Physics II: 1702
Gravity, Electricity, & Magnetism

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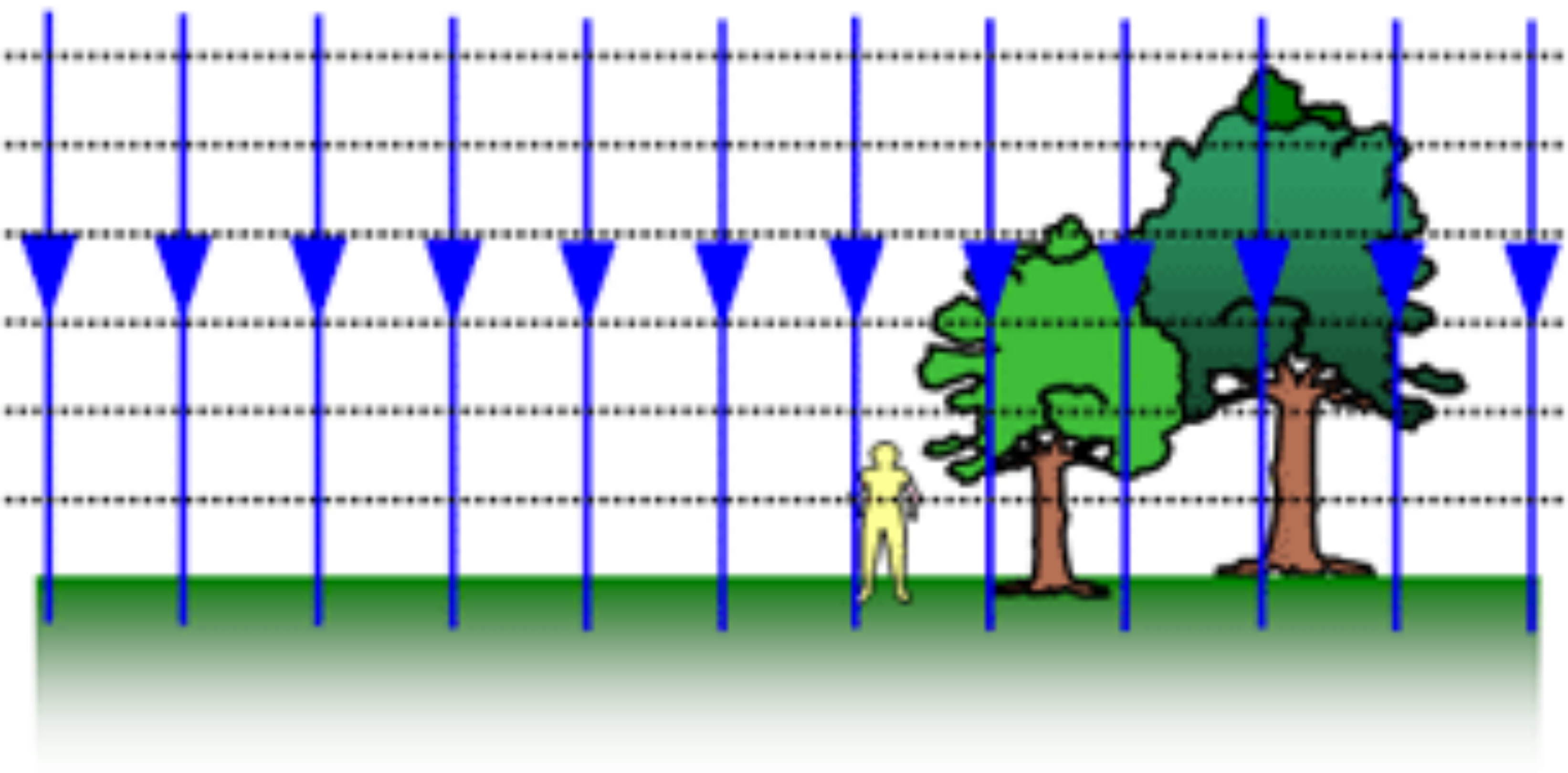
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Gravity Near Surface

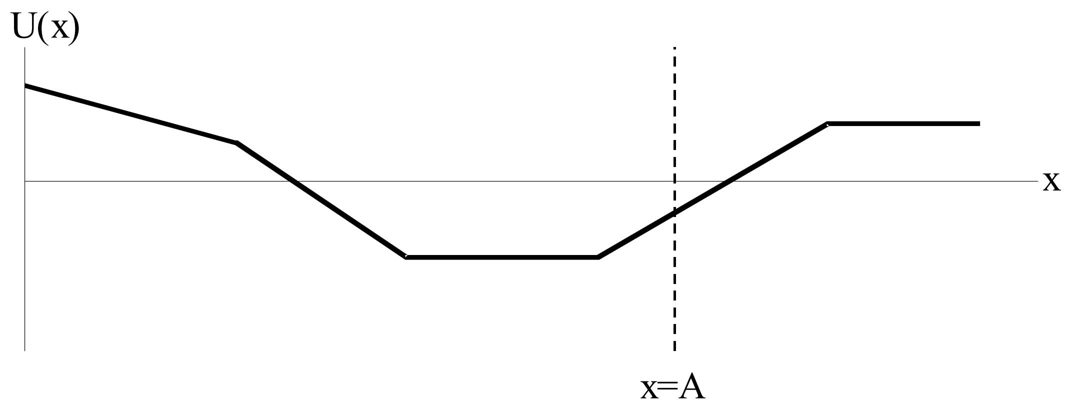
- $\vec{g}(x,y,z) = -g\,\hat{y}$
- $W = -mg\Delta y$
- $\Delta U = -W = mg\Delta y$
- If you set potential energy equal to zero at y = 0, then:
  - $U_g(x,y,z) = mgy$

Gravitational Field/Equipotentials



Concept Check

An object moves along the x-axis . The potential energy U(x) vs. position x is shown below.



When the object is at position x=A, which of the following statements must be true?

A. The velocity v_x is positive.
B. The acceleration a_x is negative.
C. The total energy is negative.
D. The total energy is positive.
E. None of these statements is always true.

Force and Potential Energy

$$\Delta U = -\int_{x_1}^{x_2} F(x)dx = area$$

If potential energy is the (negative) antiderivative of force (with respect to displacement) then how would we find the force if we were given a potential energy function?

Force and Potential Energy

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If potential energy is the (negative) antiderivative of force (with respect to displacement) then how would we find the force if we were given a potential energy function?

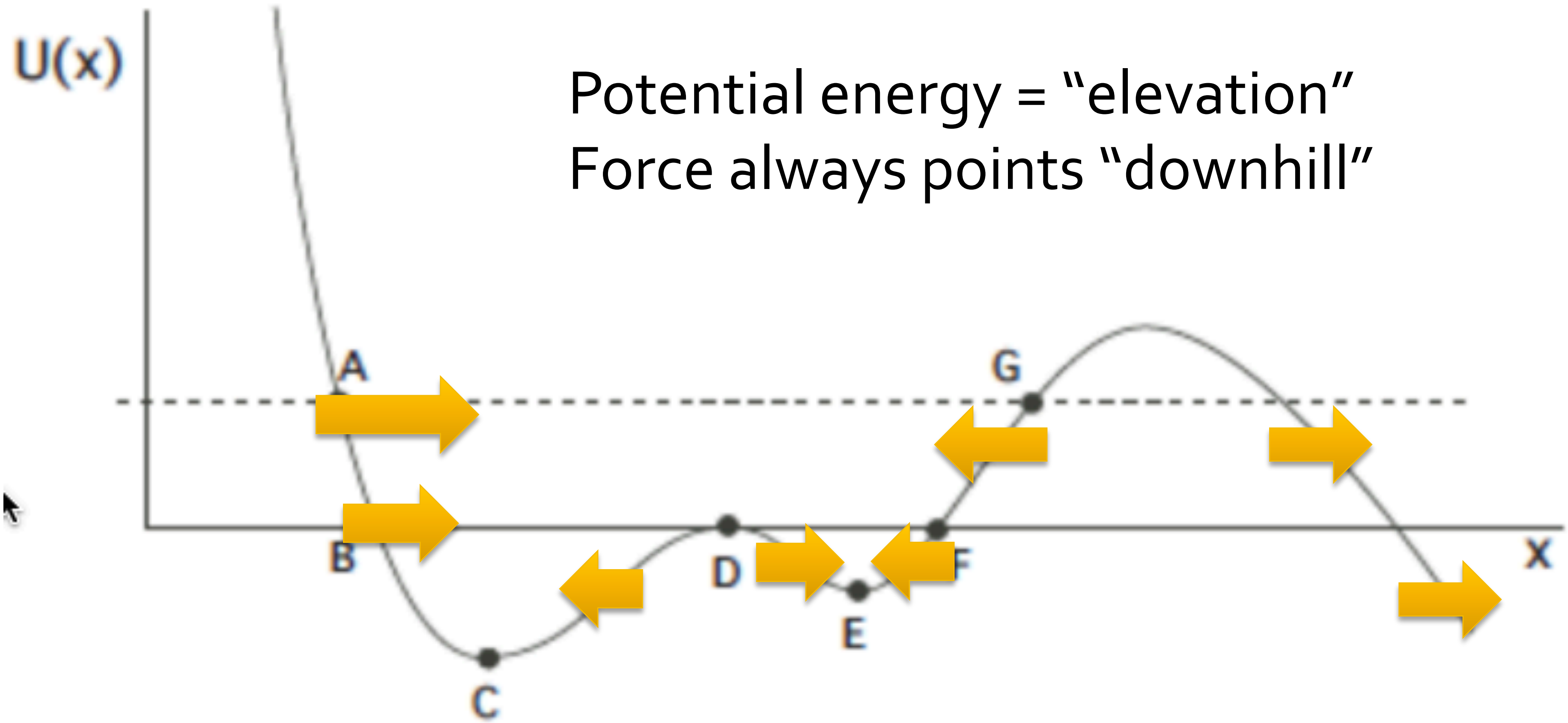
Just go the opposite way….

…the reverse process of the antiderivative is the derivative.

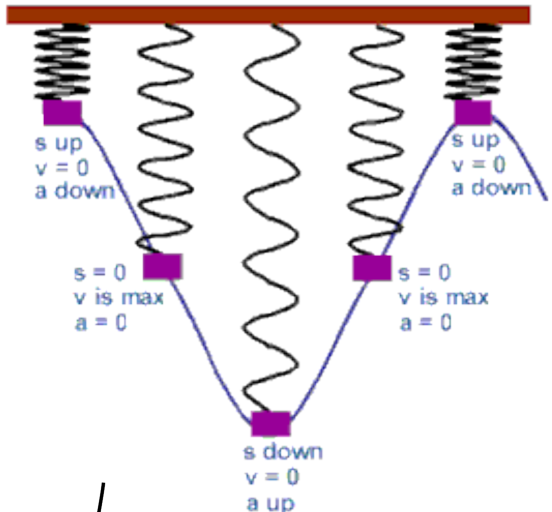
$$F(x) = \frac{-dU}{dx} = -slope$$

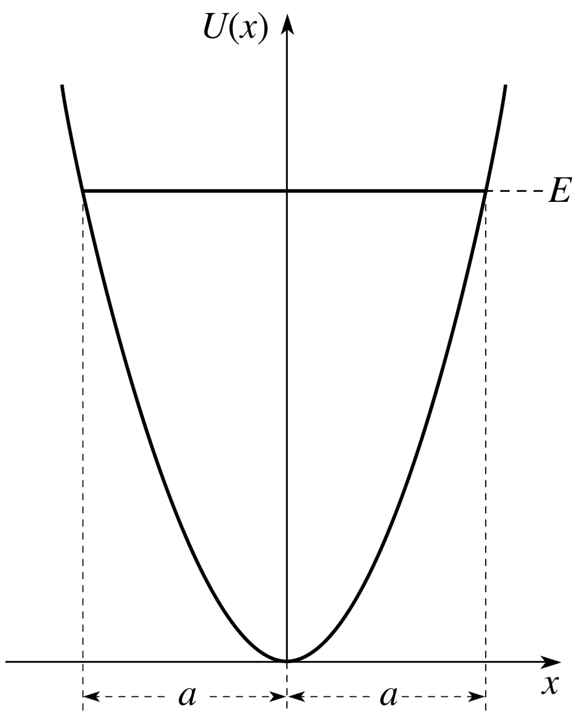
Roller Coaster

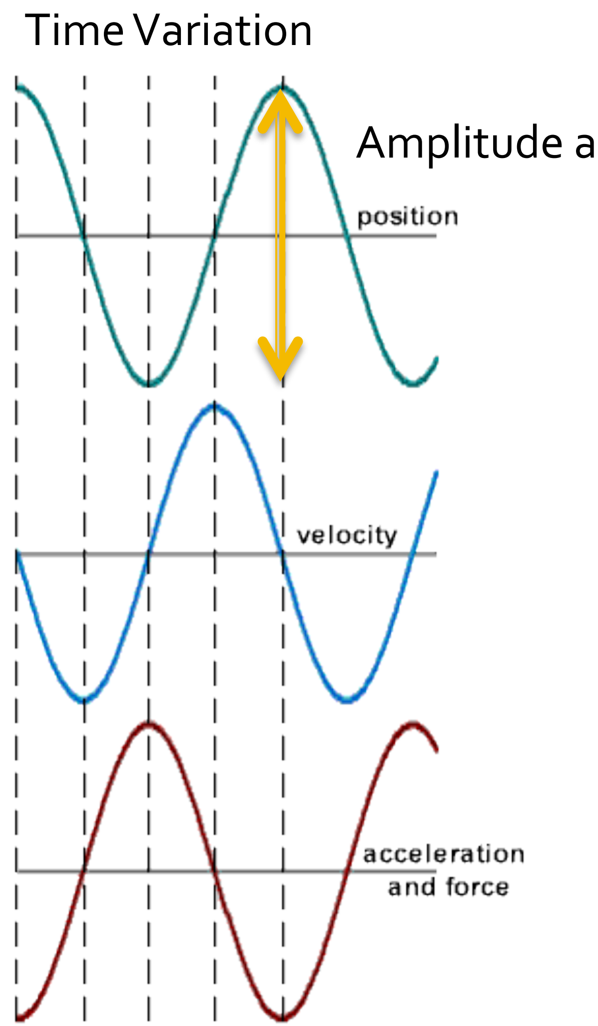
Potential energy = “elevation”
Force always points “downhill”



Harmonic Oscillator

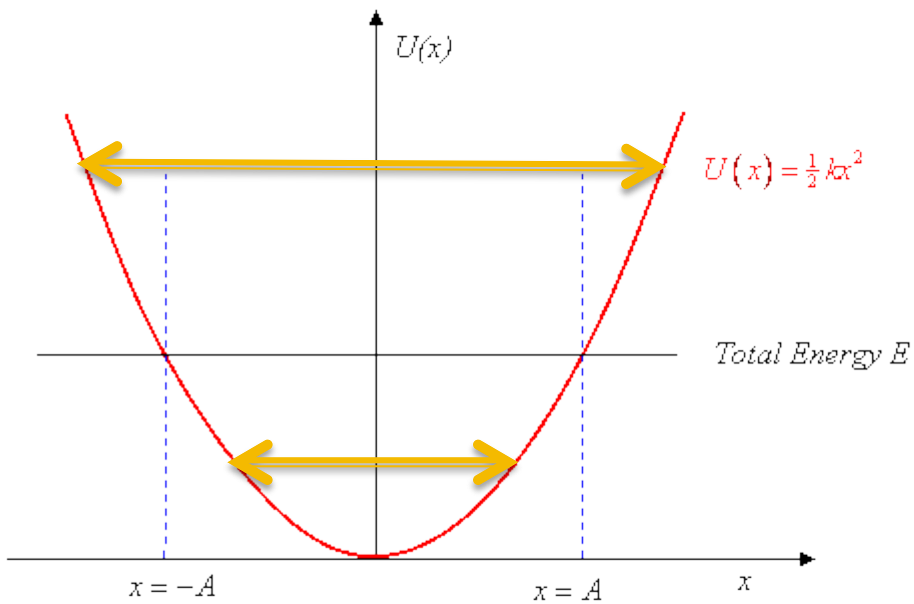






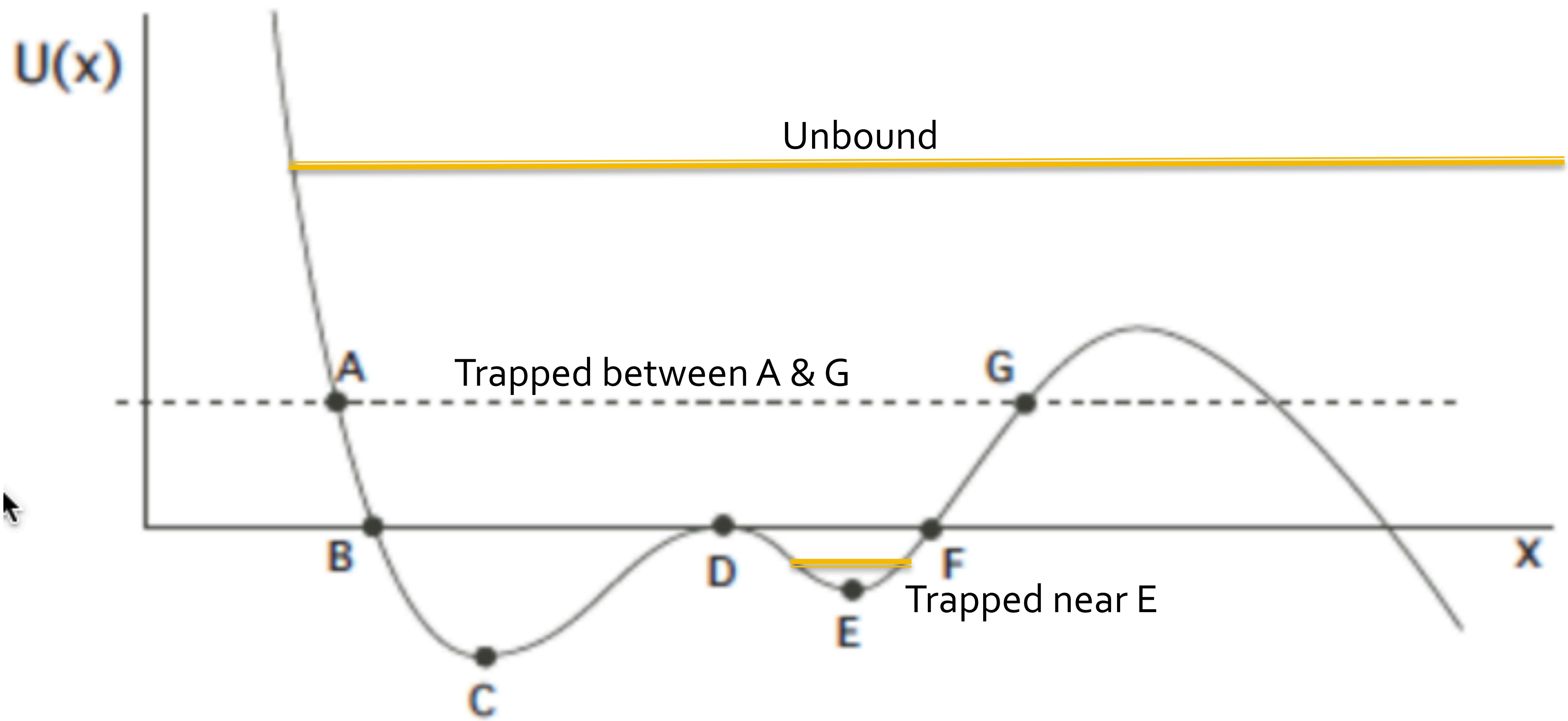
Energy Levels

- The Potential Energy curve together with the Total Mechanical Energy determine many characteristics of the motion of an object



Potential Energy *U*(*x*) for a Simple Harmonic Oscillator.
For ***total*** energy *E*, the oscillator swings back and forth between *x* = –*A* and *x* = +*A*.

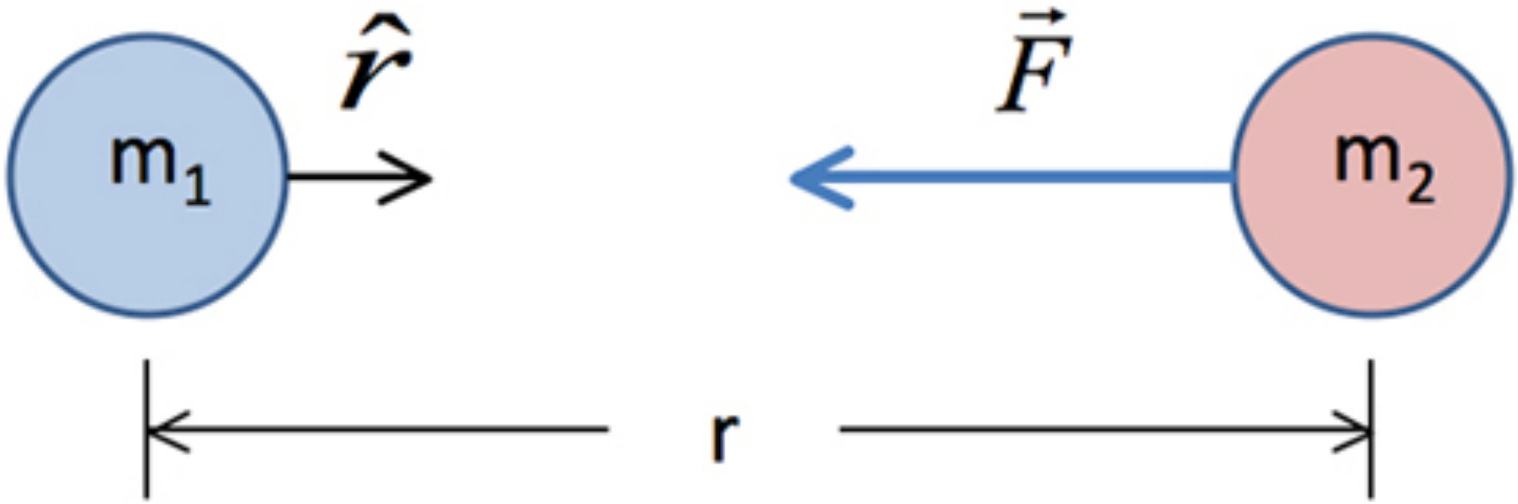
Energy Levels



Newton's Law of Gravitation

- $F = G\, m_1\, m_2 / r^2$
- $G = 6.67 \times 10^{-11}\ N\, m^2/kg^2$

Gravitational Force: Vector Form



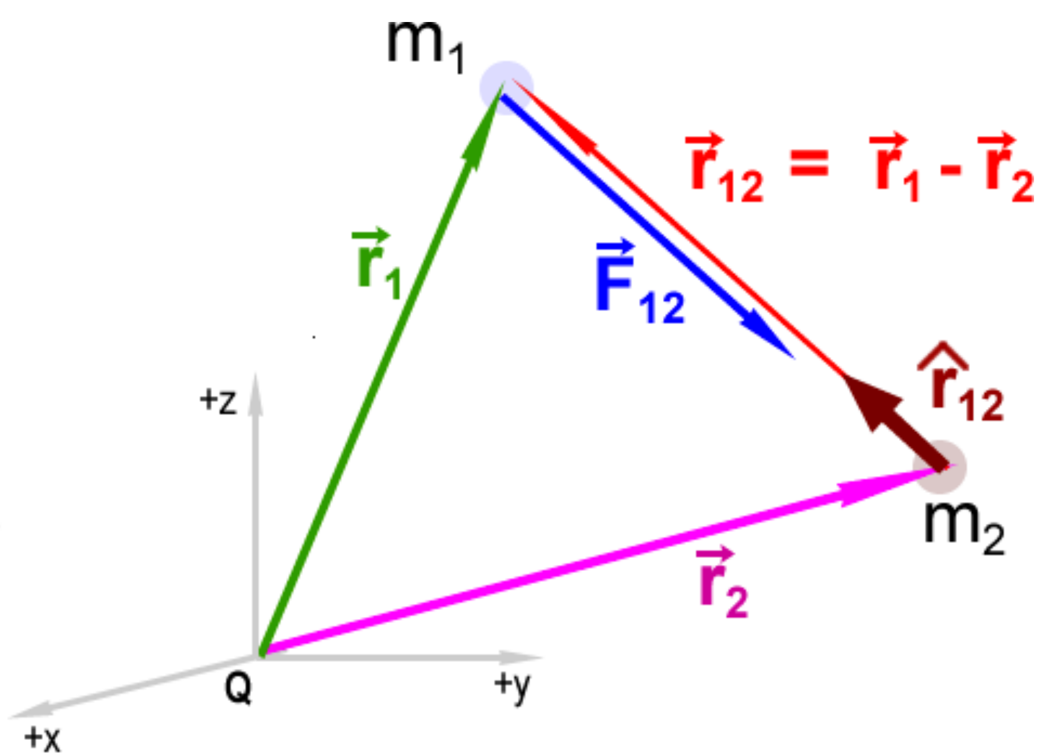
$$\vec{F}_{21} = -\frac{Gm_1m_2}{r^2}\hat{r}$$

Force on particle 2 from particle 1

More Complete Vector Form

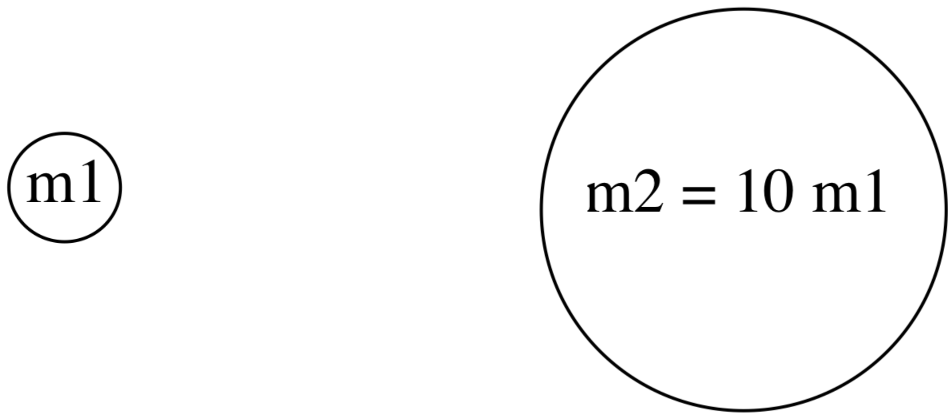
$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{|\mathbf{r}_{12}|^2}\,\hat{\mathbf{r}}_{12}$$

$$F_{12} = \frac{G m_1 m_2 (\mathbf{r_2} - \mathbf{r_1})}{|\mathbf{r_2} - \mathbf{r_1}|^3}$$



Concept Check

At some instant in time, two asteroids in deep space are a distance r=20 km apart. Asteroid 2 has 10 times the mass of asteroid 1. What is the ratio of their resulting acceleration (due to gravitational attraction to each other),



a1 / a2 =

A: 10:1 **B:** 1:1 **C:** 1:10
D: Not enough information

$$\vec{F}_{12} = \frac{-G m_1 m_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

$$|\vec{F}_{12}| = \frac{-G m_1 m_2}{|r_{12}|^2} = -\frac{G m_1 m_2}{r^2}$$

$$= m_1 a_1$$

$$\Rightarrow a_1 = -G m_2 / r^2$$

$$|\vec{F}_{21}| = -\frac{G m_1 m_2}{|r_{21}|^2} = -\frac{G m_1 m_2}{r^2}$$

$$= m_2 a_2$$

$$\Rightarrow a_2 = -G m_1 / r^2$$

$$a_1 / a_2 = \frac{-G m_2 / r^2}{-G m_1 / r^2} = \frac{m_2}{m_1} = 10$$

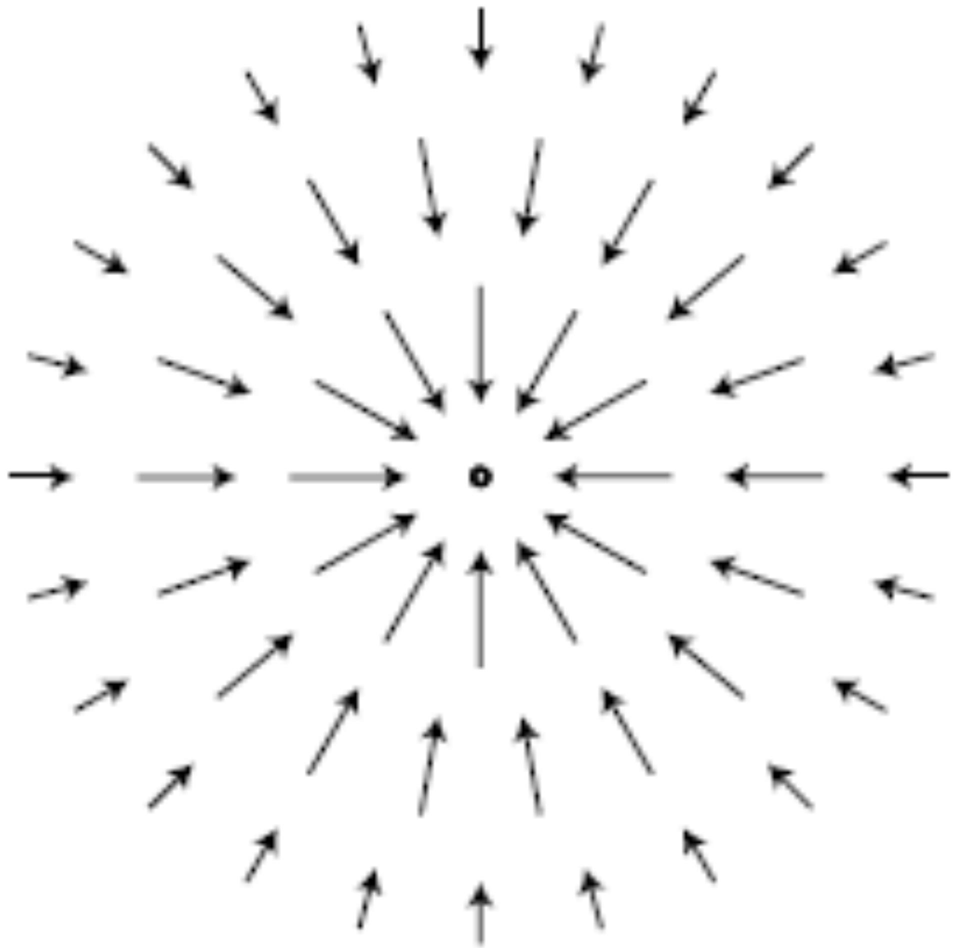
Gravitational Field

- Remember we defined the gravitational field as the force per unit mass

$$\vec{g} = \frac{\vec{F}_g}{m} = -G\frac{mM}{mr^2}\hat{r} = -\frac{GM}{r^2}\hat{r}$$

The gravitational field is *equal* to the gravitational acceleration of a freely falling body!

Gravitational Field Vectors



Convention: Length proportional to field strength

Gravity And Superposition

- Gravity has the wonderful property that the gravitational field due to multiple objects is just the sum of the individual fields

  - **For a group of interacting particles, the net gravitational force on one of the particles is**

$$\vec{F}_{1,net} = \sum_{i=2}^{n} \vec{F}_{1i}$$

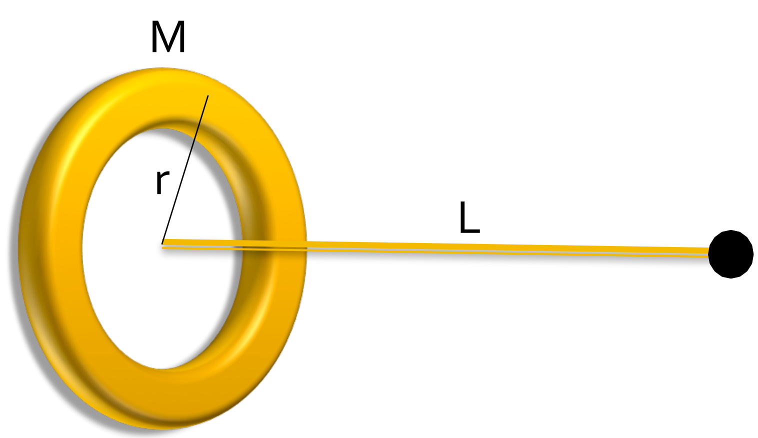
  - **For a particle interacting with a continuous arrangement of masses (a massive finite object) the sum is replaced with an integral**

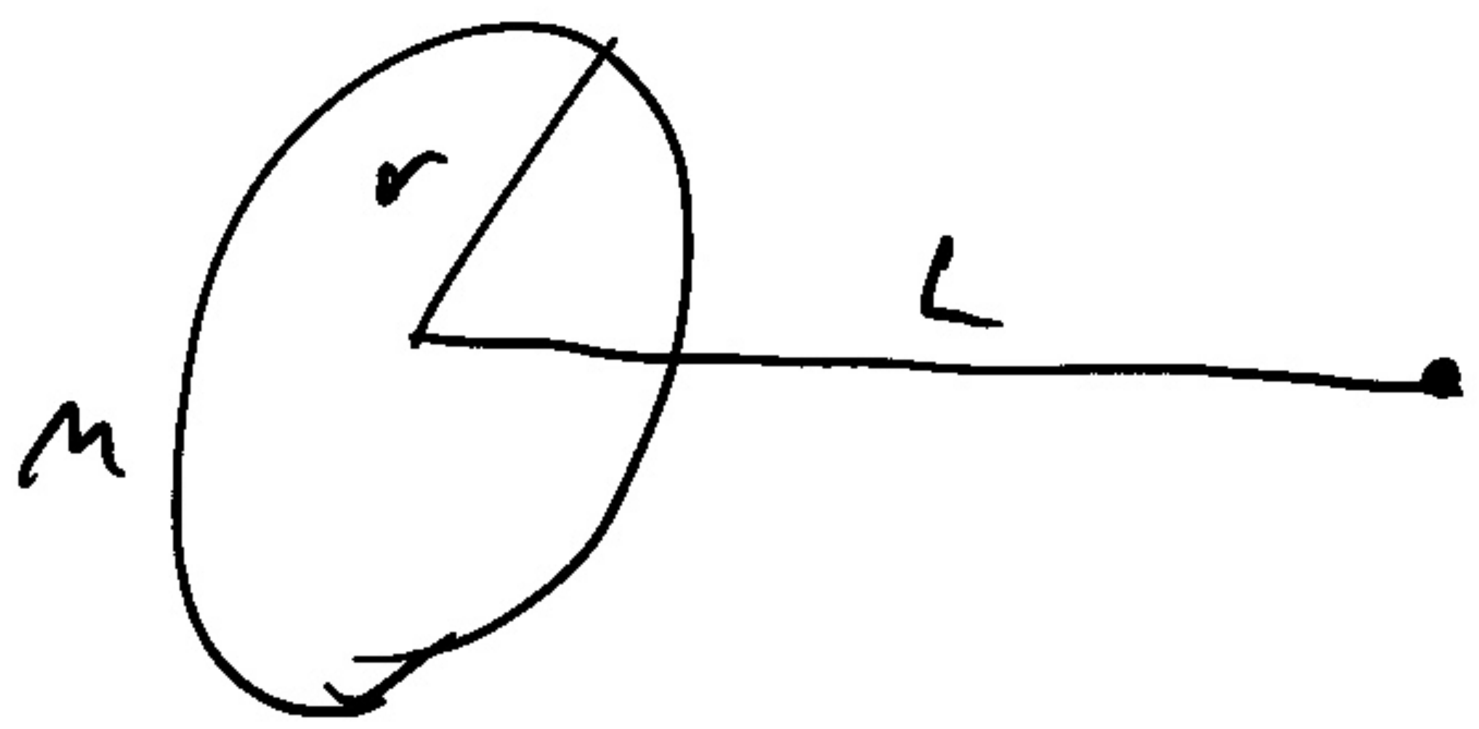
$$\vec{F}_{1,body} = \int_{body} d\vec{F}$$

Concept Check

- What is the gravitational field due to a thin ring of matter with radius r and total mass M, at a point a distance L along the line from the center of the ring?

A. $-GM/L^2$
B. $-GML/(r^2 + L^2)^{3/2}$
C. $-GM/r^2$
D. $-GM/(r^2 + L^2)$
E. $-GMr/(r^2 + L^2)^{3/2}$



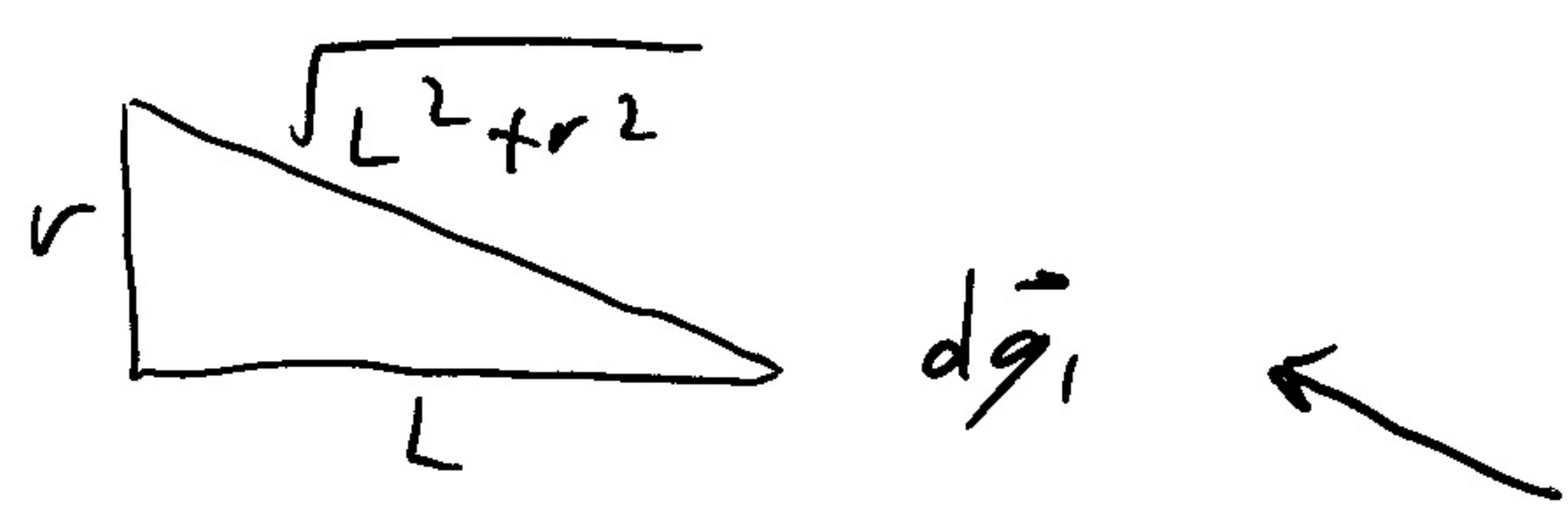


- This would be a very hard problem without symmetry.

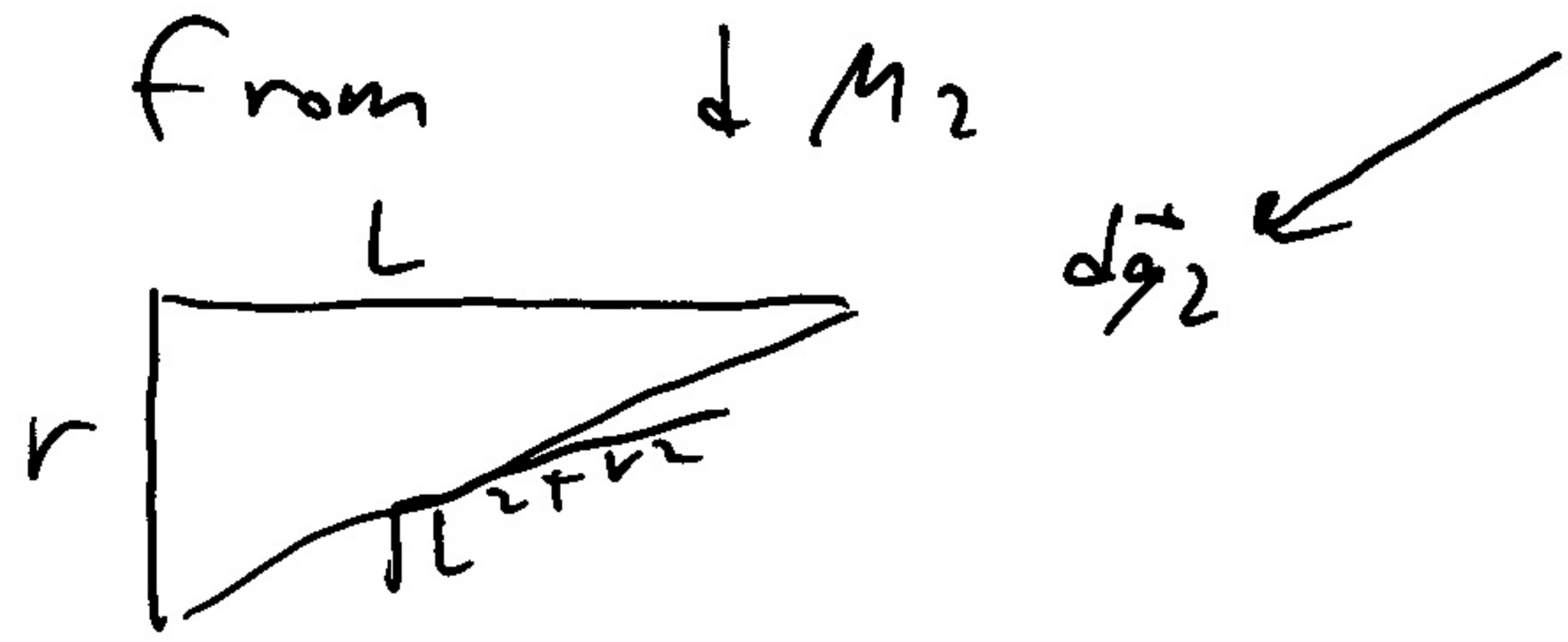
- Imagine splitting into different parts dM

Look @ fields in pairs

Field from dM_1



Field from dM_2



$$|d\vec{g}_1| = G\,dM_1 / r^2+L^2$$

$$|d\vec{g}_2| = G\,dM_2 / r^2+L^2$$

- what happens when we add them?

Along-L - components add
Along-r - components cancel

$$dg_{1L} = |d\vec{g}_1| \cos\theta$$
$$= |d\vec{g}_1| \, L / \sqrt{r^2+L^2}$$

$$dg_{1r} = |d\vec{g}_1| \, r / \sqrt{r^2+L^2}$$

$$dg_{(1+2)r} = 0$$

$$dg_{(1+2)L} = \frac{G(dM_1 + dM_2)}{r^2+L^2} \cdot \frac{L}{\sqrt{r^2+L^2}}$$

Total $\vec{g}$ is the sum of all such:

$$|\vec{g}| = \frac{GML}{(r^2+L^2)^{3/2}}$$

$$\vec{g} = \frac{-GML\hat{L}}{(r^2+L^2)^{3/2}}$$