Physics II: 1702/029:028 Electricity and Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enc}$$

Maxwell's Equations: Integral Form

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$ ✔

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$ ✔

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$ ✔-

Symmetry Strikes Again

Ampere's Law (for constant currents) is always true.

$$\oint_{loop} \vec{B} \cdot d\vec{l} = \mu_0 i_{thru}$$

However, like Gauss' Law, it is only useful if there is a nice symmetry for solving the left integral.

Ampere's Law: Wire

Use Ampere's law to obtain the magnetic field.

$$\sum B_{\parallel}\Delta\ell = \mu_o I$$

$$\Downarrow$$

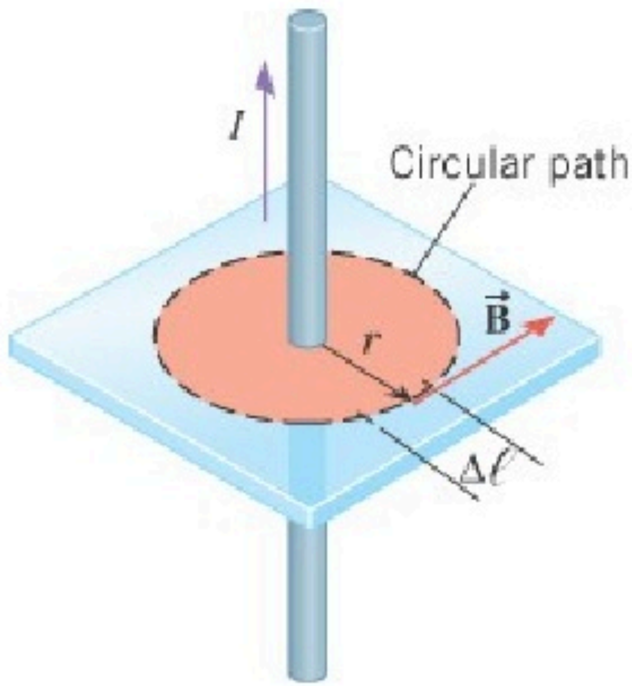
$$B\left(\sum \Delta\ell\right) = \mu_o I$$

$$\Downarrow$$

$$B 2\pi r = \mu_o I$$

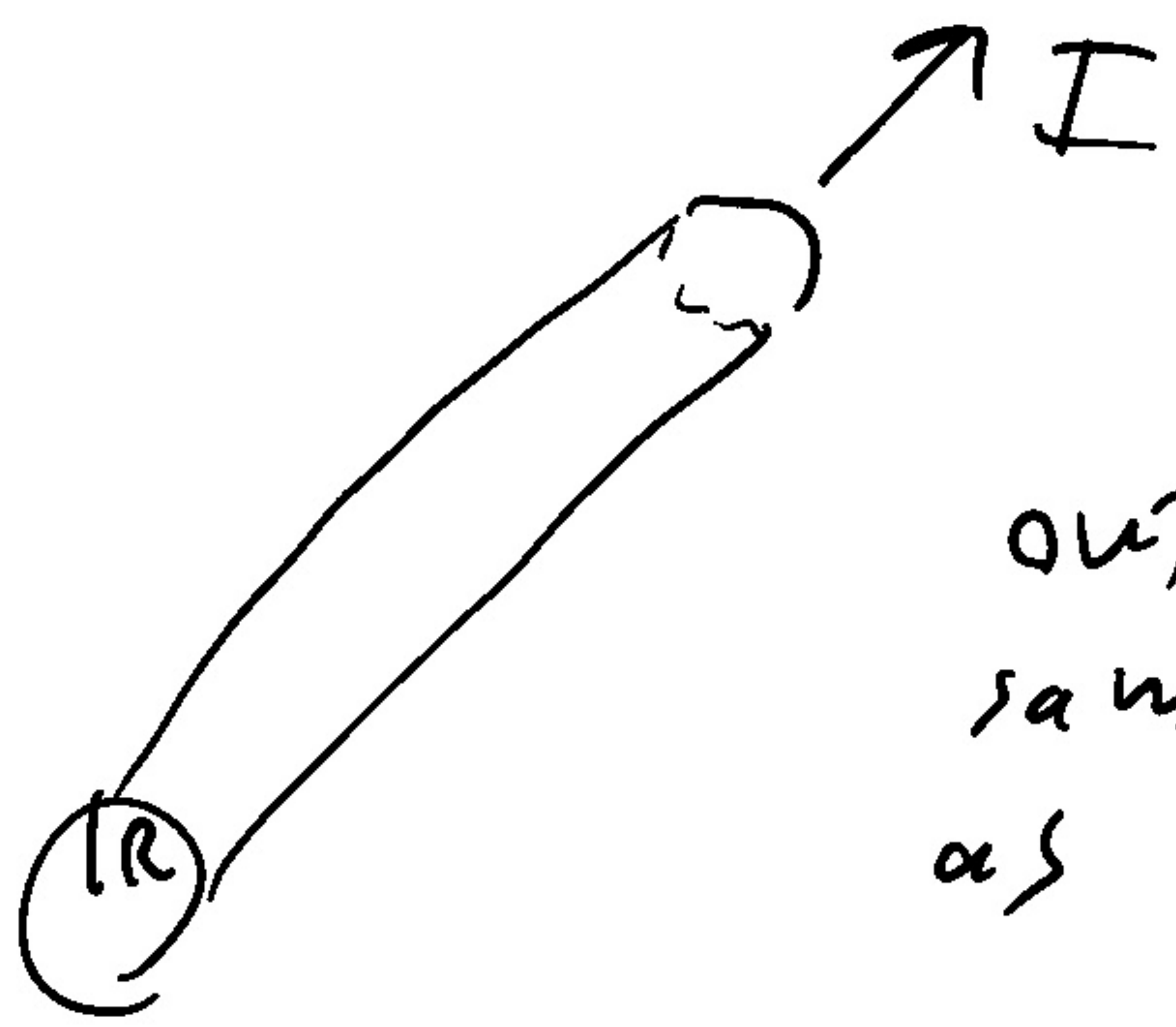
$$\Downarrow$$

$$B = \frac{\mu_o I}{2\pi r}$$



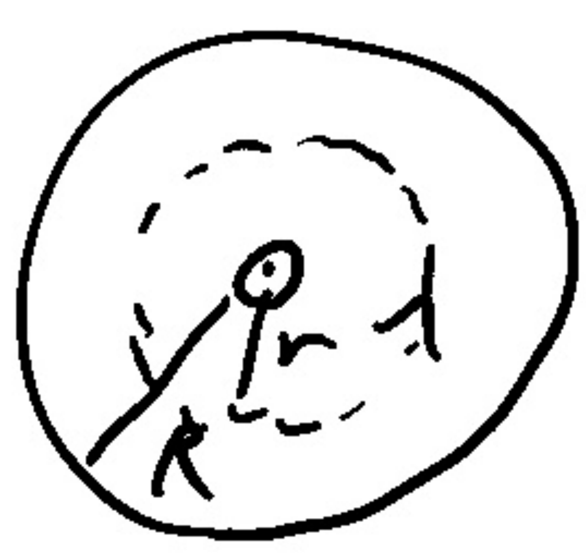
Ampere's Law : Thick Wire

$$J = I / \pi R^2$$



outside,
same solution
as thin wire

Inside



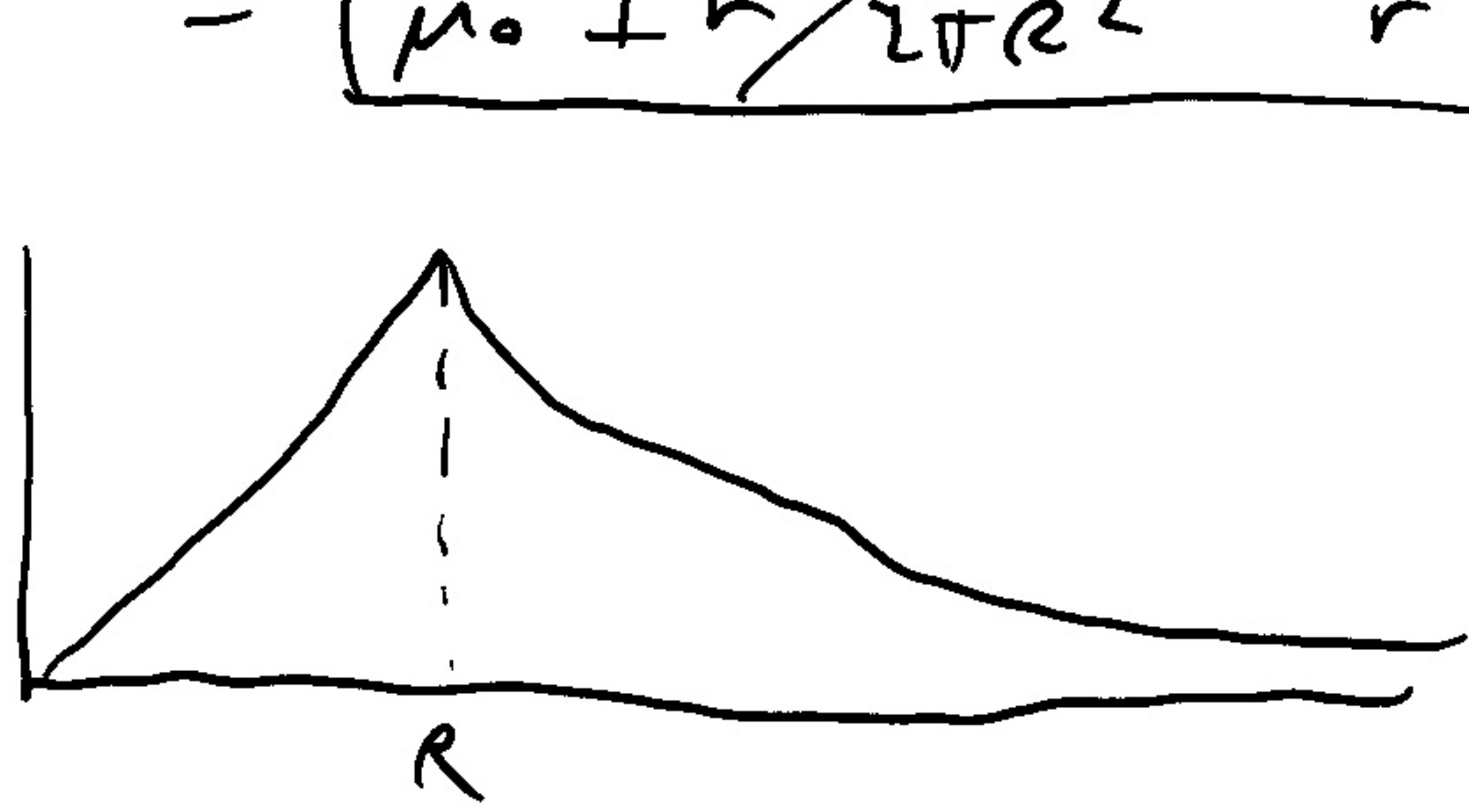
$$\oint \vec{B} \cdot d\vec{l} = B(r) \cdot 2\pi r$$

$$I_{enc} = \int \vec{J} \cdot d\vec{A} = J \cdot A$$
$$= J \cdot \pi r^2$$
$$= I \cdot \pi r^2 / \pi R^2 = I r^2 / R^2$$

$$\Rightarrow \quad B(r) = \mu_0 I \, r^2 / R^2 \cdot 1 / 2\pi r$$

$$= \boxed{\mu_0 I r / 2\pi R^2 \quad r < R}$$

B(r)

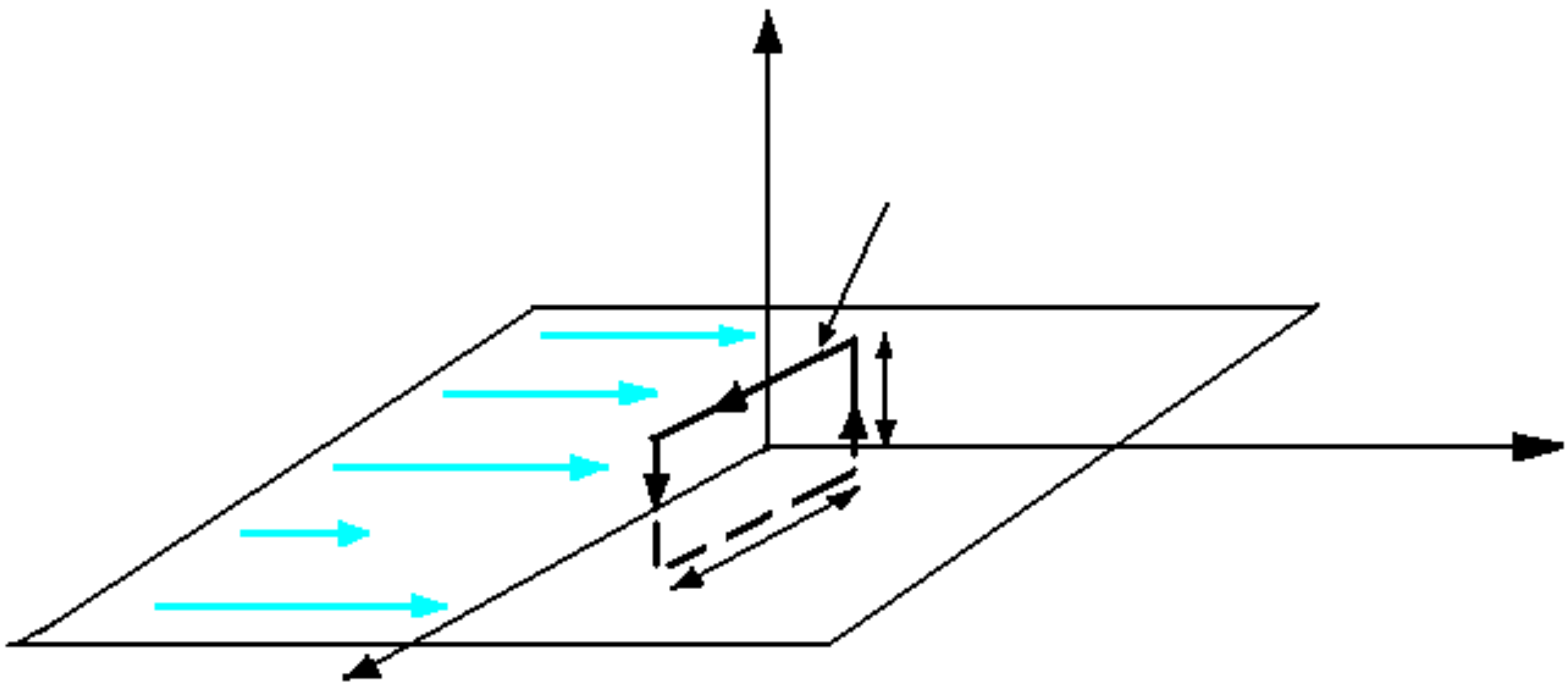


Concept Check

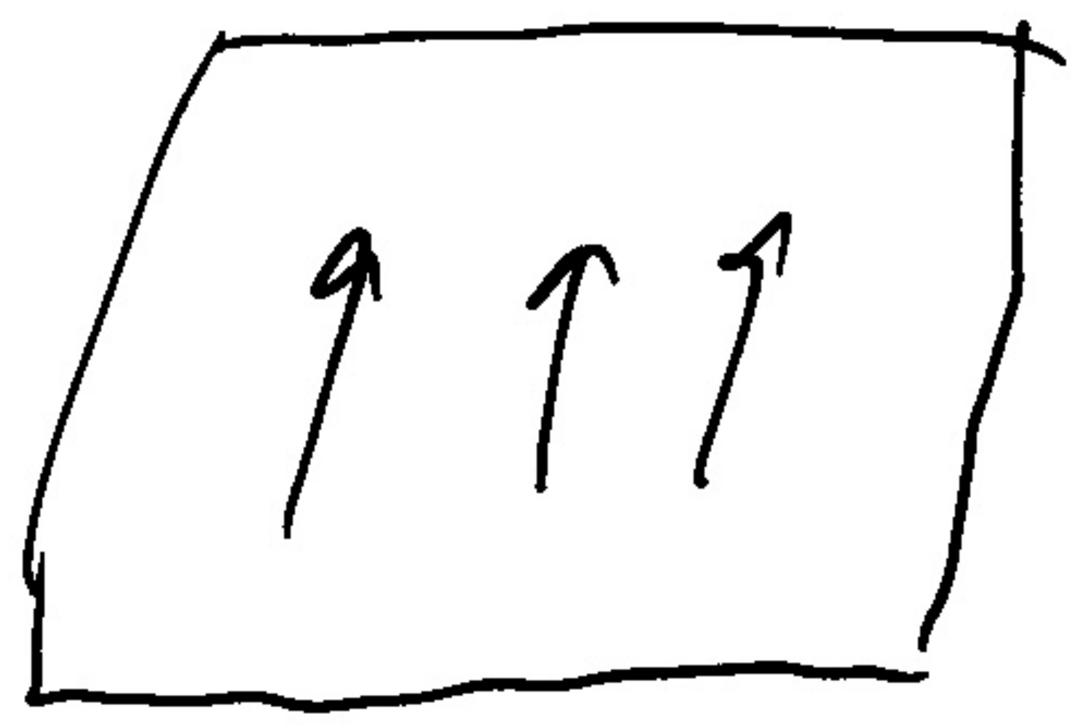
- A cross section of the magnetic field near a ribbon of current is shown. The direction of the current in the ribbon is:

A. To the left.
B. To the right.
C. Into the screen.
D. Out of the screen.

Ampere's Law: Planar Current

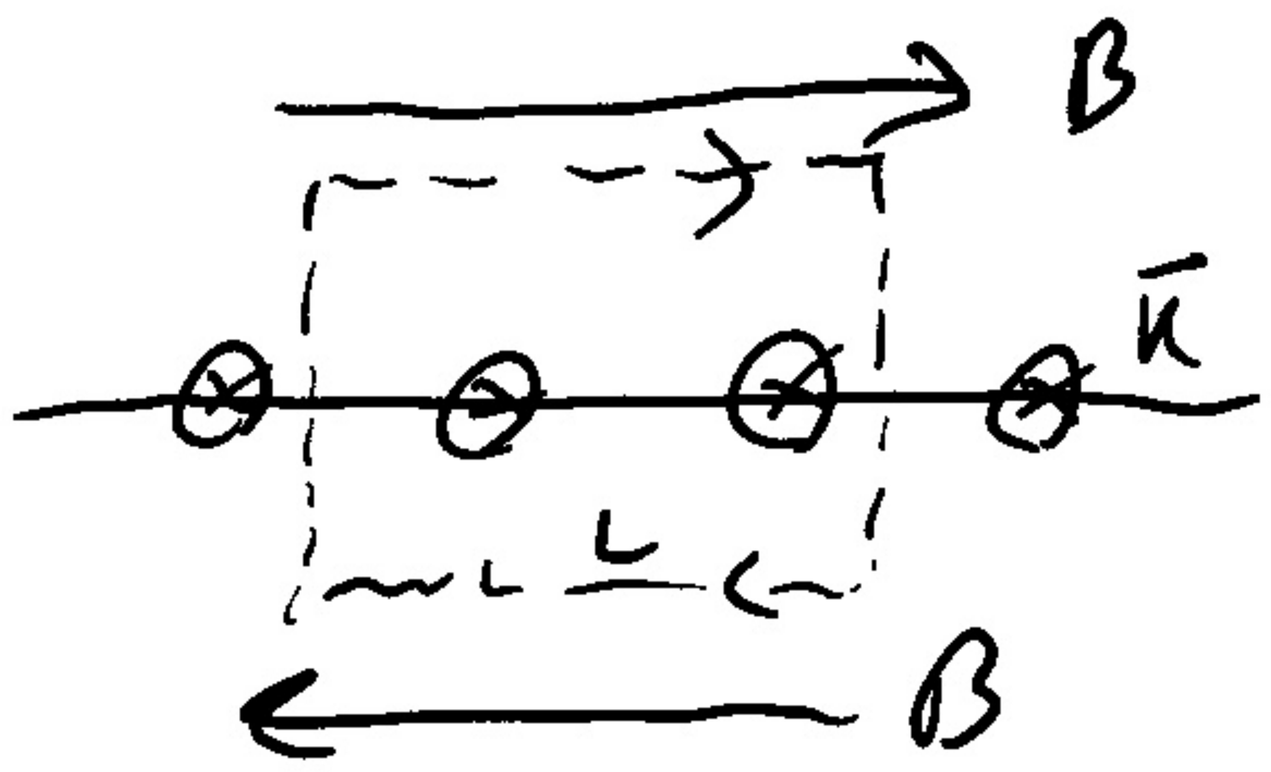


Ampere's Law: planar current "Ribbon"



$\vec{K}$ = current per length

= A/m



$$\oint \vec{B} \cdot d\vec{l} = B \cdot L + B \cdot L$$
$$= 2BL$$

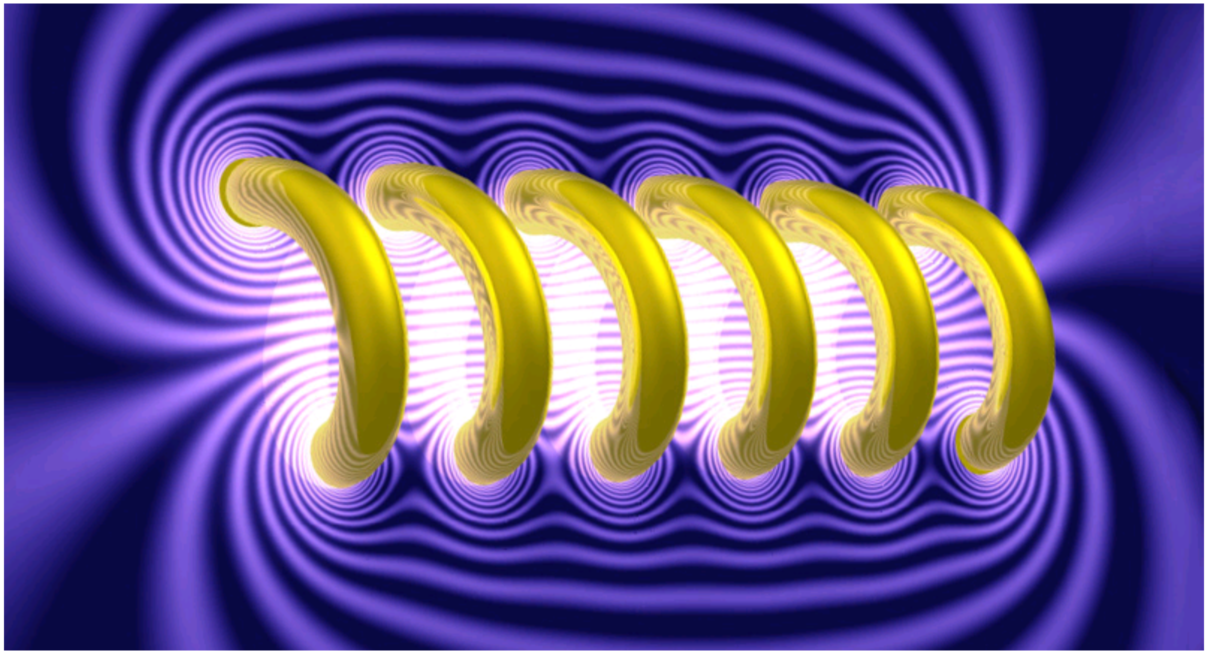
$$I_{enc} = K \cdot L$$

$$\Rightarrow B = \mu_0 KL / 2L$$
$$= \frac{\mu_0 K}{2}$$

- Compare to $E = \sigma / 2\varepsilon_0$
- But B tangential instead of out

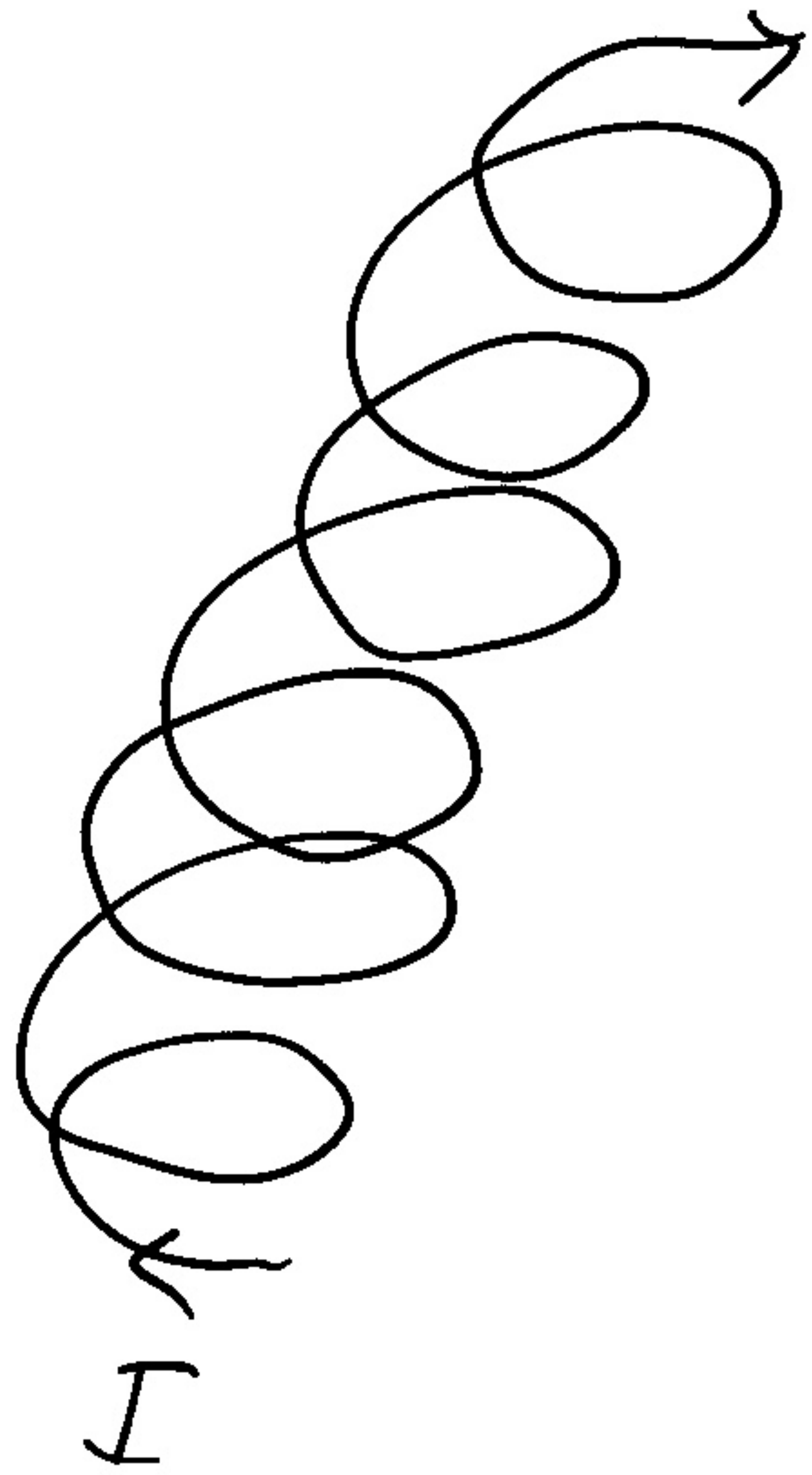
Solenoids

Solenoid

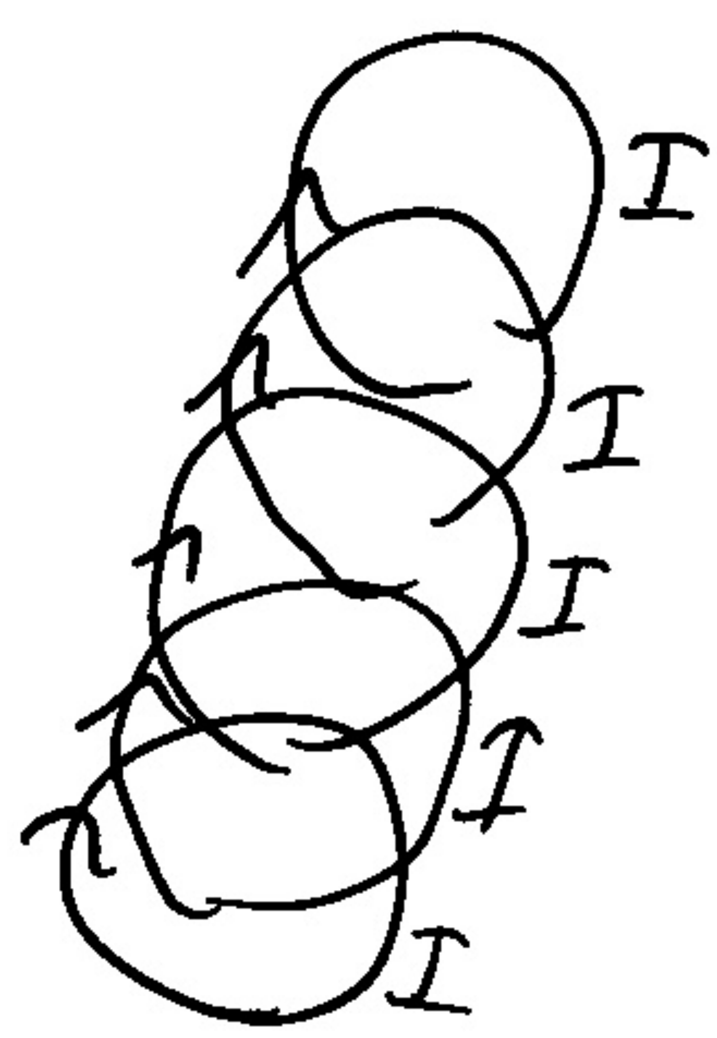


A single wire tightly coiled up into loops.
Since it is a single wire, the current magnitude is the same in all parts of the coil.

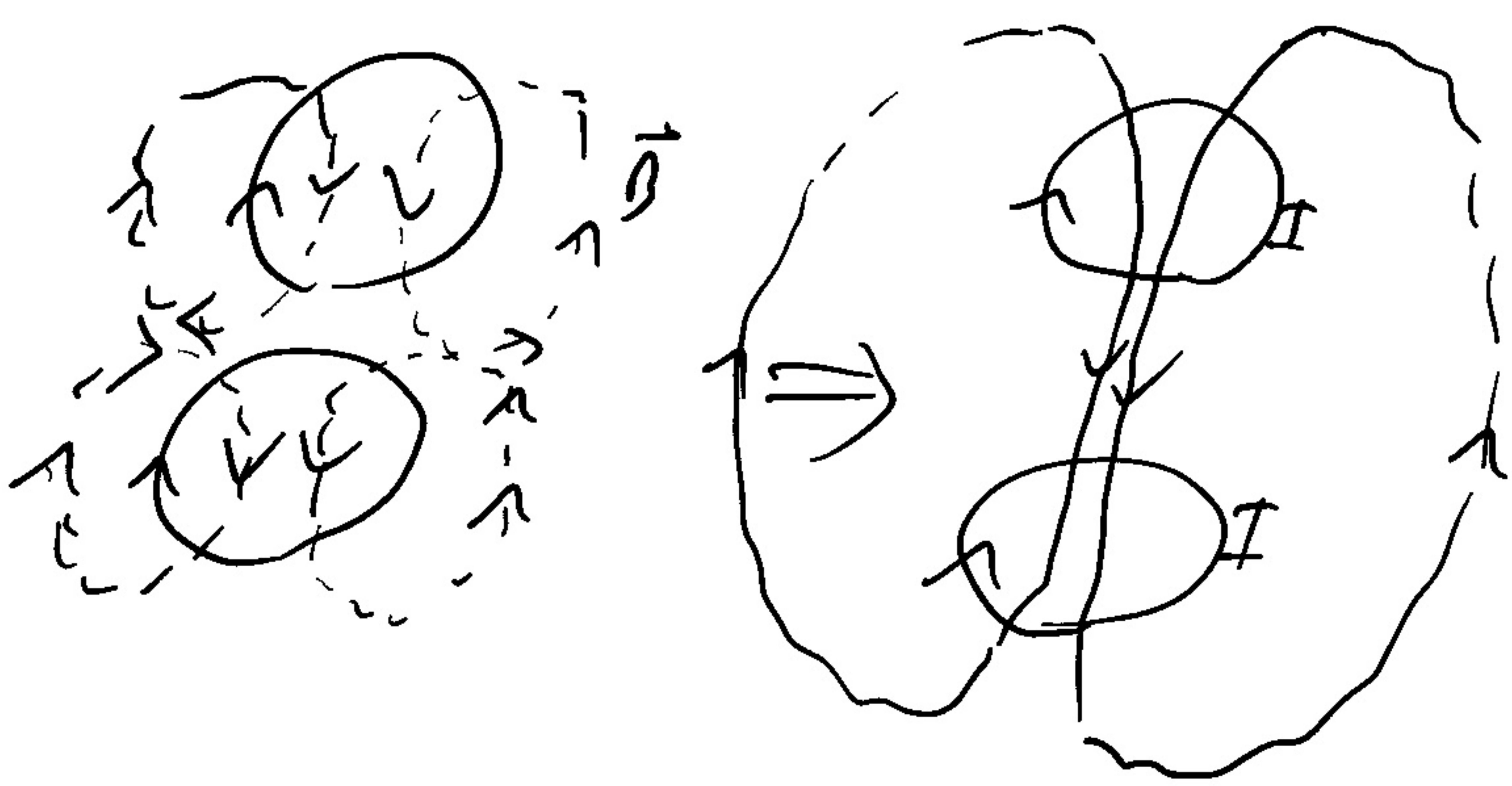
Solenoid



If tightly wound, looks like:

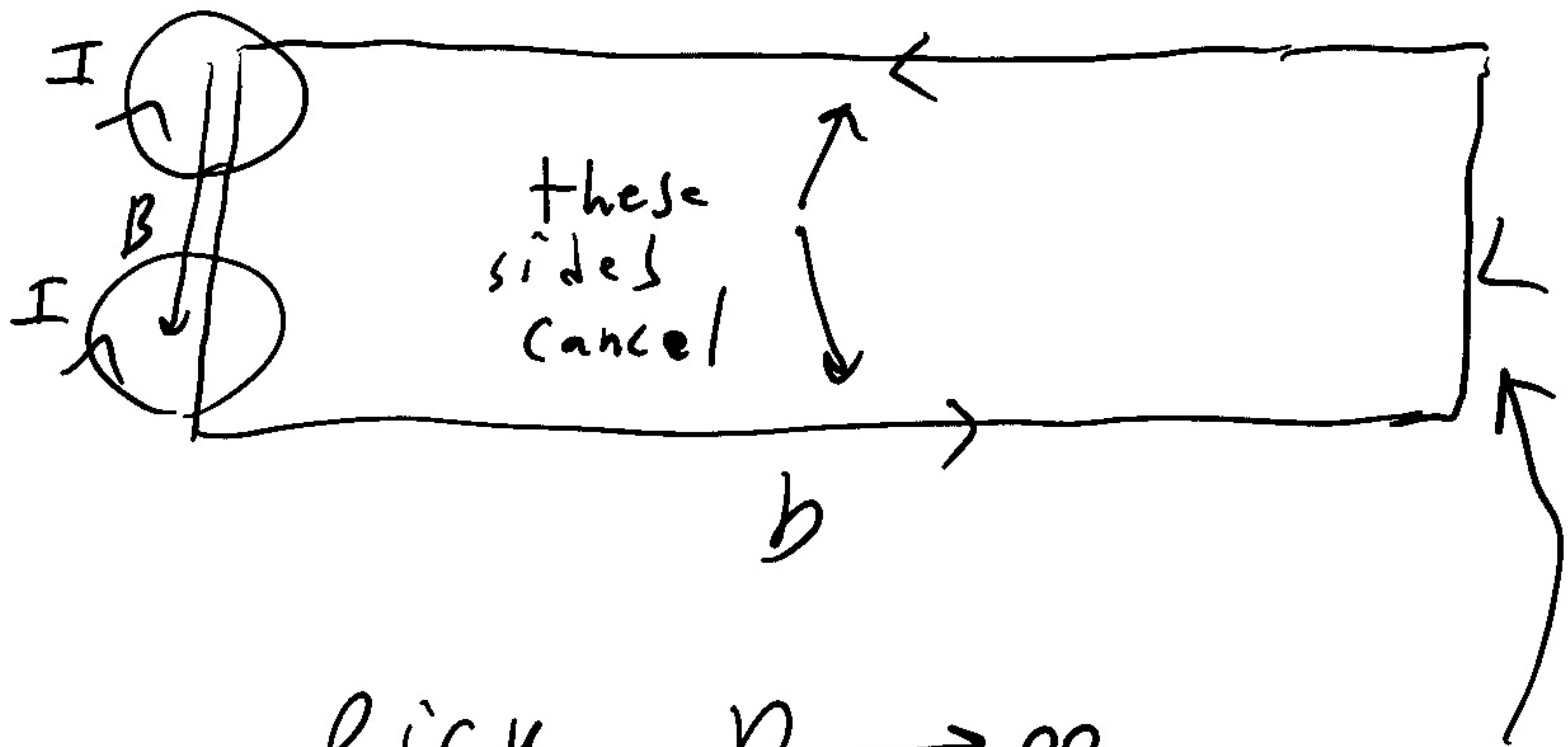


stacked rings



More tightly spaced
⇒ stronger axial field in center
(and weaker outside)

Use Ampere's Law



Pick $b \to \infty$, so B is zero here

$$\oint \vec{B} \cdot d\vec{\ell} = BL$$

$$= \mu_0 I_{enc}$$

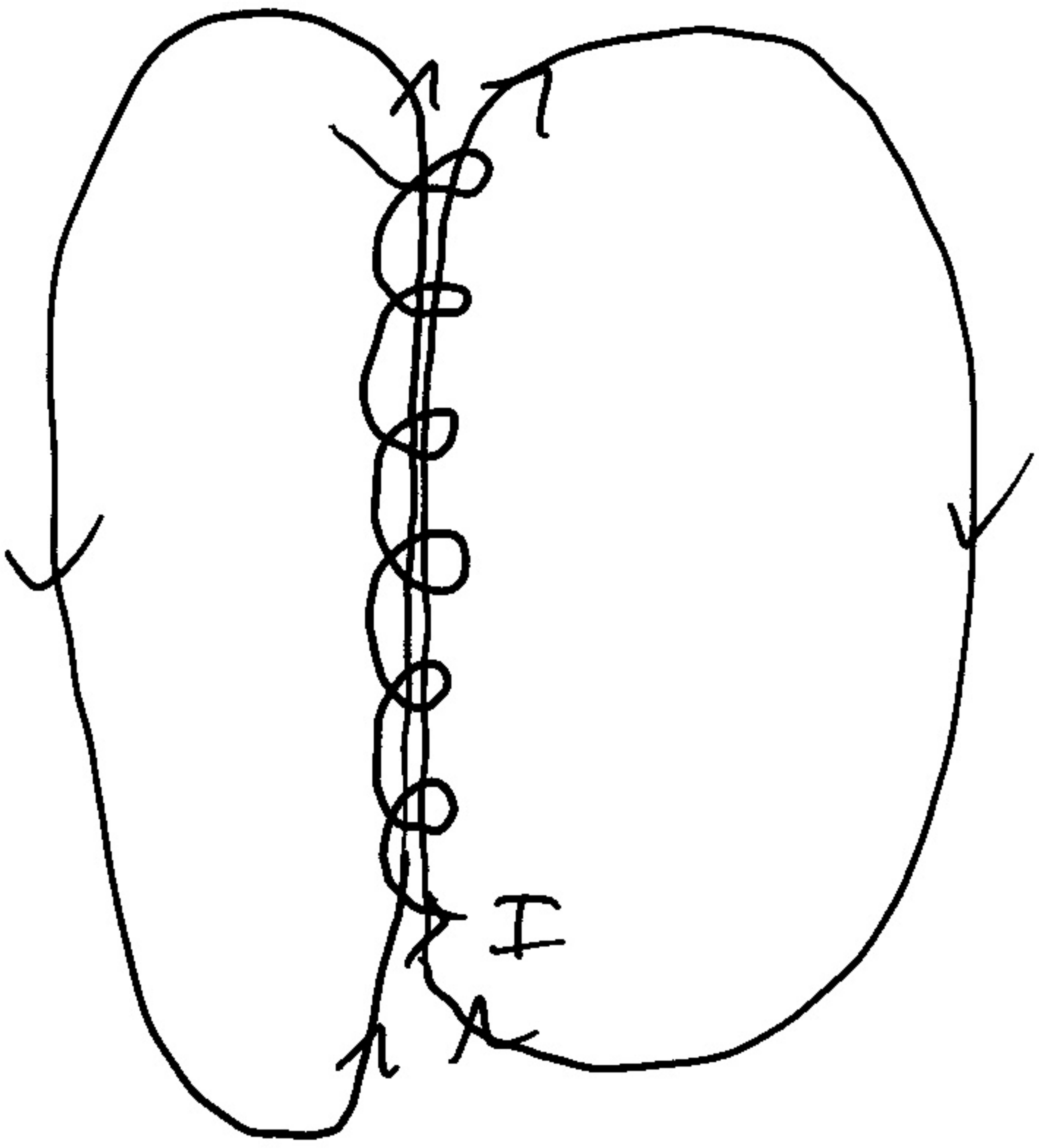
$$= \mu_0 N I$$

N = # of current loops in length L

$$\Rightarrow B = \mu_0 N I / L$$

$$= \mu_0 I n \quad \text{w/ } n = \frac{\#\text{ loops}}{\text{unit length}}$$

- Field outside solenoid



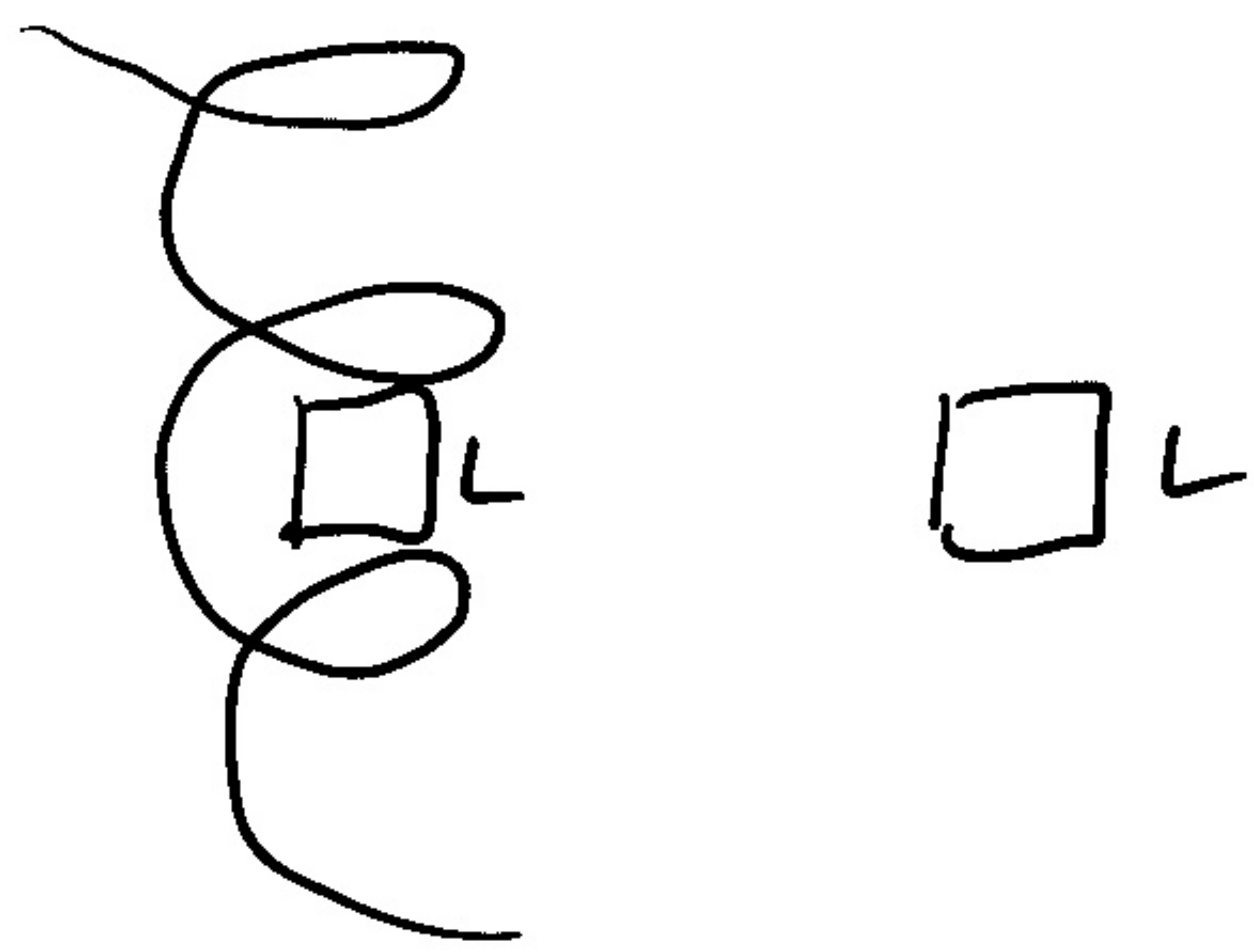
Finite solenoid

- Infinite solenoid

$B_{ext} = 0$

- Field from opposite sides of loop cancels for infinite solenoid
- All field lines contained in solenoid

Infinite solenoid



- Loops inside and out
- Top and bottom legs are zero for infinite solenoid

$B_{left} \cdot L - B_{right} \cdot L = 0$
for both

$\Rightarrow B_{right} = B_{left}$

$\Rightarrow$ field uniform inside and out

Inside $B = \mu_0 n I$ same as on axis
outside $B = 0$