Physics II: 1702
Gravity, Electricity, & Magnetism

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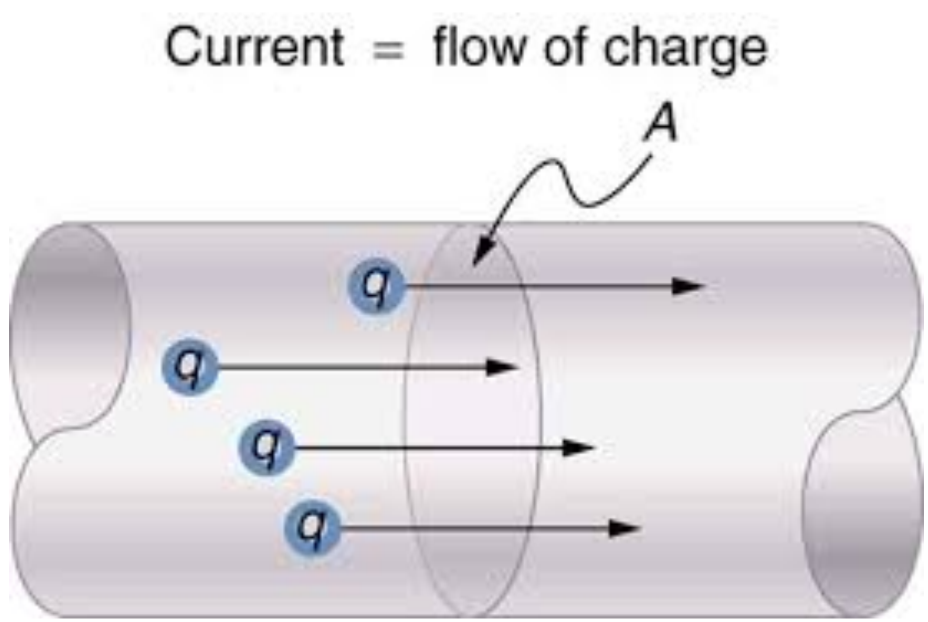
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Current

Electric Current

$$I = i \equiv \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} =$$ Rate of flow of net charge past a point



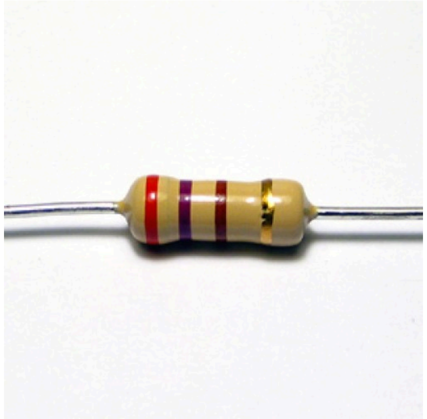
Resistors and Ohm's Law

Resistance R of a material

$$R \equiv \frac{V}{i} = \text{constant (for Ohmic materials)}$$

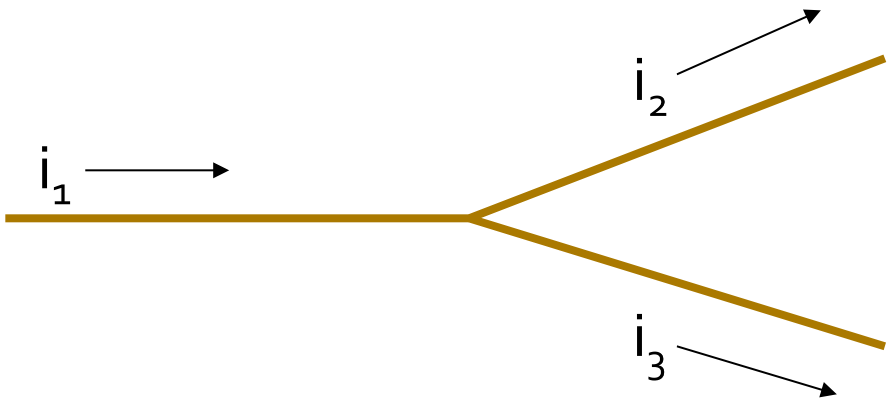
Definition of R

"Ohm's Law" $V = iR$



Current Flow in Circuits

Junction Rule



$$i_1 = i_2 + i_3$$

In a steady state, must have $\mathbf{i_{(in)} = i_{(out)}}$ at any junction, otherwise charge is building up somewhere, which cannot happen in steady state [Capacitors charging up are non-steady-state].

Current Density

<u>Current Density J</u>

$$\vec{J} = \frac{\vec{i}}{A} = \frac{current}{area}$$

J is caused by E, the electric field. In fact,

$$\vec{J} = \frac{1}{\rho}\vec{E} = \sigma\vec{E}$$

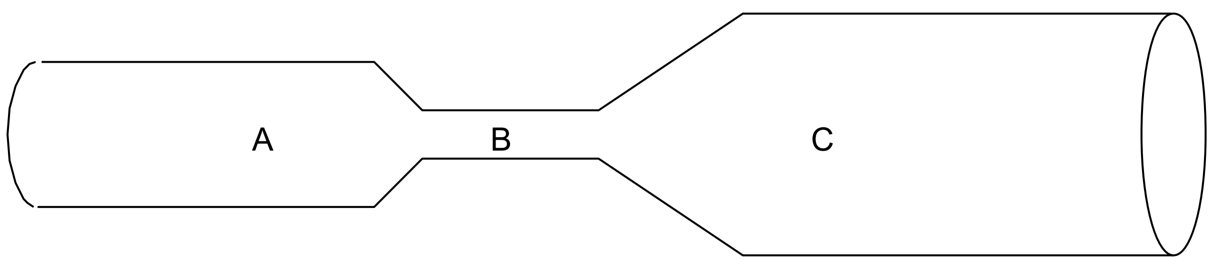
Conductivity is the inverse of Resistivity.

Resistivity

Material		Resistivity (Ohm-meter)
Aluminum	Al	2.62×10^{-8}
Copper	Cu	1.72×10^{-8}
Gold	Au	2.44×10^{-8}
Iron	Fe	9.71×10^{-8}
Lead	Pb	21.9×10^{-8}
Mercury	Hg	95.8×10^{-8}
Nickel	Ni	6.9×10^{-8}
Platinum	Pt	10.5×10^{-8}
Silver	Ag	1.62×10^{-8}
Tungsten	W	5.48×10^{-8}
Carbon		$[3\text{-}60] \times 10^{-5}$
Glass		$[1\text{-}10000] \times 10^{9}$
Hard Rubber		$[1\text{-}100] \times 10^{13}$

Concept Check

A copper cylinder is machined to have the following shape. The ends are connected to a battery so that a current flows through the copper.

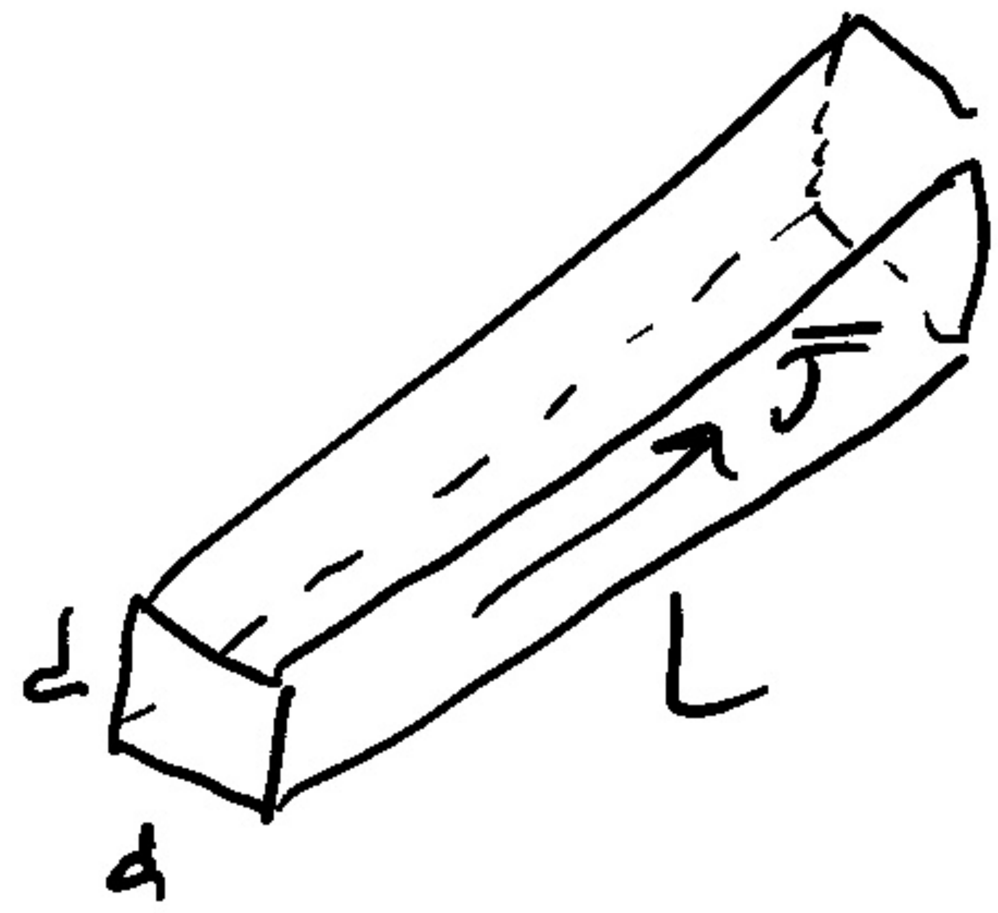


Which region A, B, or C has the greatest magnitude current density J?

A B C D) all three have the same J.

Region B has the largest current density $J = I/A$. All 3 regions have the same current I, so the region with the smallest A has the largest J.

Square wire



constant current density $\vec{J}$

$$I = \int \vec{J} \cdot d\vec{A}$$

w/ $d\vec{A}$ the cross-sectional area of wire element

$$= \int J\, dA \quad \text{since } \vec{J} \parallel d\vec{A}$$

$$= JA$$
$$= Jd^2$$

$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$
$$= -EL$$

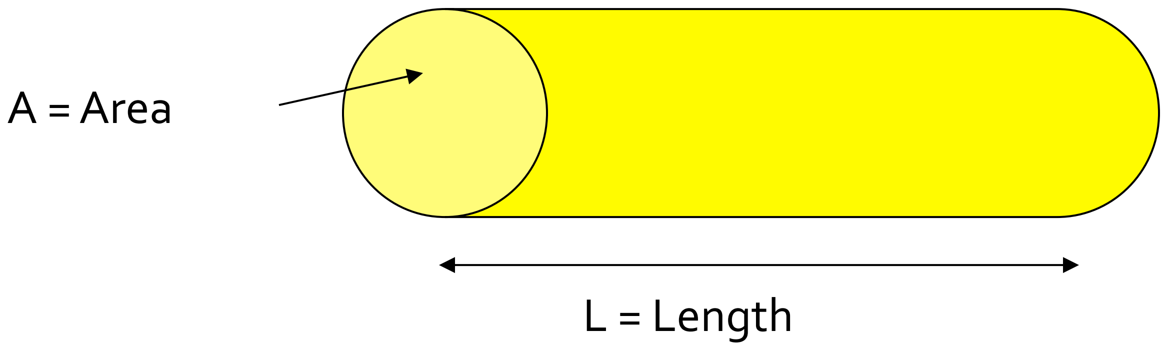
$|\Delta V| = IR$ By Ohm's Law

$$\Rightarrow EL = Jd^2R$$
$$\Rightarrow E/J = Rd^2/L$$
$$\equiv \rho$$
$$\Rightarrow \boxed{R = \rho L/A}$$

Resistance and Resistivity

Resistance R of a piece of conductor depends on

1) Composition or Material
2) Shape and Dimensions



$$R = \rho \frac{L}{A}$$

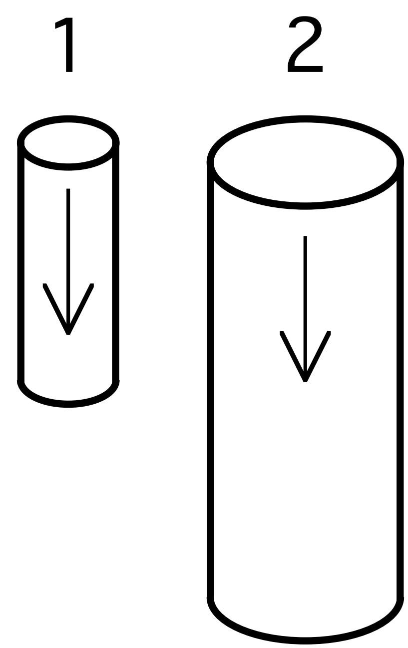
Where ρ is **resistivity** – measure of internal friction; dependent on material composition.

Motivating Resistance

- The longer a resistor is, the longer distance the current has to flow through it, and the more it is “slowed down”
- The smaller the cross sectional area of a resistor, the less easily current can flow through it
- Thus, both lengthening and shrinking the cross-sectional area increase the resistance

Concept Check

Two cylindrical resistors are made of the same material (same resistivity ρ). Resistor 2 is twice as long and has twice the diameter of resistor 1. What is the ratio $\frac{R_2}{R_1}$?



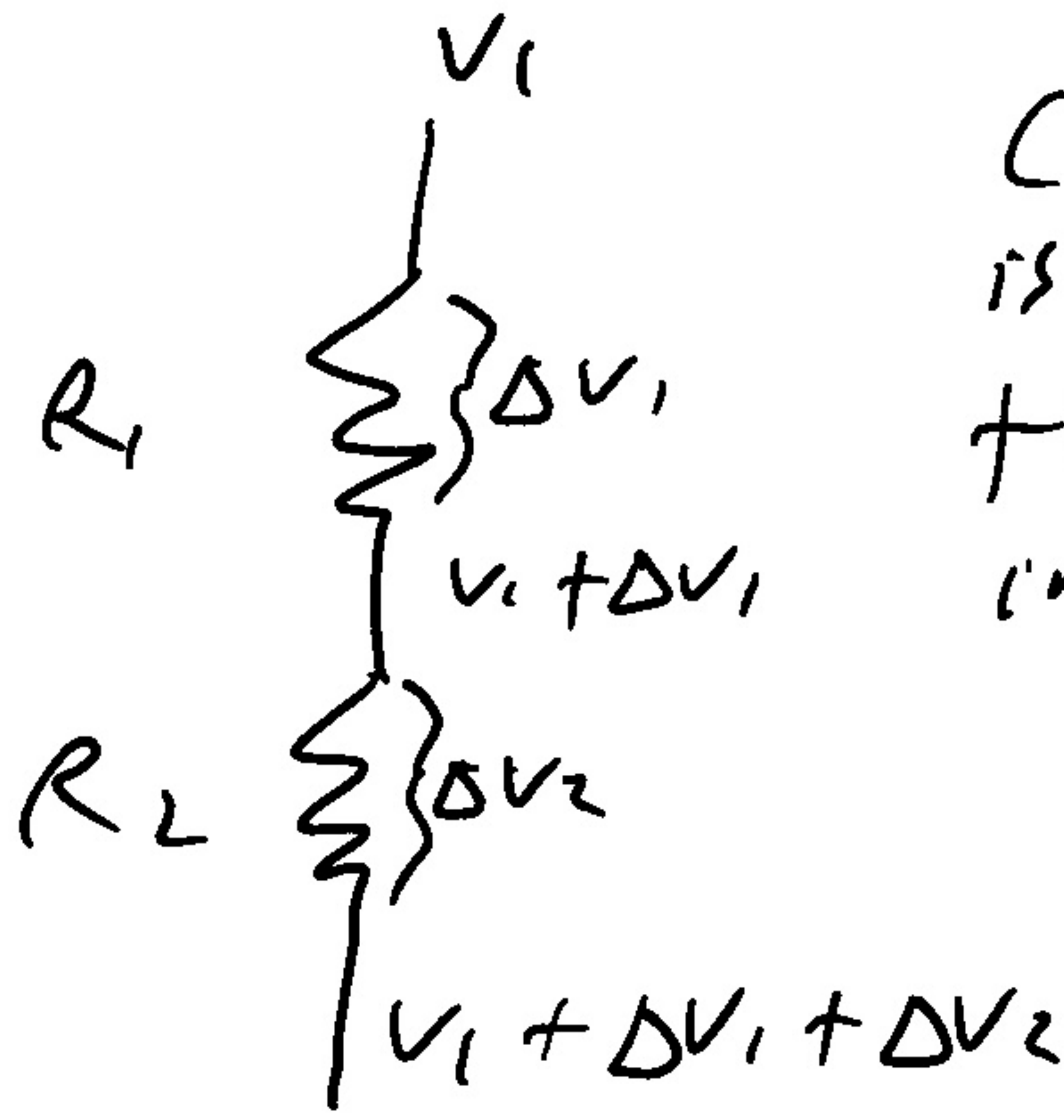
1) 2
2) 4
3) 1/2
4) 1/4
5) 1

- Recall capacitance addition

$$C_{eq} = C_1 + C_2 \quad \text{(parallel)}$$
$$1/C_{eq} = 1/C_1 + 1/C_2 \quad \text{(series)}$$

- Given resistance's different geometric dependence and relationship to Q, V you might guess different addition formulae

- Series

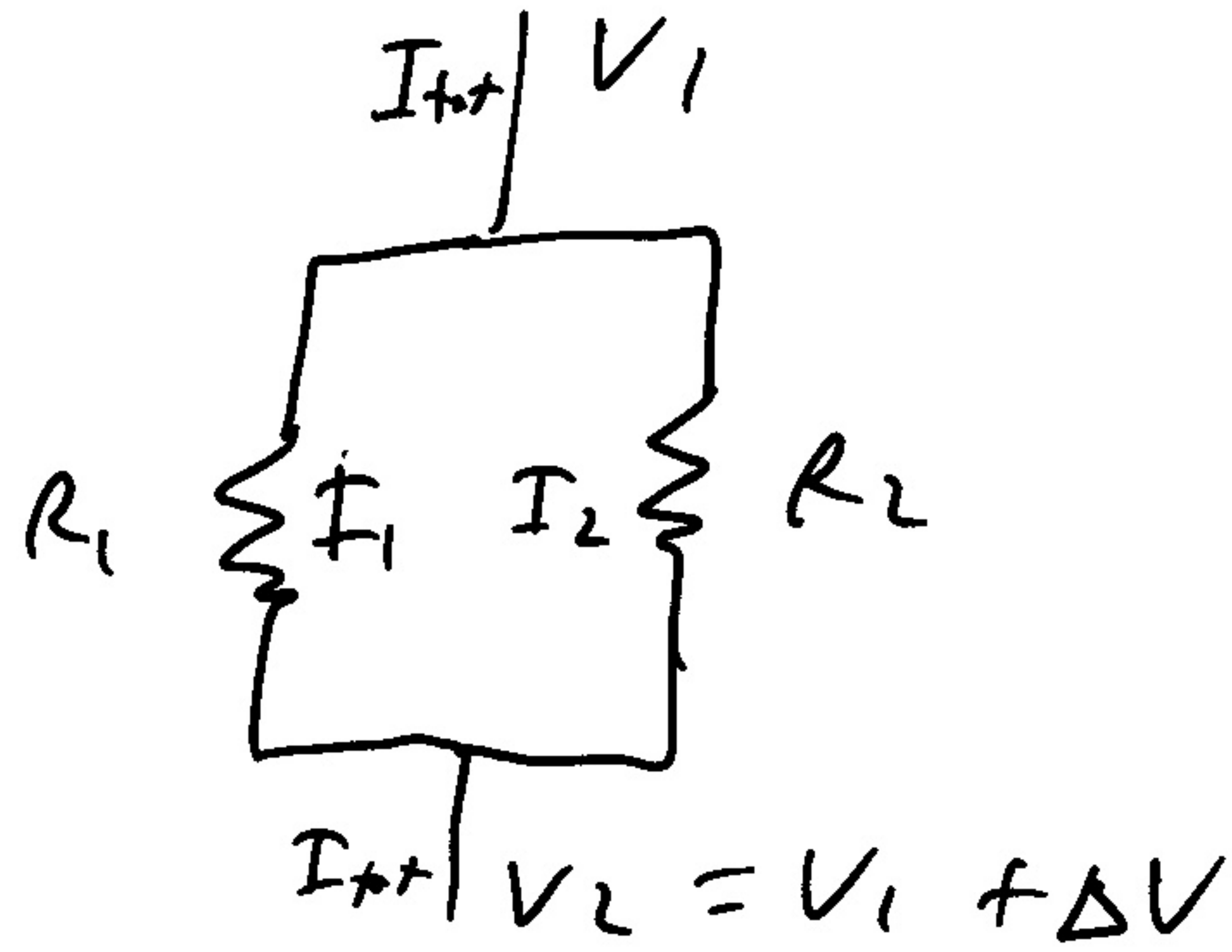


current I is constant throughout in steady state

$$\Delta V_{tot} = \Delta V_1 + \Delta V_2 = I R_{tot}$$
$$= I R_1 + I R_2$$
$$\Rightarrow R_{tot} = R_1 + R_2$$

Adding resistance in series reduces current, since more energy loss. This is why adding resistance in series eventually turned off our light bulb.

Parallel



- voltage drop same, but current splits

$$\Delta V_1 = \Delta V_2 = \Delta V = I_1 R_1 = I_2 R_2$$

Junction Rule

$$I_{tot} = I_1 + I_2$$

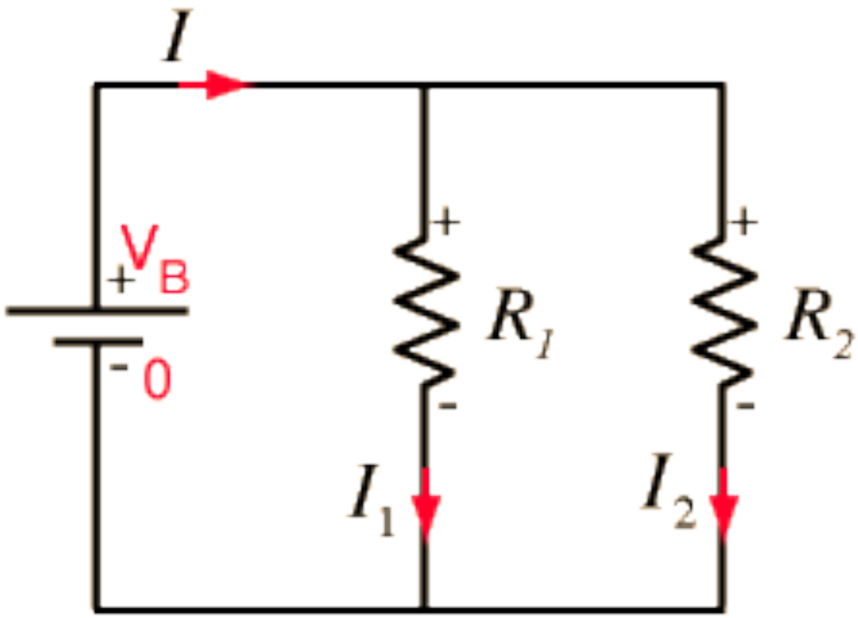
$$\Rightarrow \Delta V / R_{total} = \Delta V / R_1 + \Delta V / R_2$$

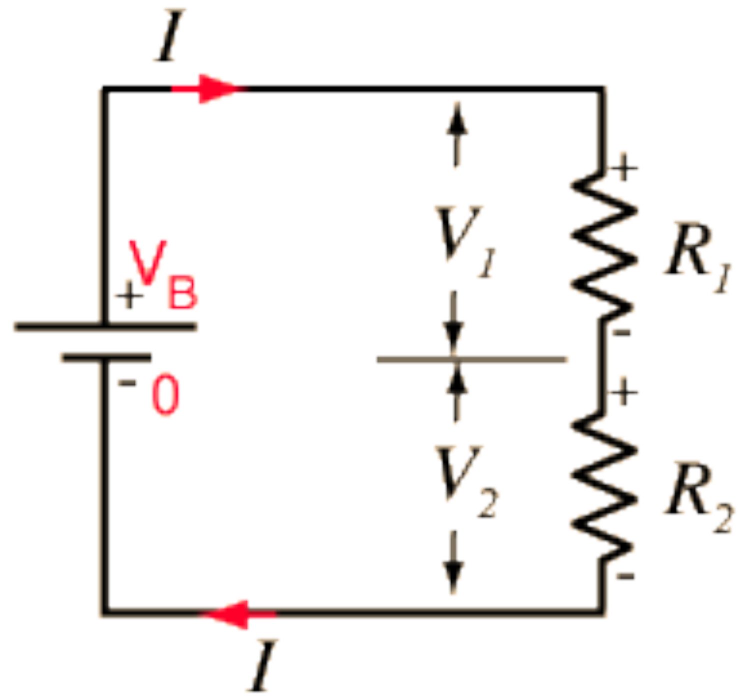
or $1/R_{total} = 1/R_1 + 1/R_2$

or $R_{total} = 1/(1/R_1 + 1/R_2)$

- series/parallel rules for resistors are inverted from those for capacitors

Series and Parallel Resistors





Parallel resistors

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Series resistors

$$R_{equivalent} = R_1 + R_2$$

Capacitance vs. Resistance

- Both material & geometrical properties
  - C proportional to A, inversely proportional to d
  - R proportional to L, inversely proportional to A
- V = IR across resistor
- V = Q/C across capacitor