# Physics II: 1702
# Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

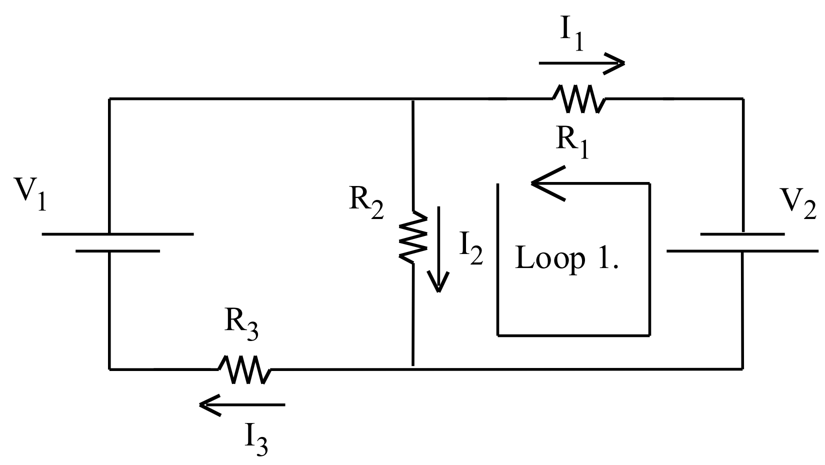
# Last Time's Concept Check

A circuit with two batteries is shown below. The directions of the currents have been chosen (guessed) as shown.

Which is the correct current equation for this circuit?

A) $I_2 = I_1 + I_3$ B) $I_1 = I_2 + I_3$ C) $I_3 = I_1 + I_2$

D) None of these.



# Last Time's Concept Check

Which equation below is the correct equation for Loop 1?

A) $-V_2 + I_1R_1 - I_2R_2 = 0$

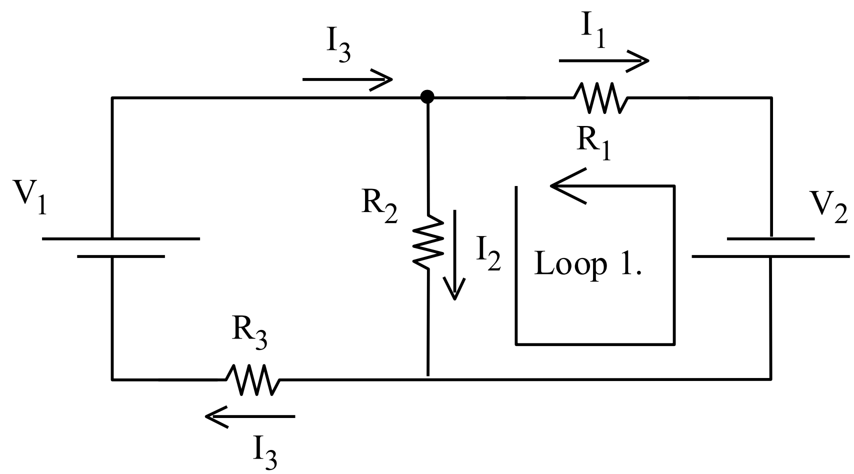
B) $V_2 + I_1R_1 - I_2R_2 = 0$

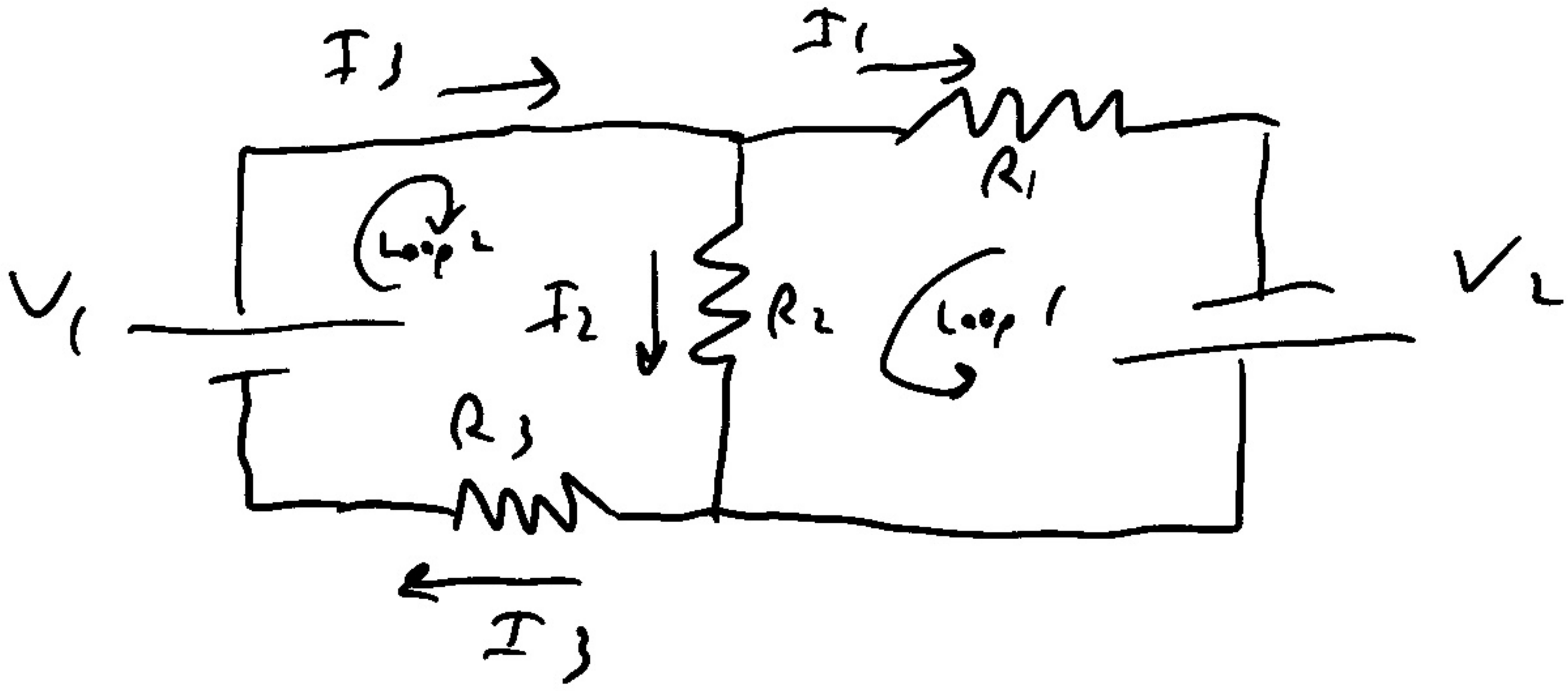
C) $-V_2 - I_1R_1 + I_2R_2 = 0$

D) $V_2 + I_1R_1 + I_2R_2 = 0$

E) None of these.

Answer: $-V_2 + I_1R_1 - I_2R_2 = 0$






$$-V_2 + I_1 R_1 - I_2 R_2 = 0 \quad \text{Loop 1}$$
$$V_1 - I_2 R_2 - I_3 R_3 = 0 \quad \text{Loop 2}$$
$$I_3 = I_2 + I_1 \quad \text{Junction}$$

- Say $V_1 = 12\ V$
  $V_2 = 24V$
  $R_1 = R_2 = R_3 = 1\ \Omega$

$$-24 + I_1 - I_2 = 0$$
$$12 - I_2 - I_3 = 0$$

$$\Rightarrow \quad -24 + I_1 - I_2 = 0$$
$$12 - I_1 - 2I_2 = 0 \quad \leftarrow \text{using junction rule}$$

add

$$-12 - 3I_2 = 0$$

$$\Rightarrow \boxed{I_2 = -4A}$$

- Our guess for current direction was wrong. $I_2$ actually up! okay as long as we are consistent.

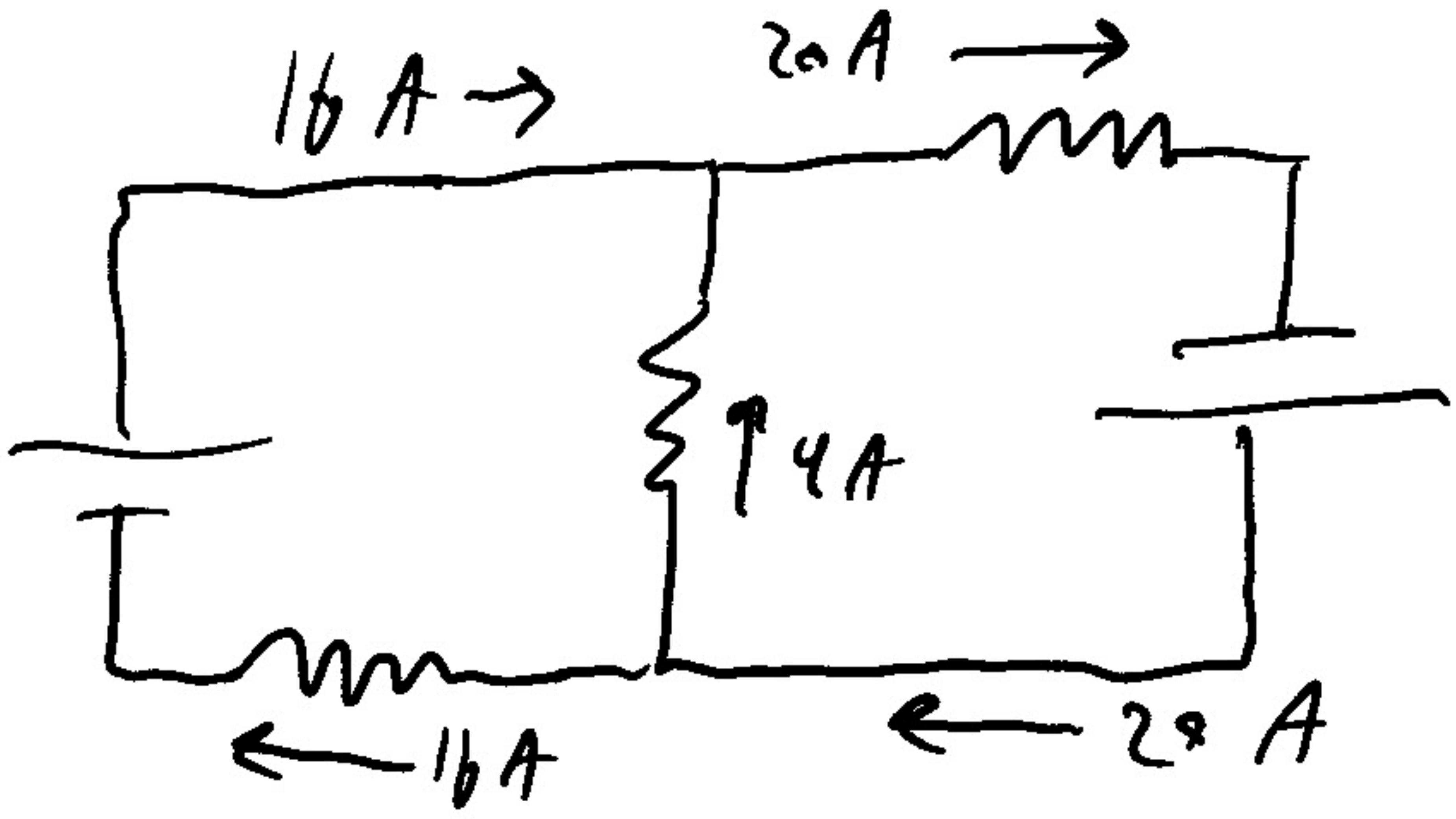
$$-24 + I_1 - I_2 = 0$$

$$\Rightarrow \quad -24 + I_1 - (-4) = 0$$

$$\Rightarrow \quad \boxed{I_1 = 20\ A}$$

$$I_3 = I_1 + I_2 = \boxed{16\ A}$$

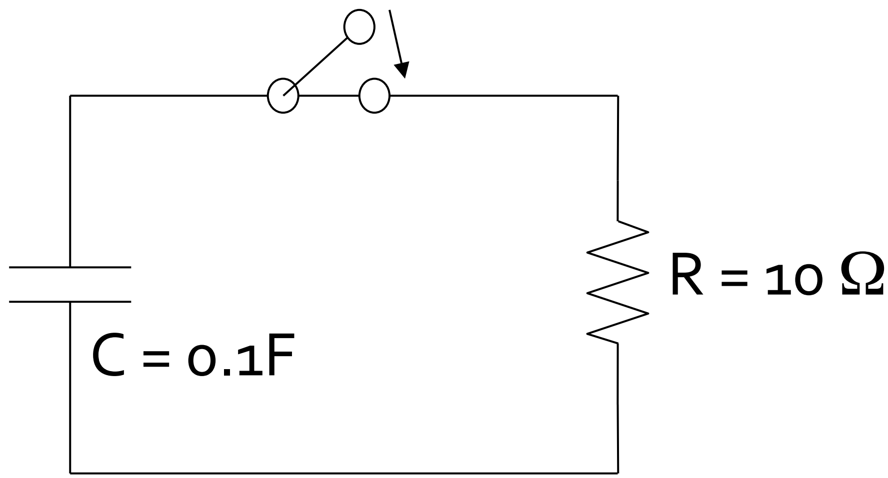
Circuit really looks like



# RC Circuits

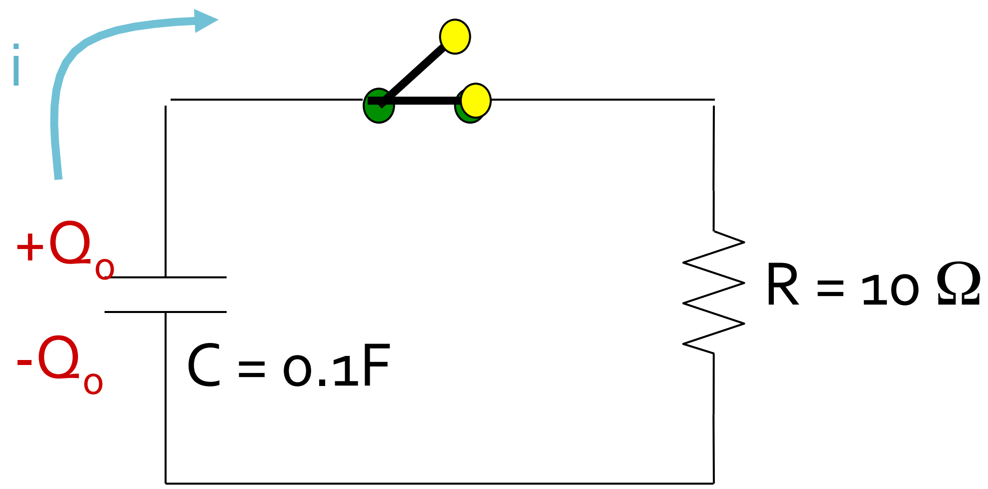
Now we will discuss circuits with batteries, resistors and capacitors. → **"RC Circuits"**

Simplest case is just a Capacitor C, charged to a Voltage $V_o=Q_o/C$, attached to a Resistor R and a switch.



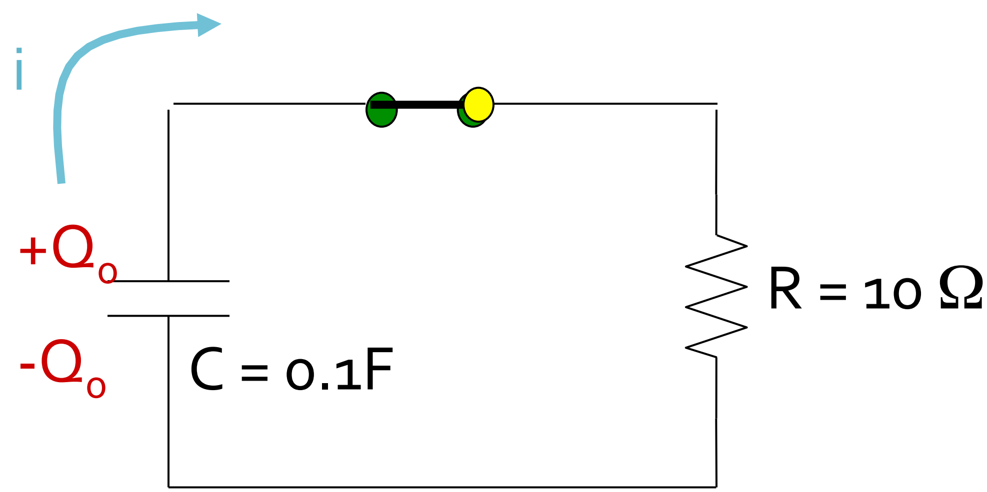
# Discharging Capacitor

Initially one has $+Q_o$ and $-Q_o$ on the Capacitor plates.
Thus, the initial Voltage on the Capacitor $V_o = Q_o/C$.
What do you think happens when the switch is closed?



# Discharging Capacitor

Close the switch at t=o, then current i starts to flow.

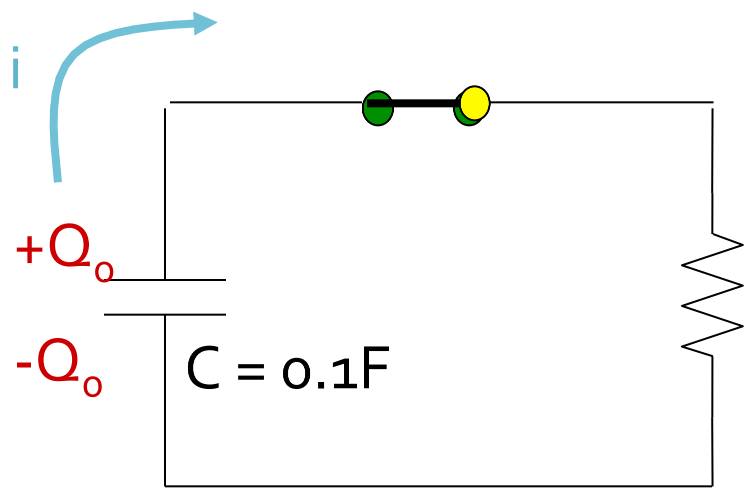



At t=o, $i_0 = \dfrac{V_0}{R}$

Later $i(t) = -\dfrac{dQ}{dt}$

* Negative sign since Q is decreasing.

# Discharging Capacitor




Voltage across C = Voltage across R

$$V_c = V_R$$

$$\frac{Q}{C} = iR = -\frac{dQ}{dt}R$$

$$\frac{dQ}{dt} = -\frac{1}{RC}Q$$

We now need to solve this differential equation.

# Discharging Capacitor

Solving the differential equation:

$$\frac{dQ}{dt} = -\frac{1}{RC}Q \quad \Rightarrow \quad Q(t) = Q_0 e^{-t/RC}$$

Check solution by taking the derivative…

$$\frac{dQ}{dt} = Q_0\left(-\frac{1}{RC}\right)e^{-t/RC} = -\frac{1}{RC}Q$$

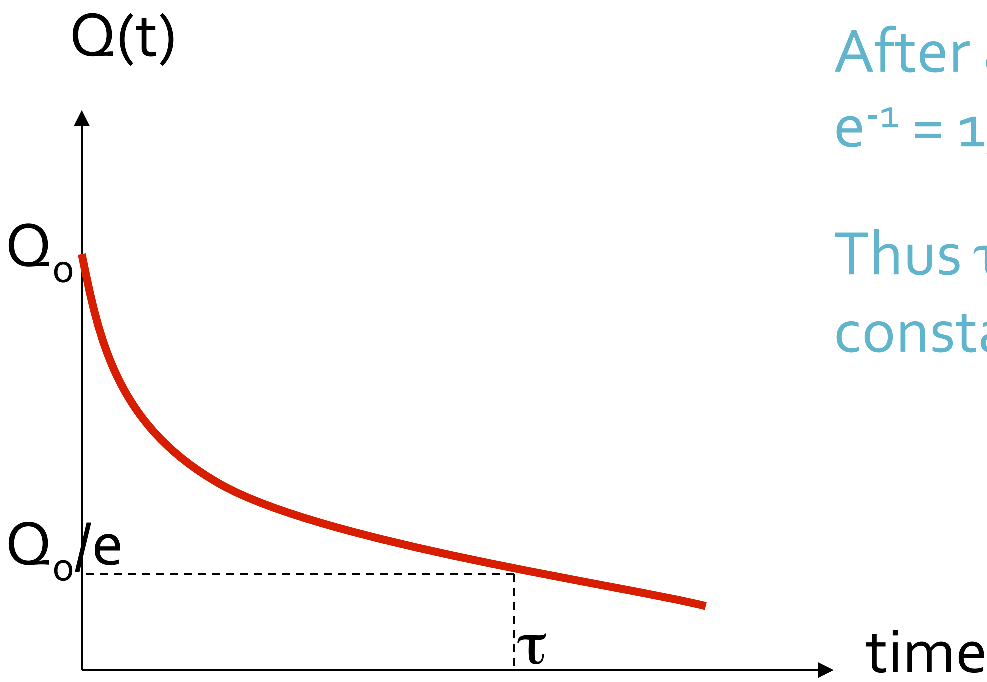
Also Q(t) = $Q_o$ at t=o.

$$Q(t=0) = Q_0 e^{-0/RC} = Q_0$$

# Discharging Capacitor

**Exponential Decay**

$$Q(t) = Q_0 e^{-t/RC}$$




After a time $\tau = RC$, Q has dropped by $e^{-1} = 1/e$.

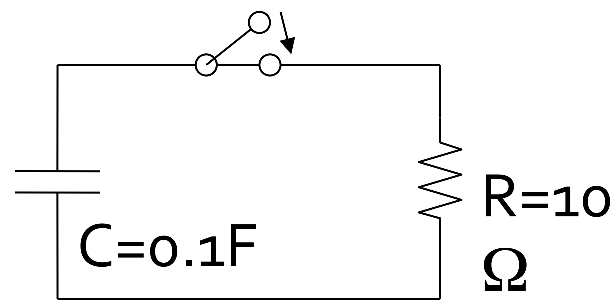
Thus $\tau = RC$ is often called the time constant and has units [seconds].

# Concept Check

A capacitor with capacitance 0.1F in an RC circuit is initially charged up to an initial voltage of $V_o$ = 10V and is then discharged through an R=10Ω resistor as shown. The switch is closed at time t=0. Immediately after the switch is closed, the initial current is $I_o = V_o$ / R=10V/10Ω.

What is the current I through the resistor at time t=2.0 s?

A) 1A
B) 0.5A
C) 0.37A
D) 0.14A
E) None of these.




Answer: $1/e^2$ A = 0.14A. The time constant for this circuit is RC=(10Ω)(0.10F) = 1.0 sec. So at time t=2.0 sec, two time constants have passed. After one time constant, the voltage, charge, and current have all decreased by a factor of e. After two time constants, everything has fallen by $e^2$. The initial current is 1A. So after two time constants, the current is $1/e^2$ A = 0.135A.

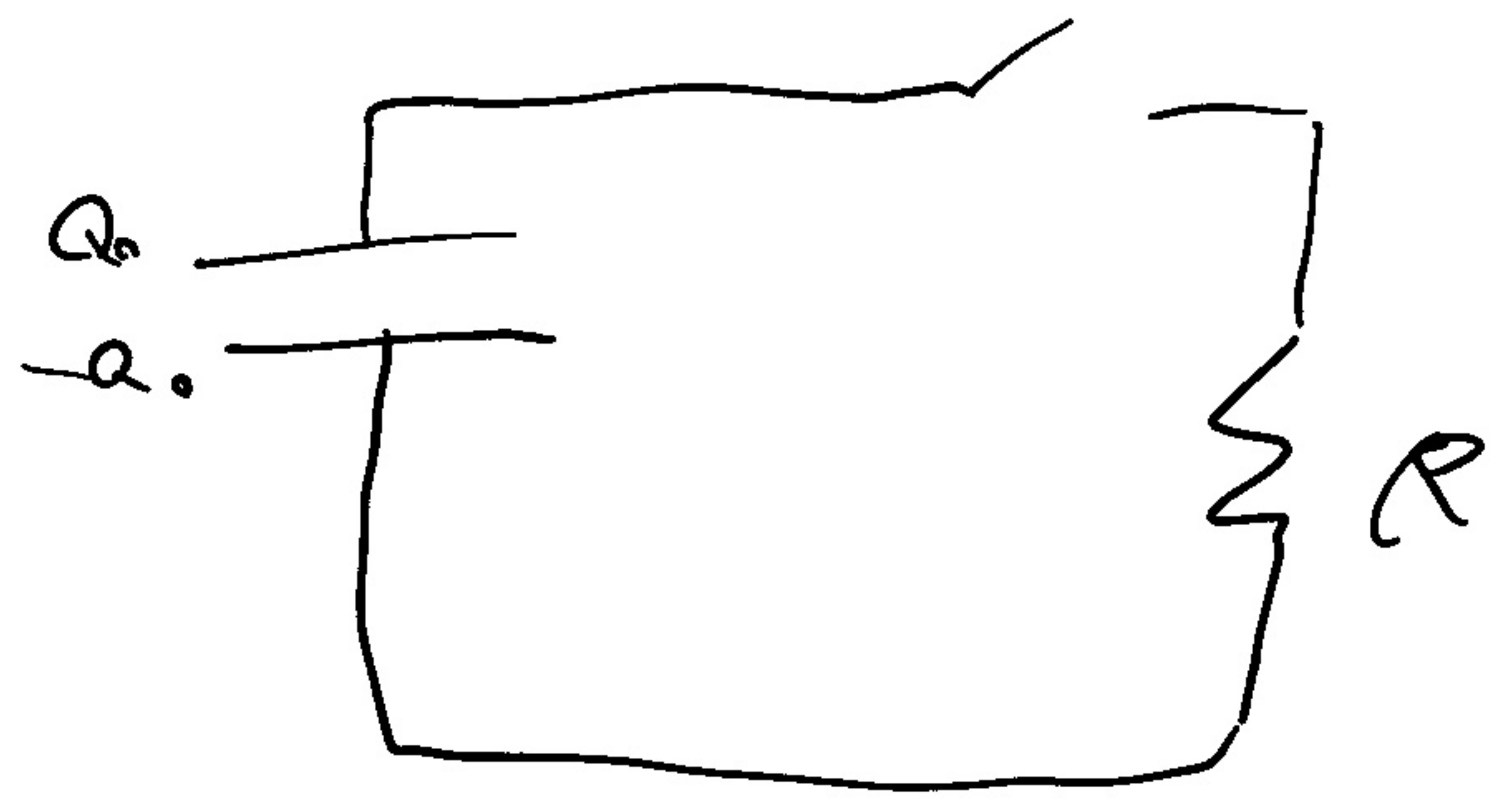
# Discharging Capacitor

$$\frac{dQ}{dt} = Q_0\left(-\frac{1}{RC}\right)e^{-t/RC} = -\frac{1}{RC}Q$$

$$|\,i(t)\,| = \left|\frac{dQ}{dt}\right| = \left(\frac{Q_0}{RC}\right)e^{-t/RC} = i_0 e^{-t/RC}$$

Thus, the current also falls with the same exponential function.

# Energy Dissipation Discharging




$$U_C = Q_0^2 / 2C$$

U dissipated in R

$$U_R = \int \frac{dU}{dt}\, dt = \int I^2 R\, dt$$

$$I(t) = I_0 e^{-t/RC} = \frac{Q_0}{RC} e^{-t/RC}$$

$$U_R = \int_0^\infty \frac{Q_0^2}{(RC)^2} e^{-2t/RC} \cdot R\, dt$$

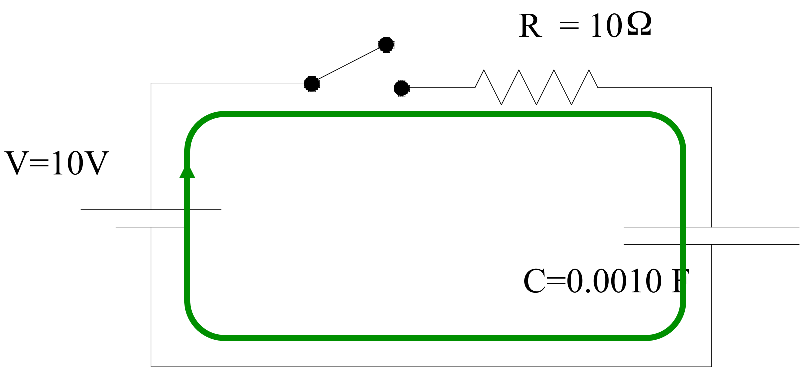
$$= \frac{Q_0^2}{RC^2} \cdot -\frac{RC}{2} e^{-2t/RC} \Big|_0^\infty$$

$$= \frac{Q_0^2}{RC^2} \cdot \frac{RC}{2} = Q_0^2 / 2C$$

- All energy stored in capacitor dissipated in resistor

# Charging Capacitor

More complex RC circuit: Charging C with a battery.




Before switch closed i=0, and charge on capacitor Q=0.

Close switch at t=0.

Try Voltage loop rule.

$$+V_b + V_R + V_C = 0$$

$$+V_b - iR - Q/C = 0$$

# Charging Capacitor

$$+V_b - iR - Q/C = 0$$

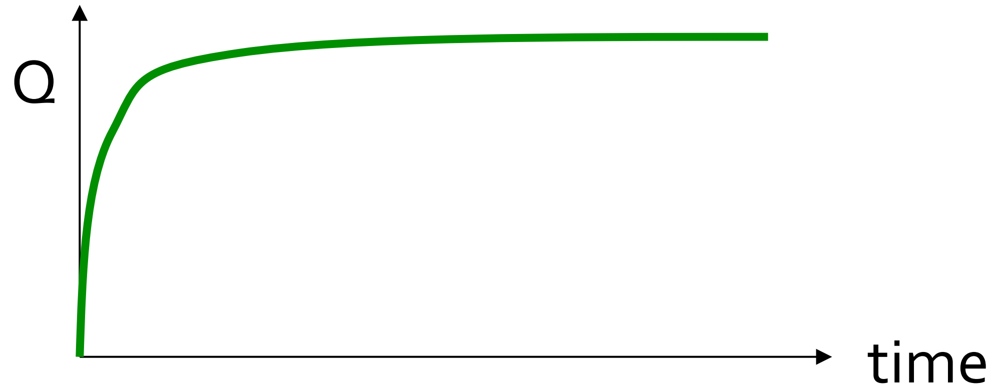
$$+V_b - \frac{dQ}{dt}R - Q/C = 0$$

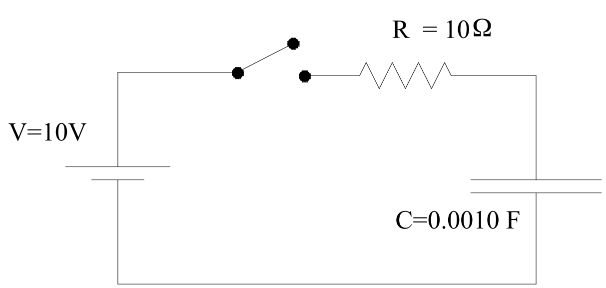
$$\frac{dQ}{dt} = +\frac{V_b}{R} - \frac{Q}{RC} \quad \Rightarrow \quad Q(t) = CV_b\left(1 - e^{-t/RC}\right)$$

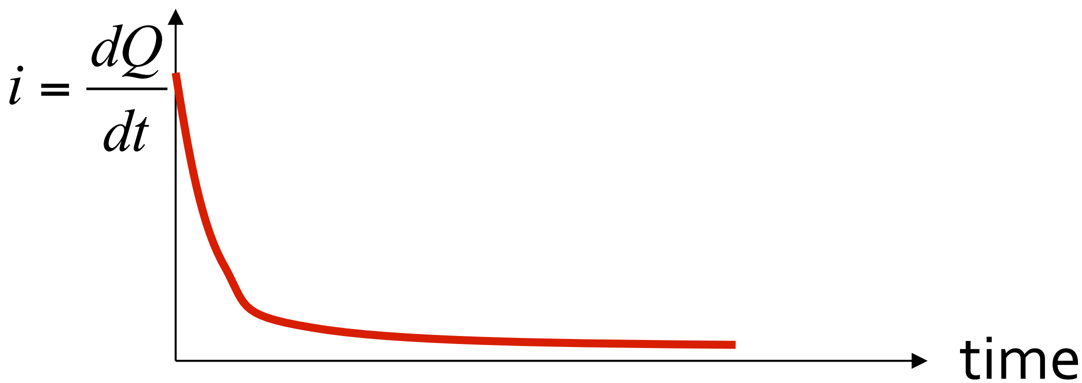
# Charging Capacitor

$$Q(t) = CV_b\left(1 - e^{-t/RC}\right)$$

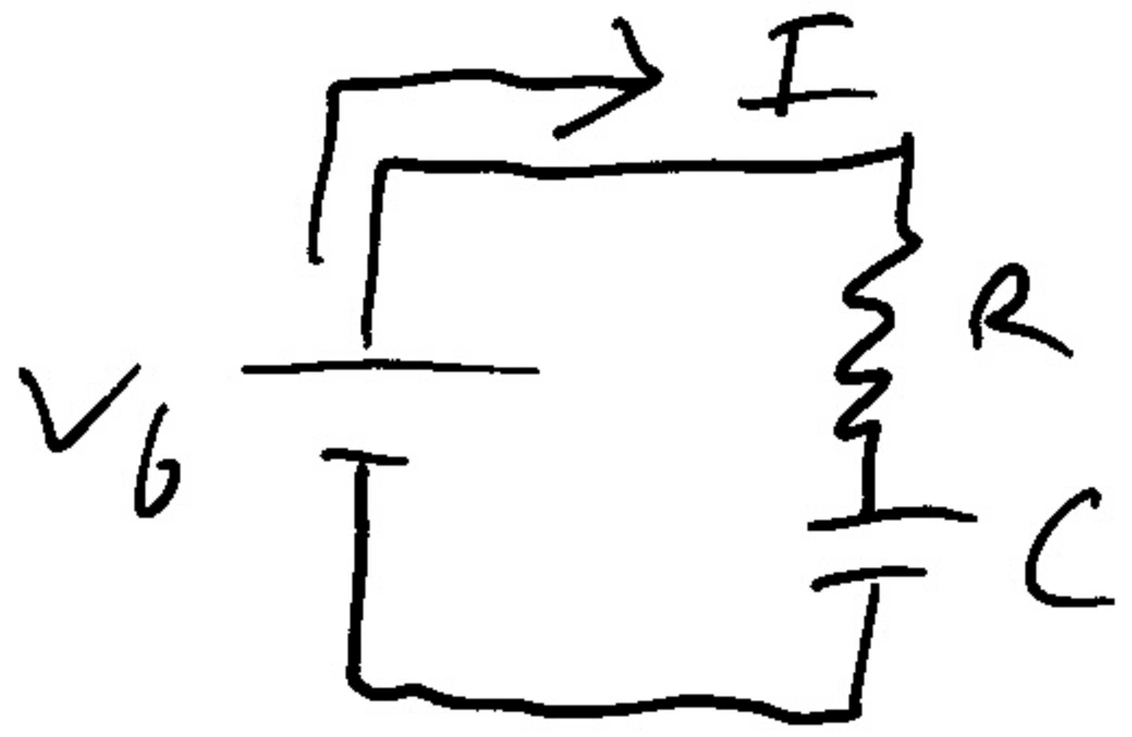
$$i(t) = \frac{dQ}{dt}(t) = \frac{V_b}{R}e^{-t/RC}$$



# Energy Dissipation Charging




$$U_{total} = \int IV\,dt$$

$$= \int_0^\infty \frac{V_0}{R} \cdot V_0\, e^{-t/RC}\,dt$$

$$= \frac{V_0^2}{R} \cdot -RC\, e^{-t/RC} \Big|_0^\infty$$

$$= CV_0^2$$

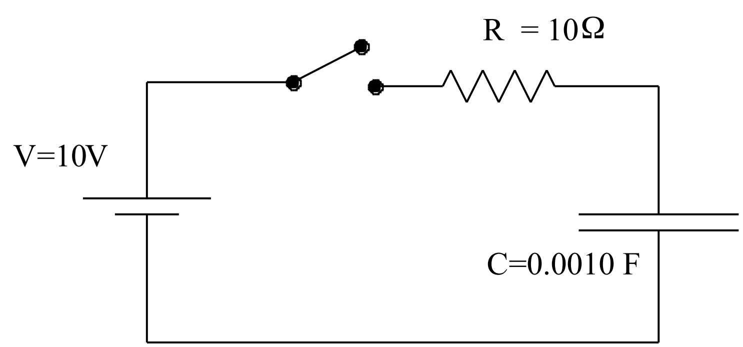
$$U_C = \tfrac{1}{2} Q_f^2/C = \tfrac{1}{2} CV_0^2 = U_{total}/2$$

- Half of power dissipated in resistor while charging, half goes to capacitor

# Concept Check

An RC circuit is shown below. Initially the switch is open and the capacitor has no charge. At time t=0, the switch is closed. What is the voltage across the capacitor ***immediately*** after the switch is closed (time = 0)?

A) Zero
B) 10 V
C) 5V
D) None of these.



# Capacitor in Circuit

Although no charge actually passes between the capacitor plates, it acts just like a current is flowing through it.

Uncharged capacitors act like a "short": $V_C=Q/C=0$

Fully charged capacitors act like an "open circuit". Must have $i_C = 0$ eventually, otherwise Q → infinity.

# Happy Spring Break!

Where I wish I was going to be…