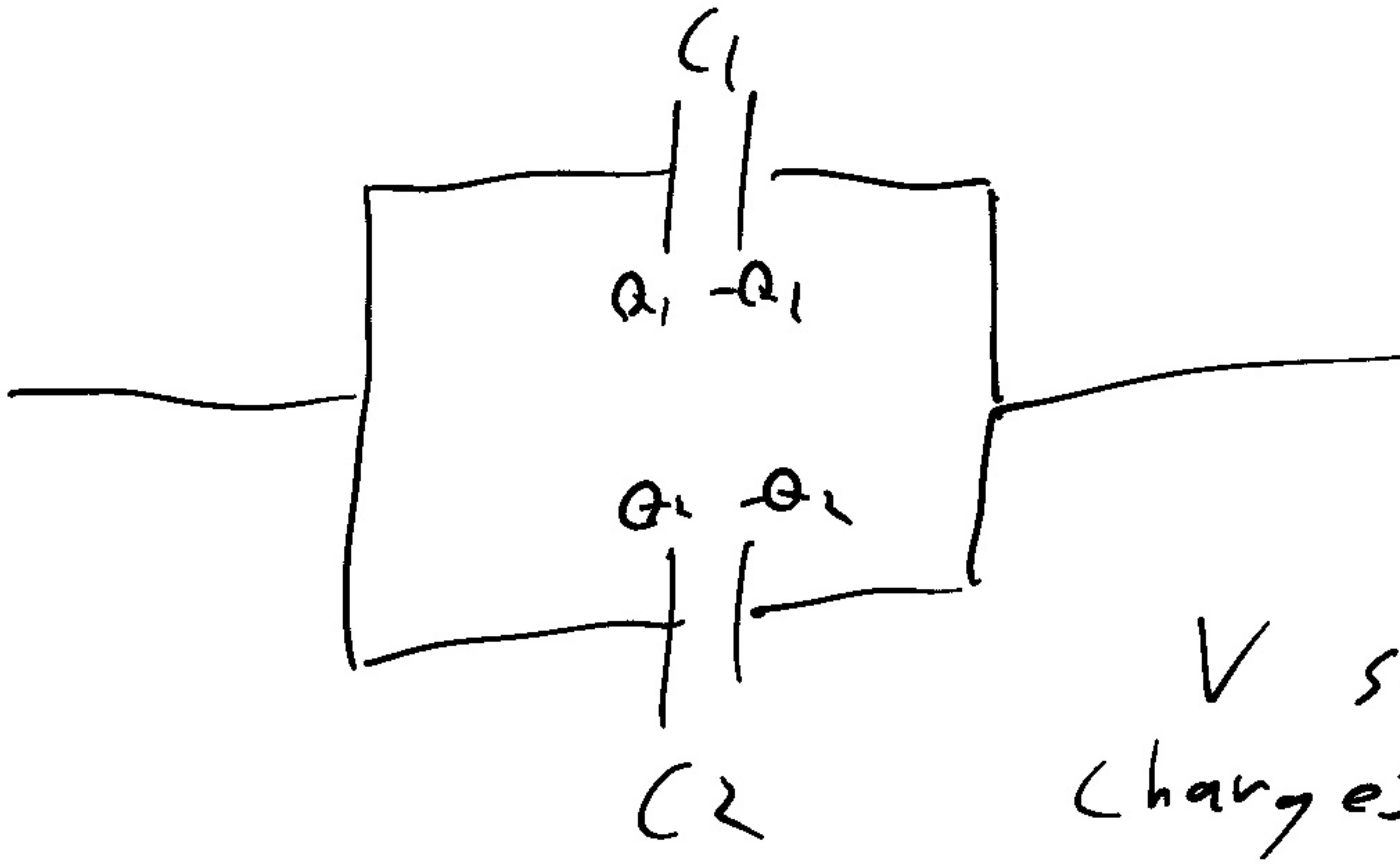
- Note that only the charge on the ends of the series capacitors are really "stored". All the charge between capacitors just flows to cancel E in conductors.

Concept Check

- A 2-mF and a 1-mF capacitor are connected in series and a potential difference is applied across the combination. The 2-mF capacitor has:

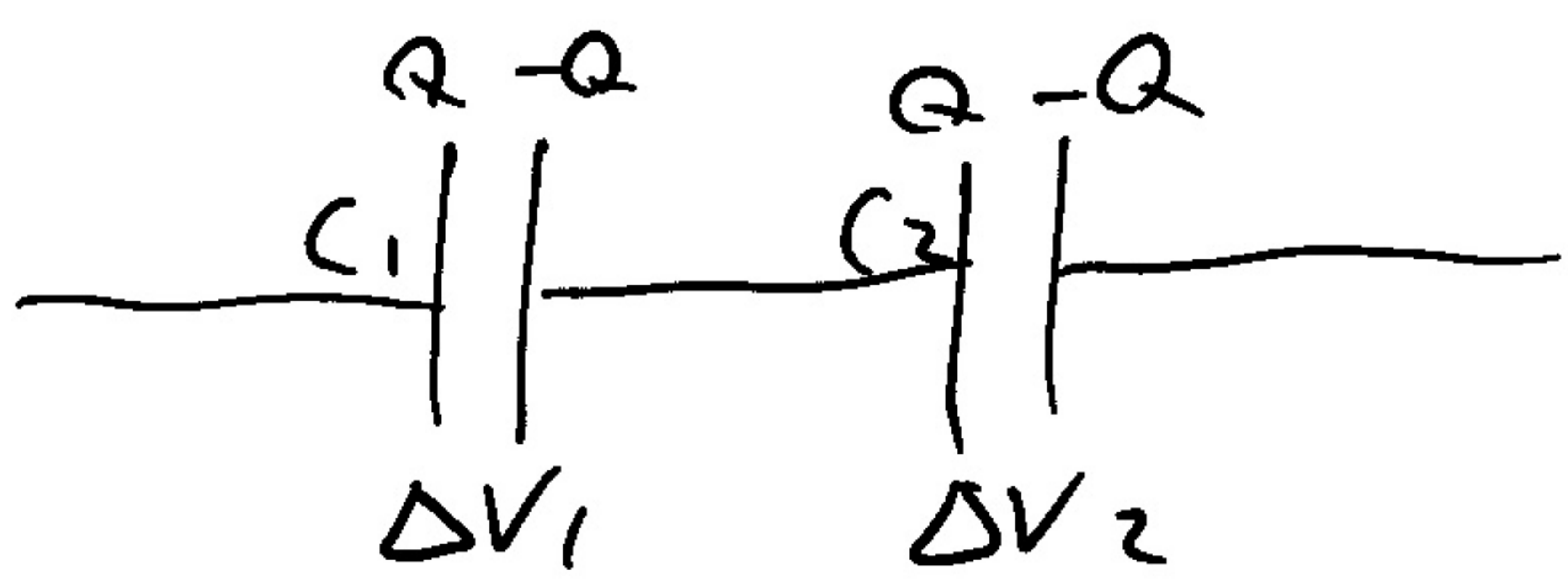
1. twice the charge of the 1-mF capacitor
2. half the charge of the 1-mF capacitor
3. twice the potential difference of the 1-mF capacitor
4. half the potential difference of the 1-mF capacitor
5. none of the above





V same
charges add

$$C_{total} = C_1 + C_2$$
$$Q_{total} = Q_1 + Q_2$$
$$Q_{total} = C_{total} V$$

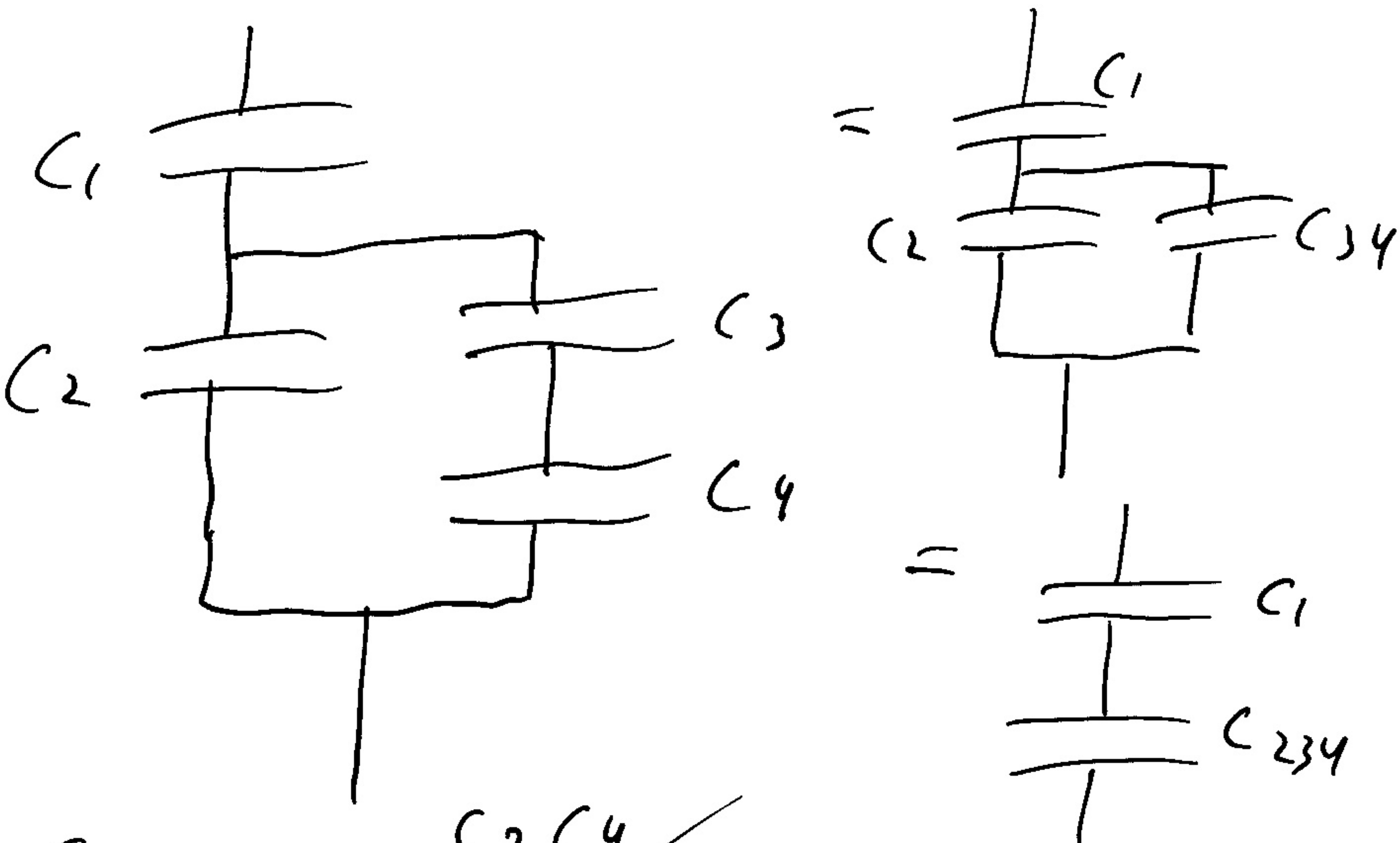


Q same
ΔV's add

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$
$$Q_1 = Q_2 = Q$$

$$C_{total} = Q/\Delta V_{total} = Q/(\Delta V_1 + \Delta V_2)$$
$$= Q/(Q/C_1 + Q/C_2)$$
$$= 1/(1/C_1 + 1/C_2)$$

– More complicated Capacitor Networks



$$C_{34} = C_3 C_4 / C_1 + C_4$$

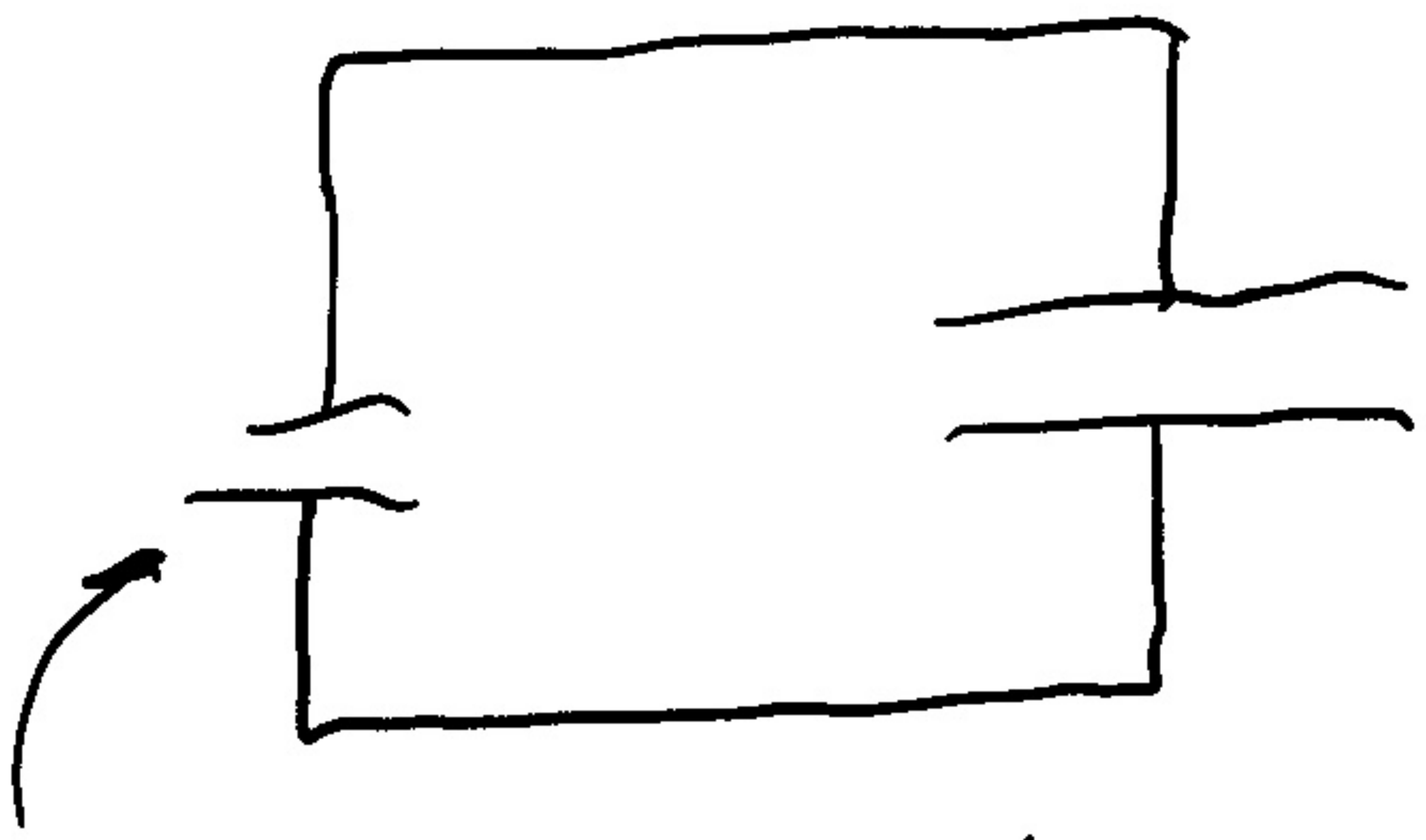
$$C_{234} = C_2 + C_{34}$$

$$C_{1234} = 1 / \left(1/C_1 + 1/C_{234} \right)$$

$$= 1 / \left(1/C_1 + 1 / \left(C_2 + \frac{C_3 C_4}{C_3 + C_4} \right) \right)$$

$$= \frac{C_1 \cdot \left[C_2 + \frac{C_3 C_4}{C_3 + C_4} \right]}{C_1 + C_2 + C_3 C_4 / (C_3 + C_4)}$$

- Energy in Capacitor
- Start w/ uncharged Capacitor



- Turn on battery
- Work must be done to move charge from one plate to the other

$V = Q/C$

(like calculating potential of group of charges)

$\Delta W_{field} = \int \vec{F} \cdot d\vec{x}$

$\Delta W_{battery} = -\Delta W_{field} = V \Delta Q$
$= \Delta Q \cdot Q/C$

$W_{total} = \int dW = U_{total}$

$= \int_0^Q q\, dq/C = q^2/2C \Big|_0^Q$

$= \boxed{Q^2/2C}$

$= \boxed{\frac{1}{2} C V^2}$

Note $U \neq QV$ which would be energy of point charge in potential V

$E = \sigma/\varepsilon_0 = Q/\varepsilon_0 A$

so $U = Q^2/2C$

$= (E\varepsilon_0 A)^2/2C$

$= E^2\varepsilon_0^2 A^2 / 2\varepsilon_0 A/d$

$= \dfrac{\varepsilon_0 E^2}{2} \cdot A \cdot d$

$= \dfrac{\varepsilon_0 E^2}{2} \cdot \text{Volume}$

U/volume = energy density

$= u = \boxed{\tfrac{1}{2}\varepsilon_0 E^2}$

True for any electric field in free space!

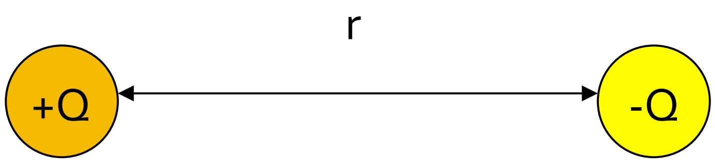
Energy Stored in a Capacitor

- Storing charge on a capacitor also implies storing energy, since it takes work to put the charge on a capacitor
- Generally speaking, the energy density stored in an electric field is:

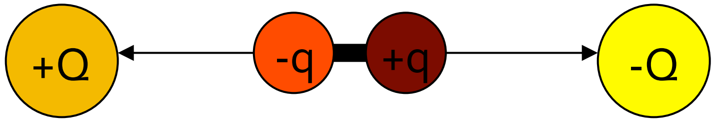
$$u_{ES} = \frac{1}{2}\varepsilon|\mathbf{E}|^2$$

Adding a Dielectric

Dielectric Material – what if the gap between the capacitor plates is not empty?



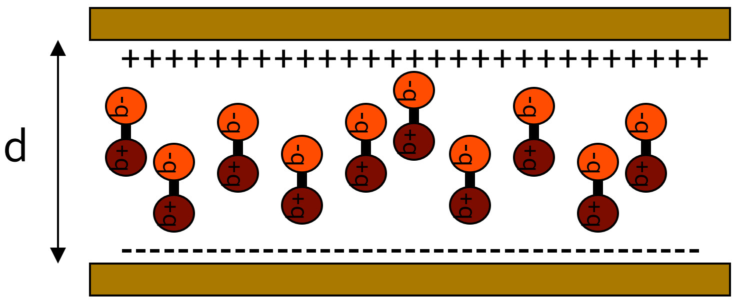
$$|\vec{F}| = +\frac{1}{4\pi\varepsilon_0}\frac{QQ}{r^2}$$



$$|\vec{F}| = +\frac{1}{4\pi\varepsilon_0}\frac{QQ}{r^2} - \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\frac{qqd}{(r/2)^3}$$

Weakens the force (screening)!

Adding a Dielectric



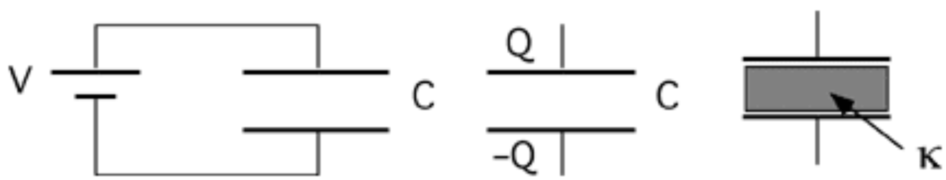
Add a dielectric material.

One parameter – κ (Kappa) the dielectric constant – can describe the modifications.

$$|\vec{F}| = +\frac{1}{4\pi\kappa\varepsilon_0}\frac{QQ}{r^2}$$

Concept Check

Q37) A capacitor with capacitance C is connected to a battery until charged, then disconnected from the battery. A dielectric having constant κ is inserted in the capacitor. What changes occur in the charge, potential and stored energy of the capacitor after the dielectric is inserted?



1) V stays same, Q increases, U increases
2) V stays same, Q decreases, U stays same
3) V increases, Q decreases, U increases
4) V decreases, Q stays same, U decreases
5) None of the above

$U = \frac{1}{2} C V^2$

$= \frac{1}{2} Q^2 / C$

Q same

C goes up by factor κ

$\varepsilon_0 A / d \rightarrow \kappa \varepsilon_0 A / d$

$U = Q^2 / C$ goes down by factor κ

$V = Q / C$ goes down by factor κ

$U = Q^2 d / 2 \varepsilon_0 A \rightarrow Q^2 d / 2 \kappa \varepsilon_0 A$

in dielectric

$E \rightarrow \sigma / \kappa \varepsilon_0$ in dielectric

$= Q / \kappa \varepsilon_0 A$

$$U = \frac{E^2 \kappa \varepsilon_0 A d}{2}$$

$$u = U / \text{volume} = \frac{E^2 \kappa \varepsilon_0}{2} = \boxed{\frac{1}{2} \varepsilon E^2}$$

w/ $\varepsilon = \kappa \varepsilon_0$

Dielectric Constant

Easy rule to remember…

Everywhere there is an ε_o, change it to $\kappa\varepsilon_o$.

$$|\vec{F}| = +\frac{1}{4\pi\kappa\varepsilon_0}\frac{QQ}{r^2}$$

$$\oint \kappa\vec{E}\cdot d\vec{a} = \frac{Q}{\varepsilon_0}$$

$$C = \frac{\varepsilon_0 A}{d} \Rightarrow \frac{\kappa\varepsilon_0 A}{d}$$

Increases the capacitance!

Dielectric Constant

- κ = dielectric constant
  - κ is the degree to which a dielectric is polarized by an external electric field
- Multiply ε_o by κ in every equation to get correct equations in dielectric
- Could also just write $\varepsilon = \kappa\varepsilon_o$
- In this case ε is the permittivity of the dielectric
  - Recall ε_o is the permittivity of free space

Dielectric Constants

A few values:

Vacuum	K = 1.0000000
Air	K = 1.00054
Paper	K = 3.5
Water	K = 80
Titanium Ceramic	K = 130
Perfect Conductor	$K = \infty$

Electric Fields in an Insulator

A dielectric in an electric field becomes polarized; this allows it to reduce the electric field in the gap for the same potential difference.