Physics II: 1702/029:028
Electricity and Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Announcements

- Hardcopy homework due Wednesday
- Midterm II next Wednesday
  - Sample questions now posted on notes page

The Mystery of the Rogue Clickers

- If you have a clicker with device number
  - 90A57D
  - 9544C8
  - 61AD86

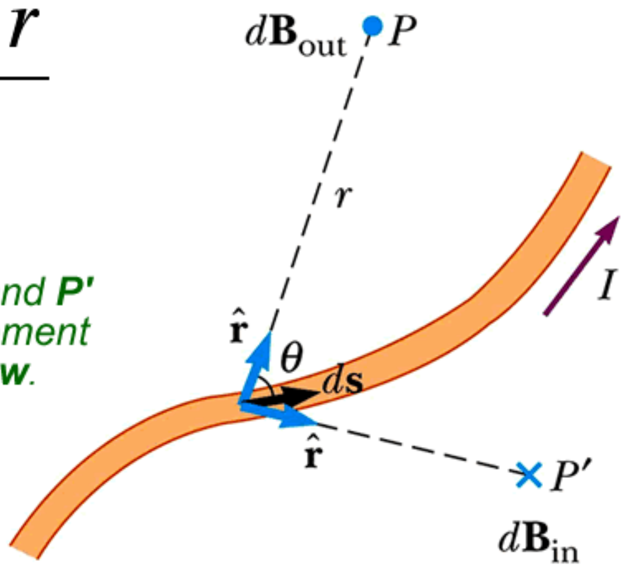
  - Please let me know, and make sure it is registered on ICON

Biot-Savart Law

The Biot-Savart Law

The Magnetic Field produced by the current in the wire

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2}$$



The magnetic field d**B** at a point **P** and **P'** due to the current **I** thru a length element d**s** is given by the **Biot-Savart law**.

This Law is based upon experimental observation

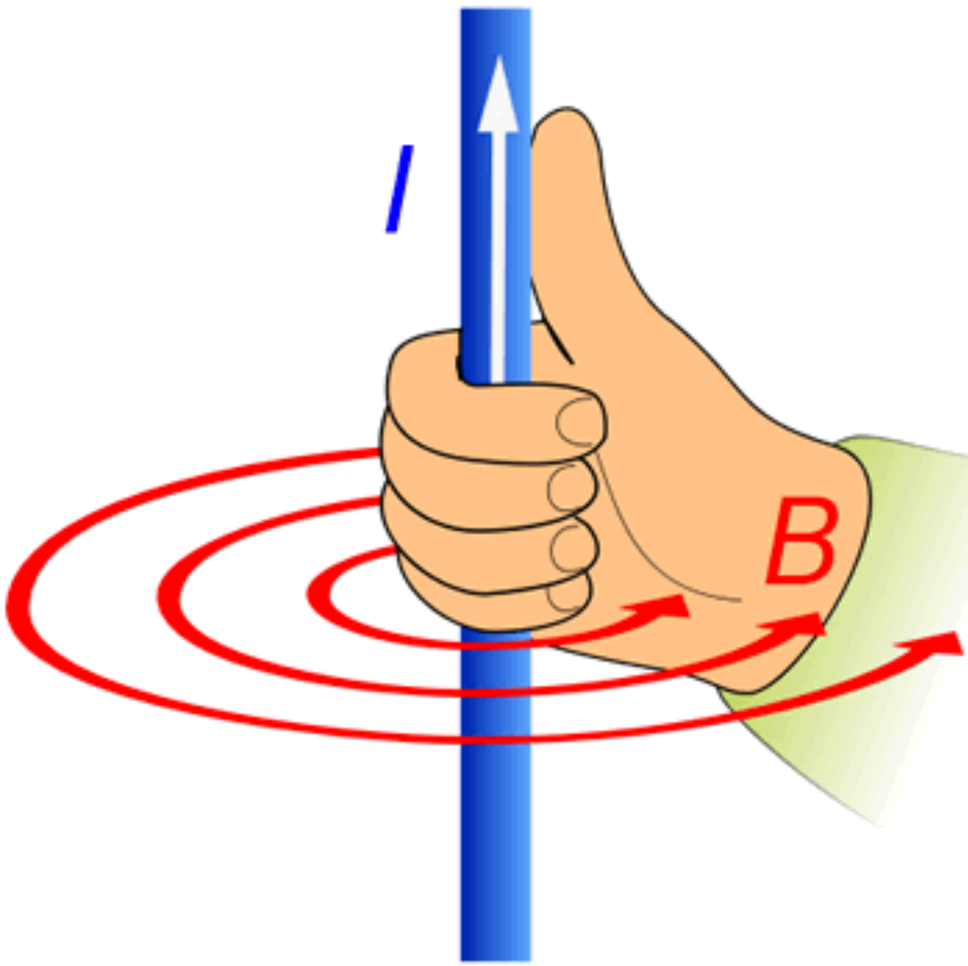
The vector d**B** is perpendicular to both d**s** and to the unit vector ***r*** directed toward the point **P.**

The magnitude of d**B** is proportional to the current ***I***, to the length element d**s** and to the sine of the angle between d**s** and **r.** It is also inversely proportional to r^2.

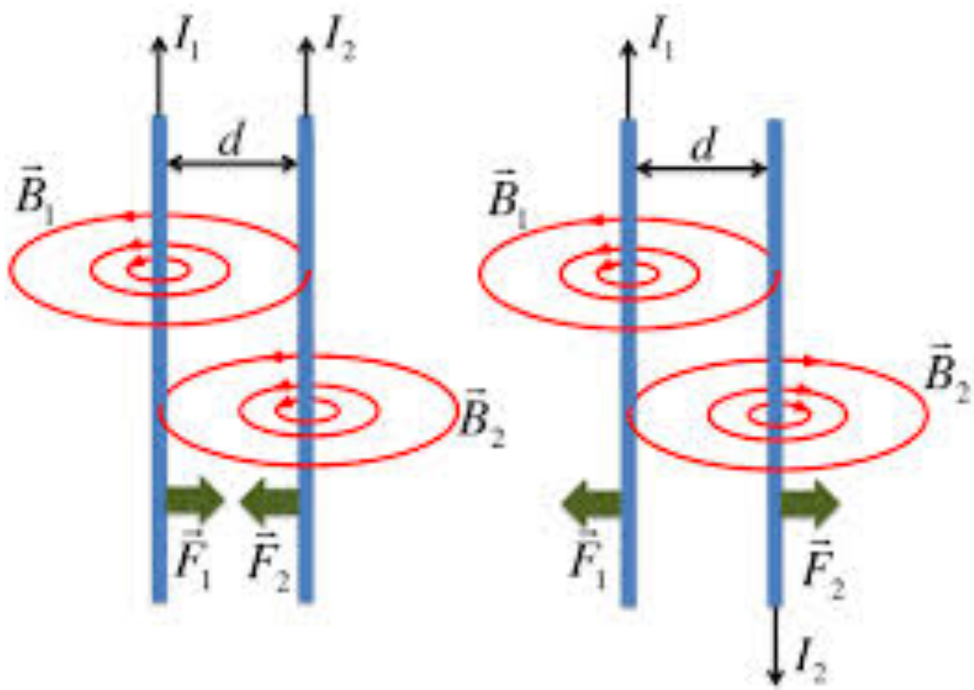
Obviously we will Integrate over the entire current distribution.

$$\vec{B} = \frac{\mu_o I}{4\pi}\int \frac{d\vec{s} \times \hat{r}}{r^2}$$

Magnetic Fields Generated by Wires

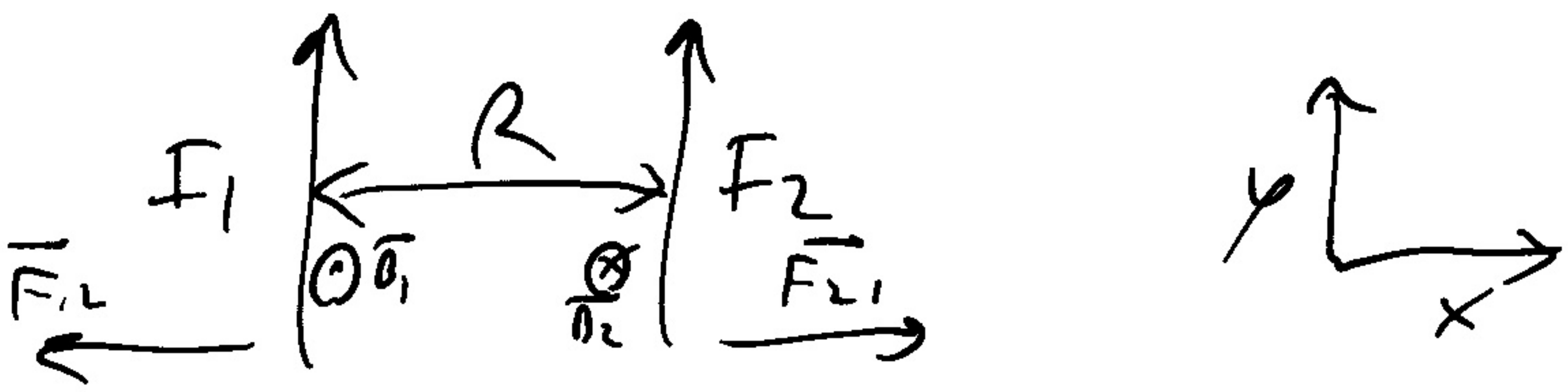


Forces Between Wires



Like Currents Attract, Opposite Currents Repel
Contrast w/ Case for Like and Opposite Charges

Force between wires



$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi R}\hat{k}$$

$$\vec{F}_{21} = I_2 \vec{L}_2 \times \vec{B}_1$$

$$= I_2 L \hat{j} \times \frac{-\mu_0 I_1}{2\pi R}\hat{k}$$

$$= -\frac{\mu_0 I_1 I_2 L}{2\pi R}\hat{i} \quad \text{left}$$

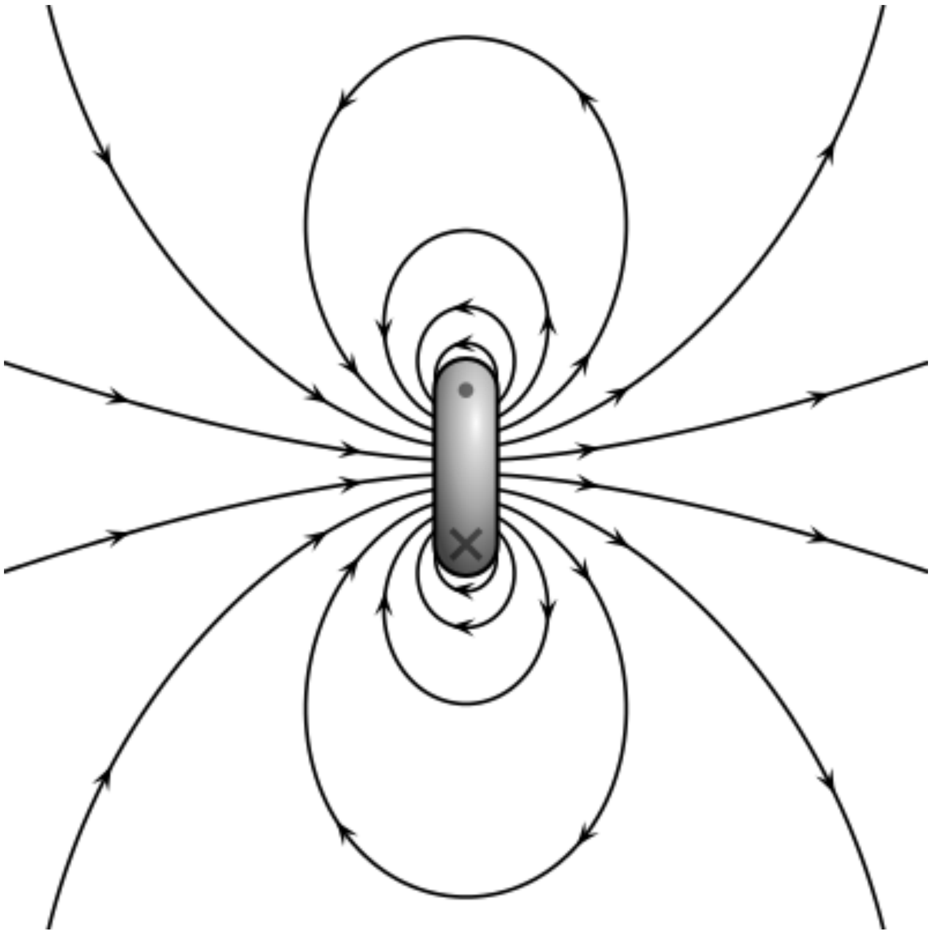
$$\vec{F}_{12} = \frac{\mu_0 I_1 I_2 L}{2\pi R}\hat{i} \quad \text{right}$$

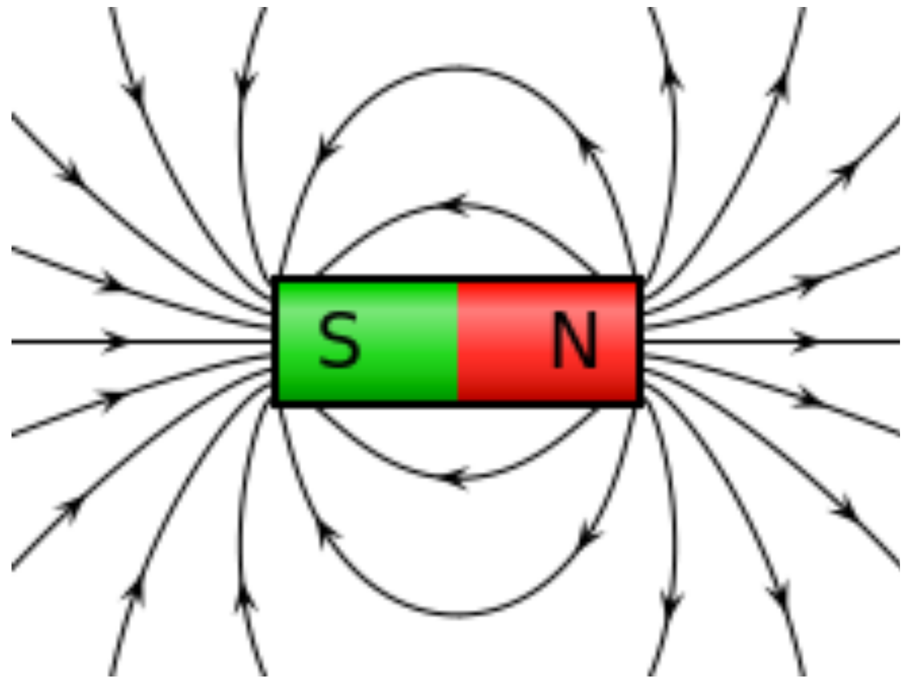
- Like currents attract
- Opposites repel

F/L = Force per length

$$= \frac{\mu_0 I_1 I_2}{2\pi R}$$

Magnetic Dipoles



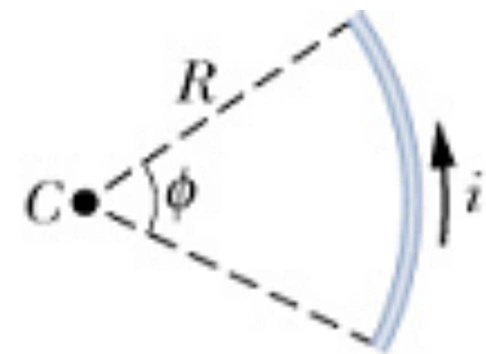


Concept Check

- To build up the magnetic field of an arc, we have to add up the fields caused by each portion of the arc

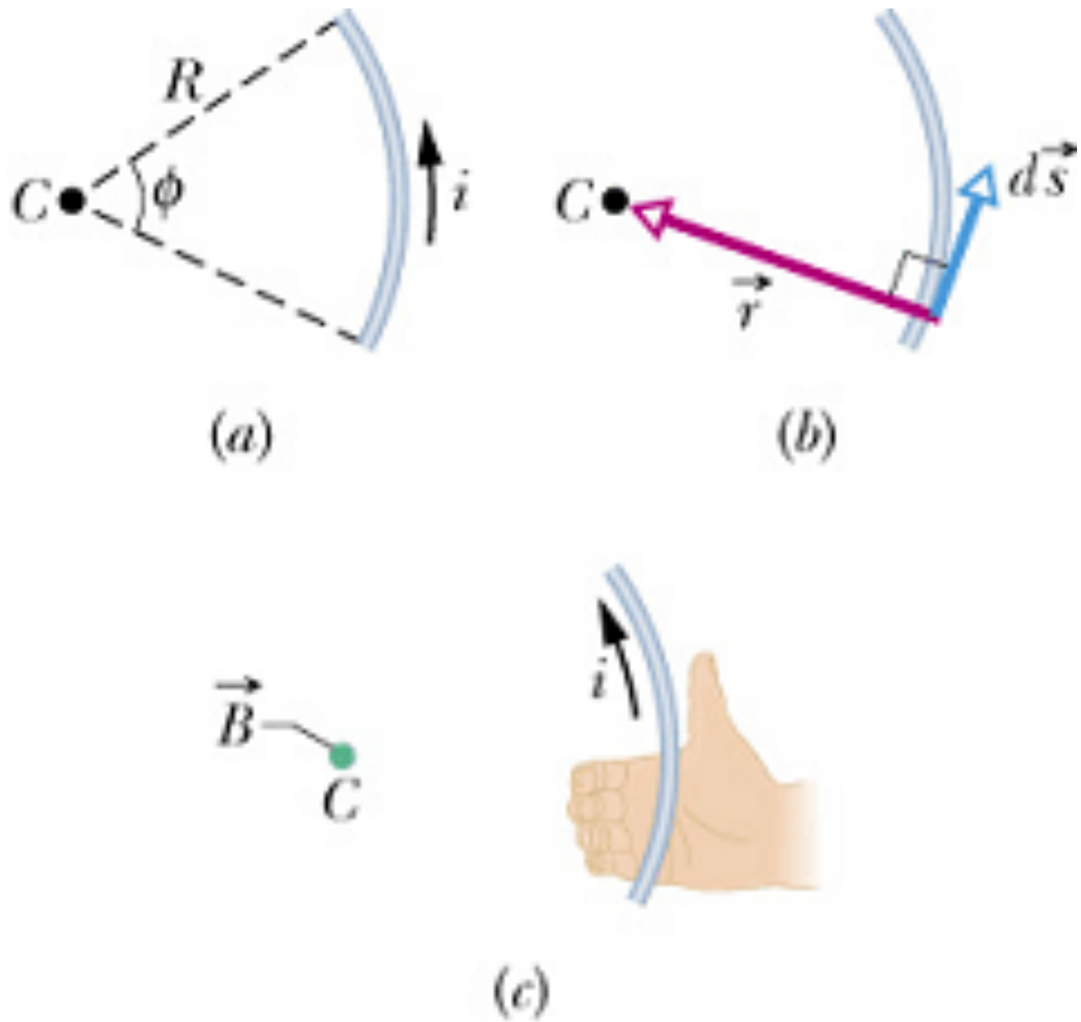
$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$

- How does the direction of the magnetic field at point C caused by the top of the arc of current shown compare to that at point C caused by the bottom of the arc?

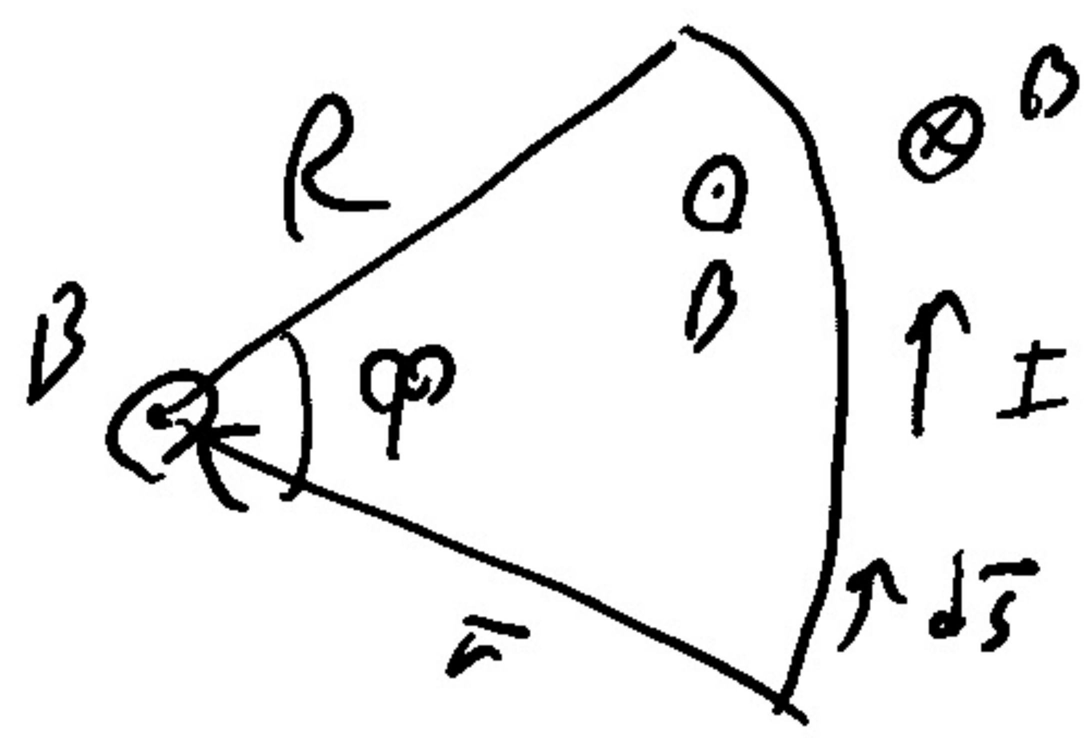


A. Parallel
B. Anti-parallel
C. At an angle of Φ from each other
D. Both are equal to zero

Magnetic Field of Arc, At Center



- Magnetic Field of Arc



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$d\vec{s} = dL\, \hat{\varphi}$$
$$= r\, d\varphi\, \hat{\varphi}$$

$$\hat{\varphi} \times \hat{r} = \hat{k}$$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, r\, d\varphi\, \hat{k}}{r^2}$$

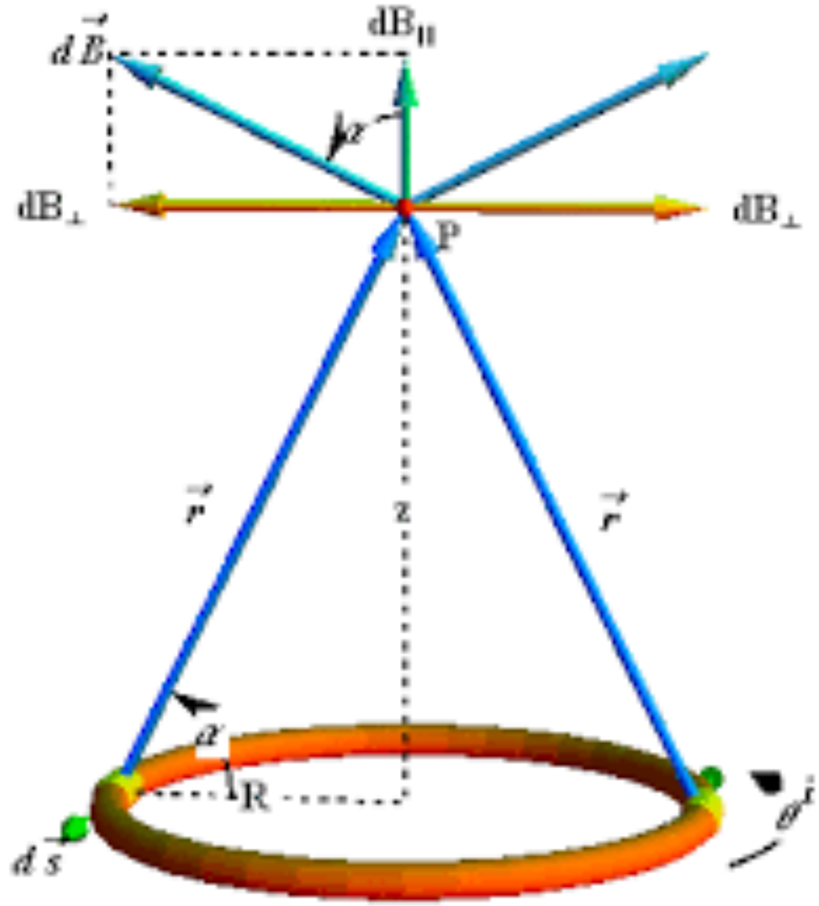
$$= \frac{\mu_0 I\, d\varphi\, \hat{k}}{4\pi r}$$

$$\vec{B} = \int d\vec{B}$$
$$= \int_{\varphi_{min}}^{\varphi_{max}} \frac{\mu_0 I\, d\varphi}{4\pi R} \hat{k}$$

$$= \frac{\mu_0 I}{4\pi R} (\varphi_{max} - \varphi_{min})$$

$$= \boxed{\frac{\mu_0 I}{4\pi R} \Delta\varphi}$$

Magnetic Field of Loop, On Axis

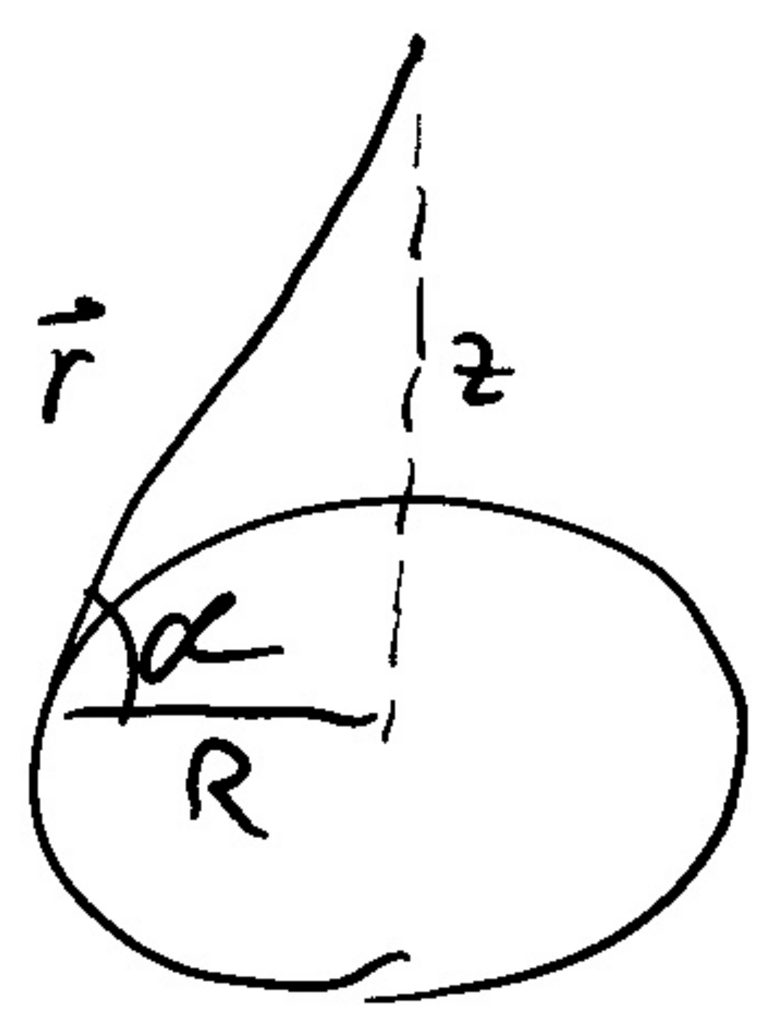


- Magnetic Field @ Center of circle

$$\Delta\varphi = 2\pi$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2R}}$$

- What about along axis?



$$|\vec{r}| = \sqrt{R^2 + z^2}$$

$$\cos\alpha = R/\sqrt{R^2+z^2} = R/r$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$|d\vec{B}| = \frac{\mu_0}{4\pi} \frac{I \cdot R\,d\varphi \cdot}{r^2}$$

$$dB_z = |d\vec{B}| \cos\alpha$$

$$= \frac{\mu_0 I R\,d\varphi}{4\pi r^2} \cdot R/r$$

$$= \frac{\mu_0 I R^2\,d\varphi}{4\pi (R^2+z^2)^{3/2}} \Rightarrow \boxed{B_z = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}}$$

Dipole magnetic field on axis:

$$1/(R^2+z^2)^{3/2} \sim 1/z^3$$

for $z \gg R$

$$\Rightarrow B = \frac{\mu_0 I R^2}{2 z^3}$$

but $\mu = IA = I \cdot \pi R^2$

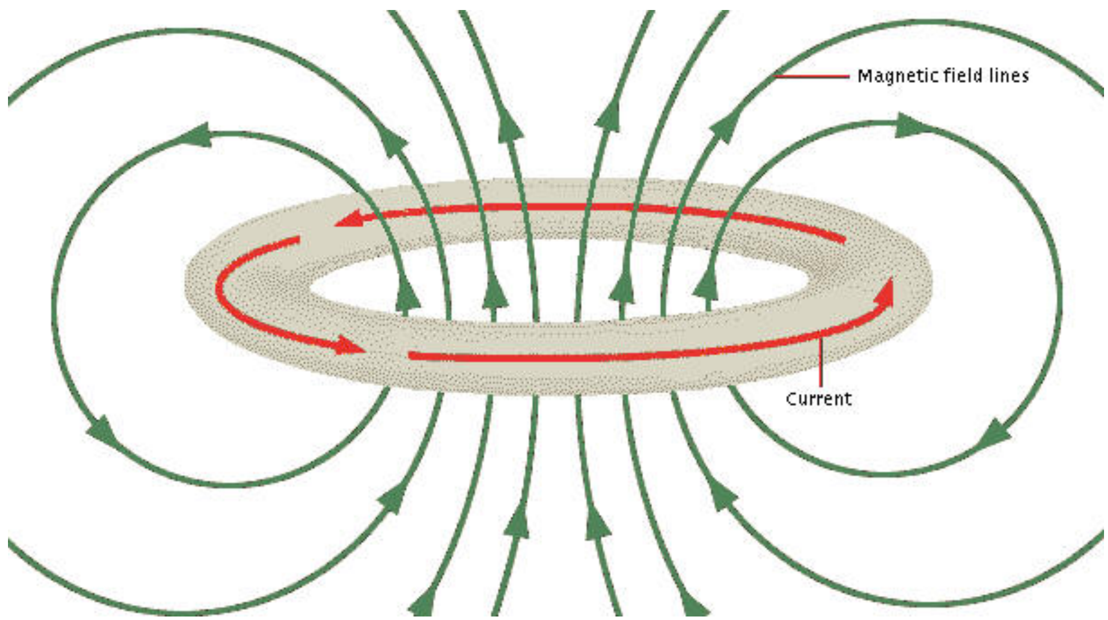
$$\Rightarrow B = \frac{\mu_0 \mu}{2\pi z^3}$$

or $$\boxed{\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}}$$

compare to

$$\vec{E} = \frac{1}{2\pi\varepsilon_0} \frac{\vec{p}}{z^3}$$ for electric dipole

Magnetic Field of Current Loop

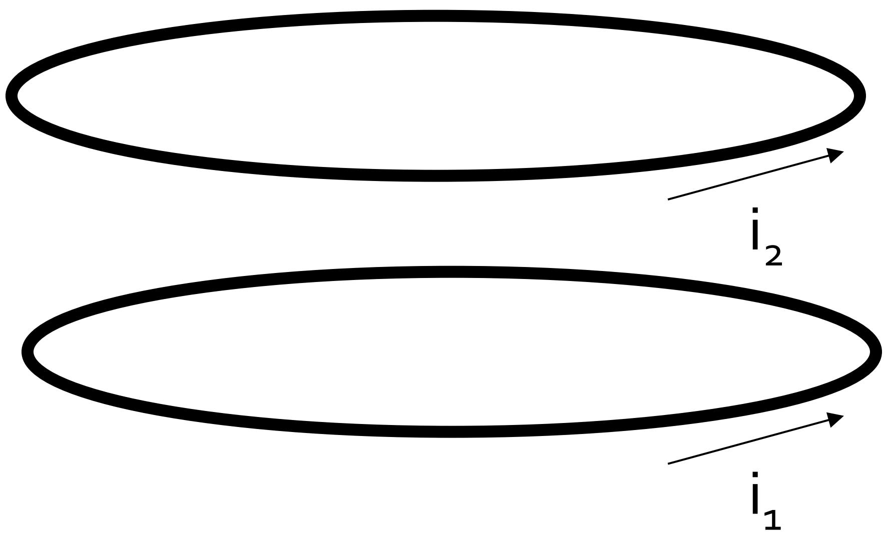


Concept Check

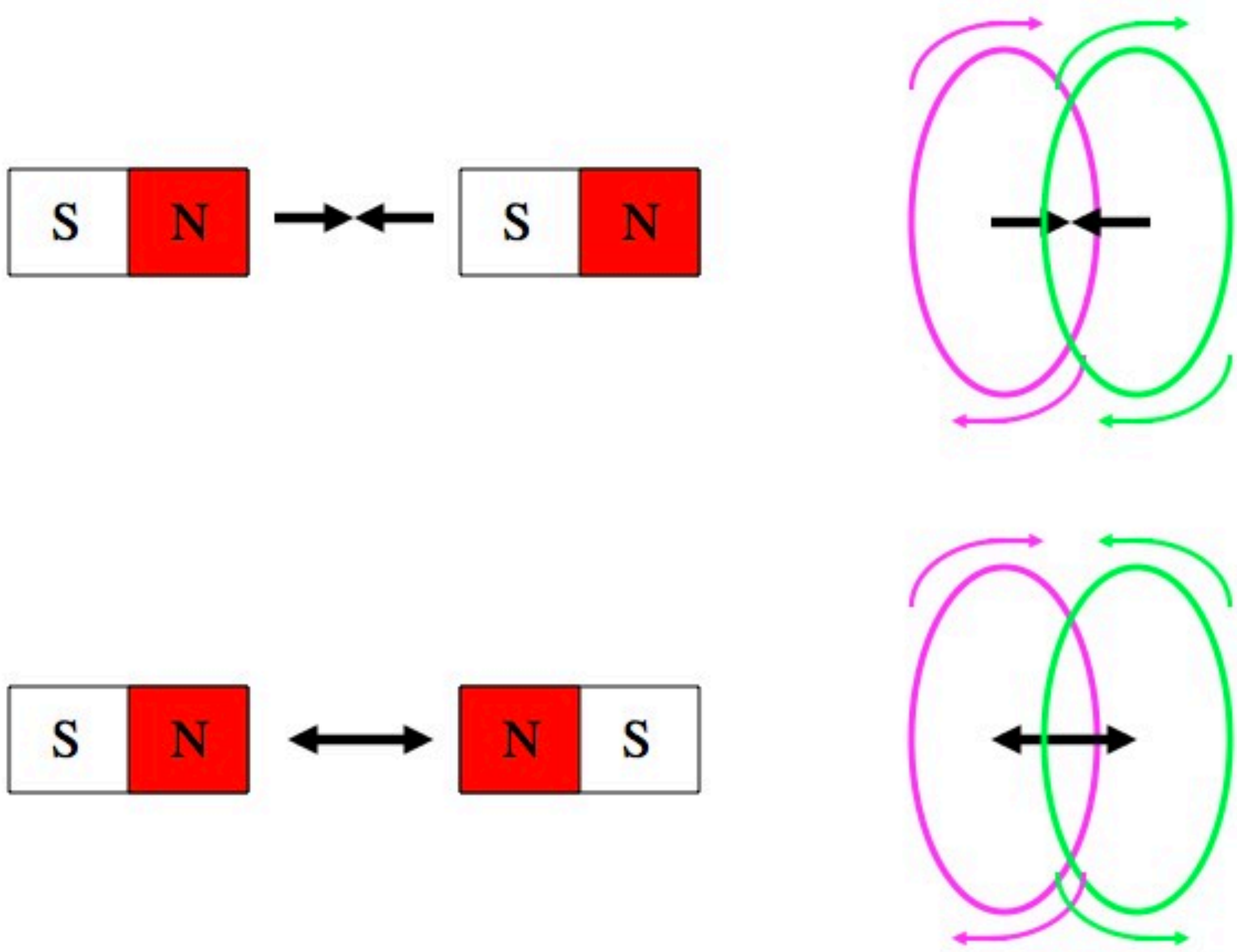
Two loops of wire have current going around in the same direction.

The forces between the loops is:

A)Attractive
B)Repulsive
C)Net force is zero.



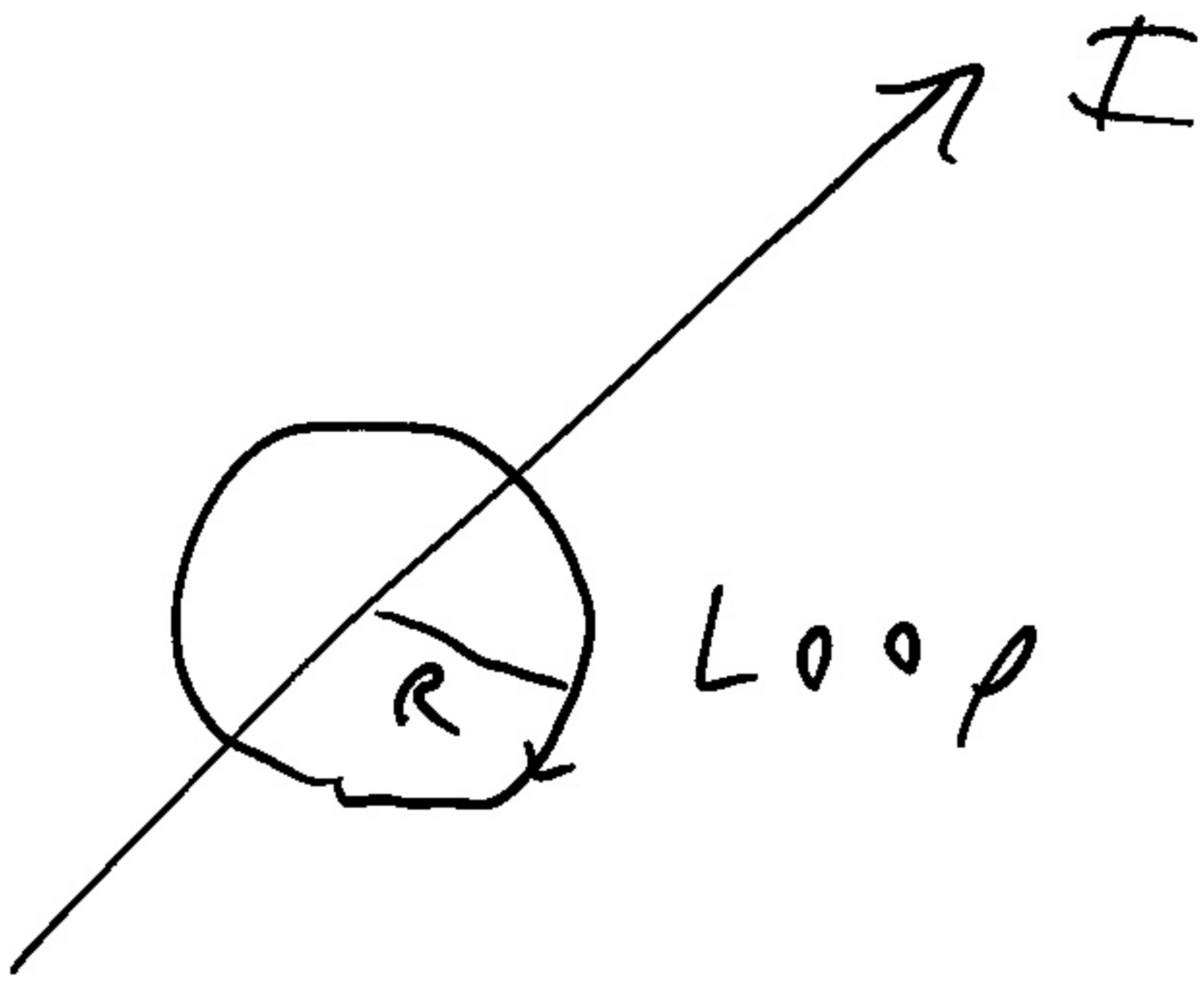
Current Loops & Bar Magnets



Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enc}$$

Ampere's Law : Thin Wire



R - hand rule

$\Rightarrow$ B circulates around wire

$$\oint \vec{B} \cdot d\vec{l} = \oint B \, dl$$

since $\vec{B} \parallel d\vec{l}$

$$\Rightarrow B \cdot 2\pi R = \mu_0 I_{enc}$$
$$= \mu_0 I$$

$$\Rightarrow \boxed{B(r) = \mu_0 I / 2\pi r}$$

- compare to $E(r) = \lambda / 2\pi\varepsilon_0 r$
- But B circulating rather than radial