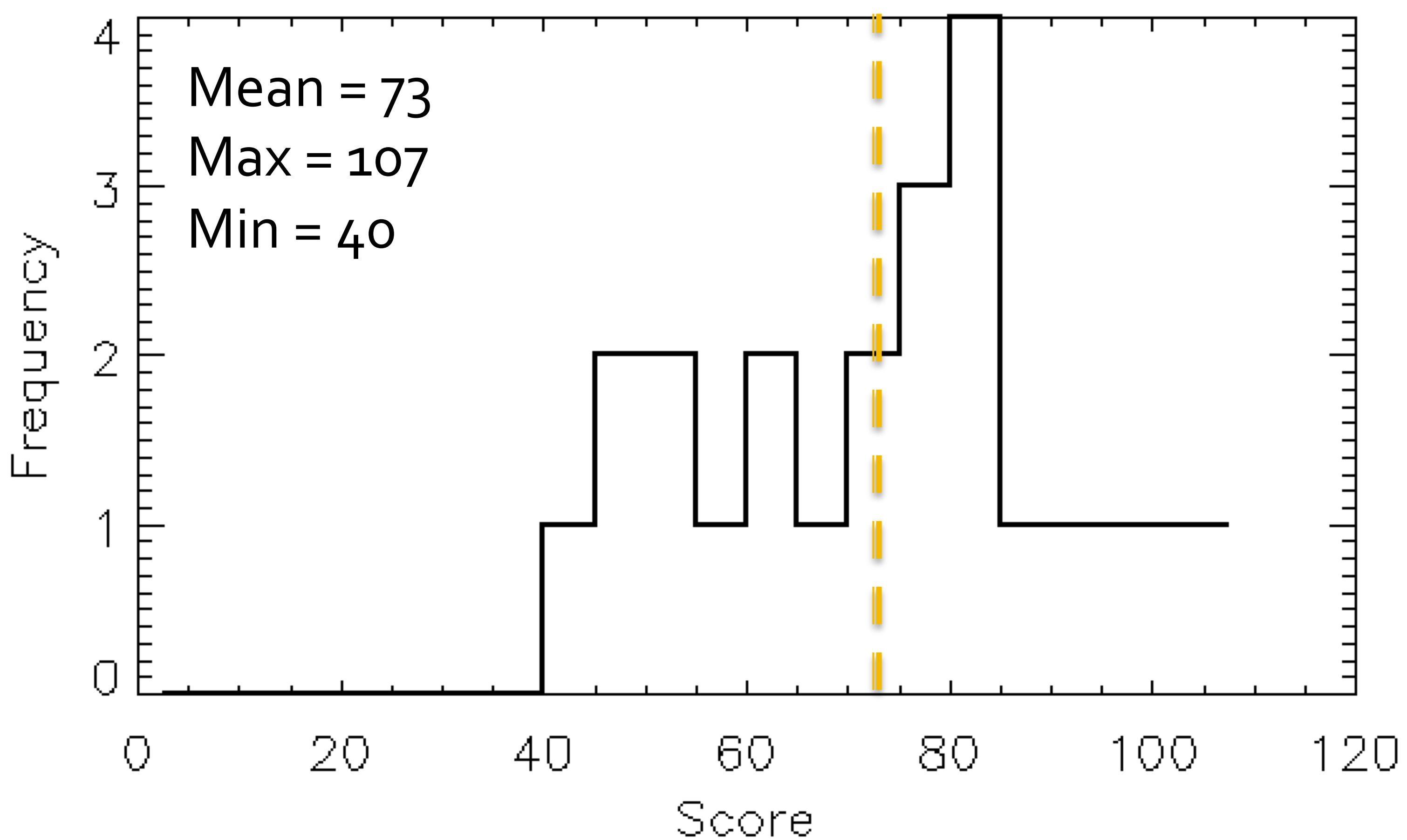
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Exam 2



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Max = 107
Min = 40
Frequency
Score
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2
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60
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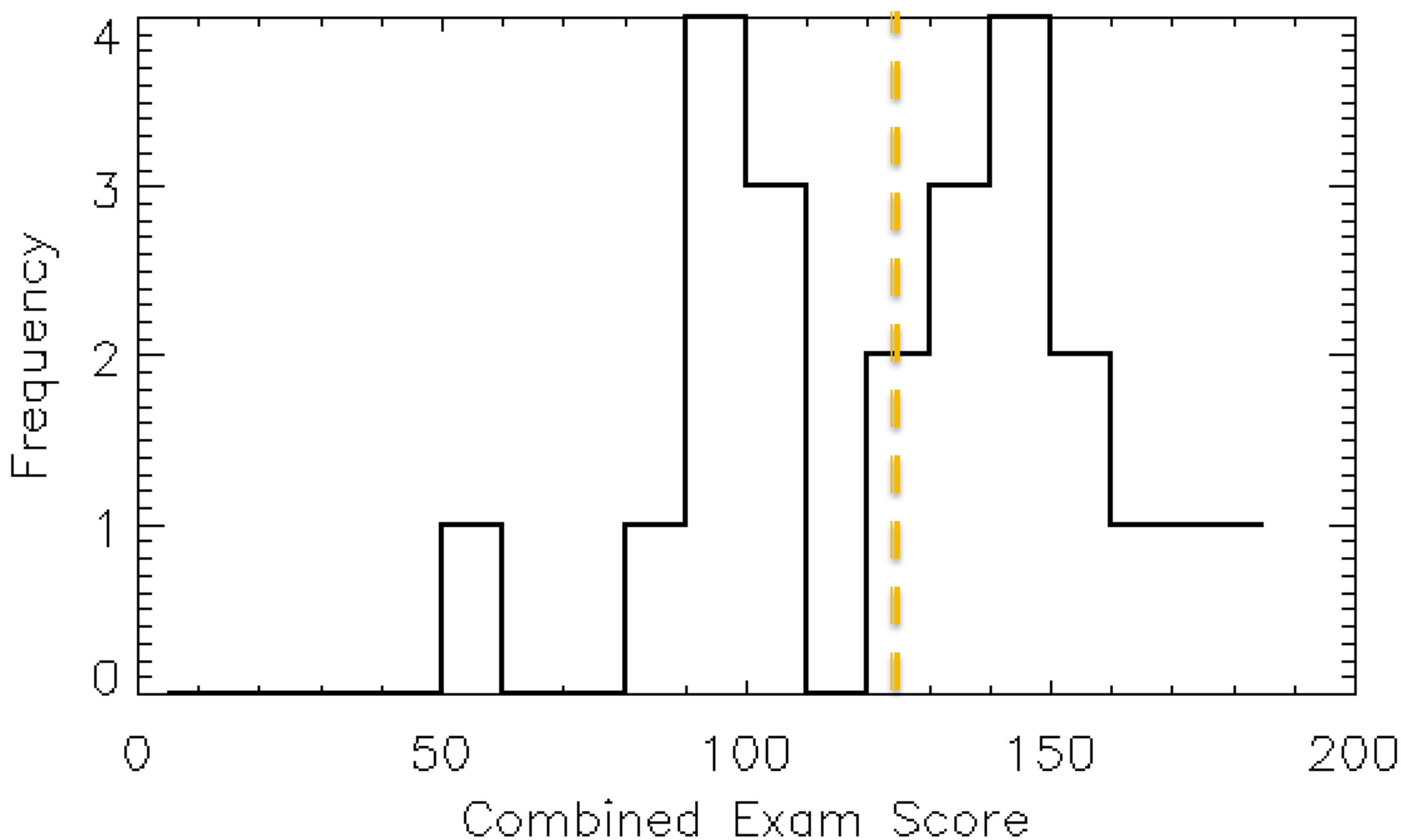
120

Combined Exam 1 + Exam 2



E&M So Far

Electric charges create E-fields

$$\vec{E} = \frac{kQ}{r^2}\hat{r}$$

Moving electric charges create B-fields

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

Next Step

Electric Fields can also be created by a changing Magnetic Field.

This is described by Faraday's Law.

Magnetic Fields can also be created by a changing Electric Field.

This will help complete the set of equations called Maxwell's equations of electricity and magnetism.

Faraday's Law

Faraday's Law of Induction

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Two familiar quantities, but in a new context.

1. Magnetic Flux = Φ_B
2. Electro Motive Force (EMF) = ε

Magnetic Flux

Recall Electric Flux

$$\Phi_E = \int_{surf} \vec{E} \cdot d\vec{A}$$

Magnetic Flux is quite similar except for Magnetic Fields.

$$\Phi_B = \int_{surf} \vec{B} \cdot d\vec{A}$$

ElectroMotive Force

Technically, the EMF around a closed Loop is defined as:

$$\varepsilon = \oint_{Loop} \vec{E} \cdot d\vec{l}$$

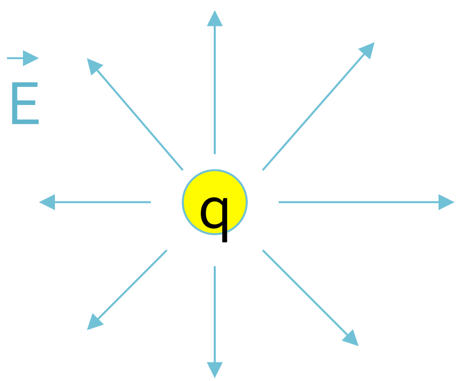
Recall that Voltage is defined as:

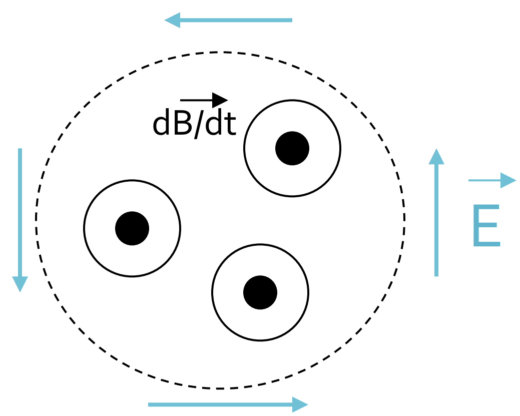
$$\Delta V = -\int \vec{E} \cdot d\vec{l}$$

ε = a Voltage difference capable of generating power.

Motional Vs. Potential EMF

The (non-Coulomb) E-field is very different from the Coulomb E-field created by single electric charges.





$$\Delta V = -\oint \vec{E} \cdot d\vec{l} = 0$$

$$\varepsilon = \oint_{Loop} \vec{E} \cdot d\vec{l} \neq 0$$

Two Kinds of Electric Fields?

- Electric fields we have seen before can be expressed as the gradient of a scalar potential
- Electric fields from changing magnetic fields cannot be expressed as the gradient of a scalar potential
  - Exactly equivalent to saying the loop integral of the electric field is non-zero
- This form of electric field comes from a different kind of potential – the magnetic vector potential

Electrostatics is a Special Case

If there are only stationary charges and only constant currents, then there are no changing B-fields.

In this case there is no changing magnetic flux through a given area.

$$d\phi_B/dt = 0 \Rightarrow \varepsilon = \oint_{Loop} \vec{E} \cdot d\vec{l} = 0 \text{ (special case only)}$$

Back to Faraday's Law

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

$$\oint_{Loop} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{surf} \vec{B} \cdot d\vec{A}$$

Maxwell's Equations: Integral Form

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$ ✔

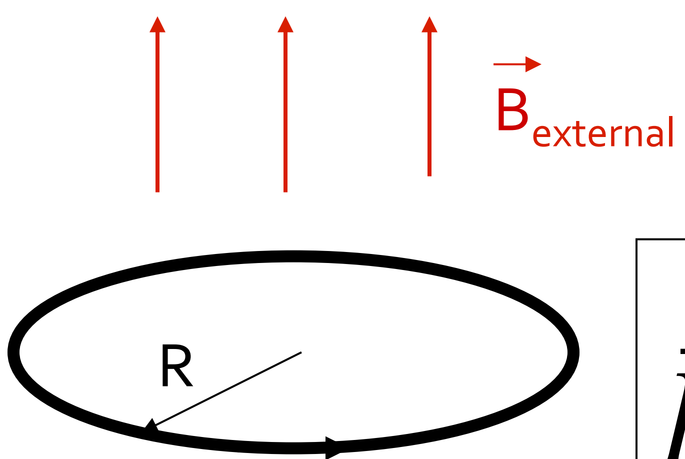
$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$ ✔

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$ ✔

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$ ✔-

Faraday's Law

$$\oint_{Loop} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{surf} \vec{B} \cdot d\vec{A}$$



$$\vec{E}(2\pi R) = -\frac{d}{dt}\left[\vec{B}_{ext}(\pi R^2)\right]$$

Changing Magnetic Flux

Many different ways to change the Magnetic Flux

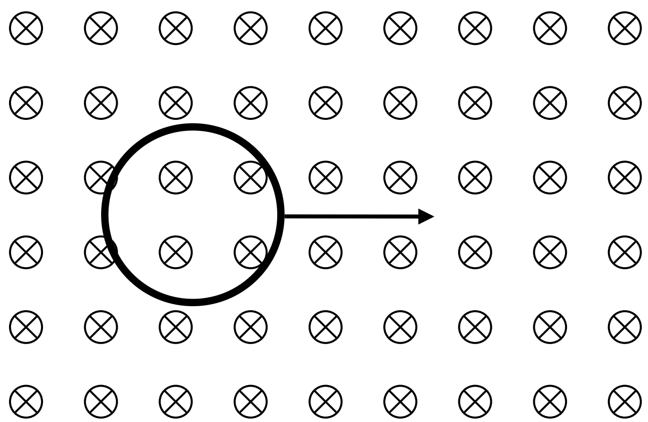
1. Change the strength of the Magnetic Field

2. Change the area of the loop

3. Change the orientation of the loop ($\vec{A}$ vector) and the $\vec{B}$-field

Concept Check

A loop of wire is moving rapidly through a uniform magnetic field as shown. Is a non-zero EMF induced in the loop?



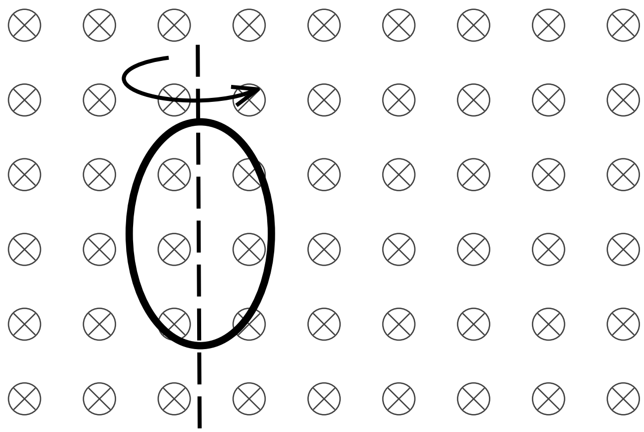
A) No, there is no EMF

B) Yes, there is an EMF

Concept Check

A loop of wire is spinning rapidly about a stationary axis in uniform magnetic field as shown.

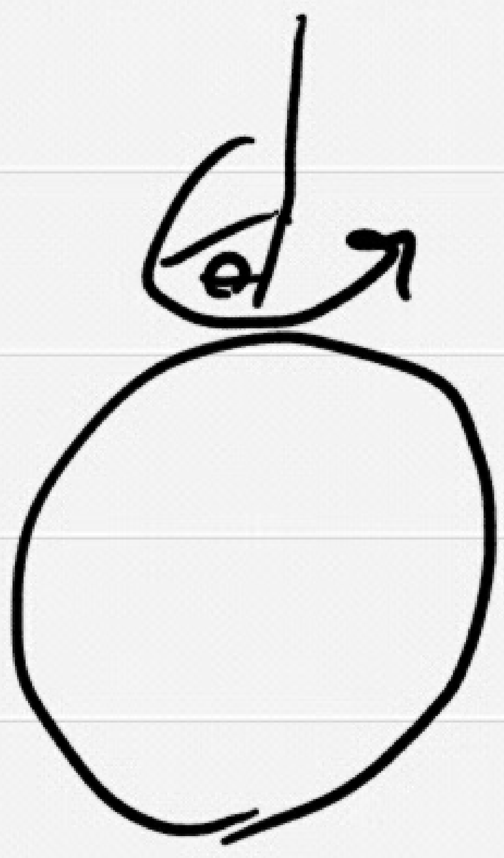
True or False: There is a non-zero EMF induced in the loop.



A) False, zero EMF

B) True, there is an induced EMF

Rotating loop



$$\theta(t) = \omega t$$

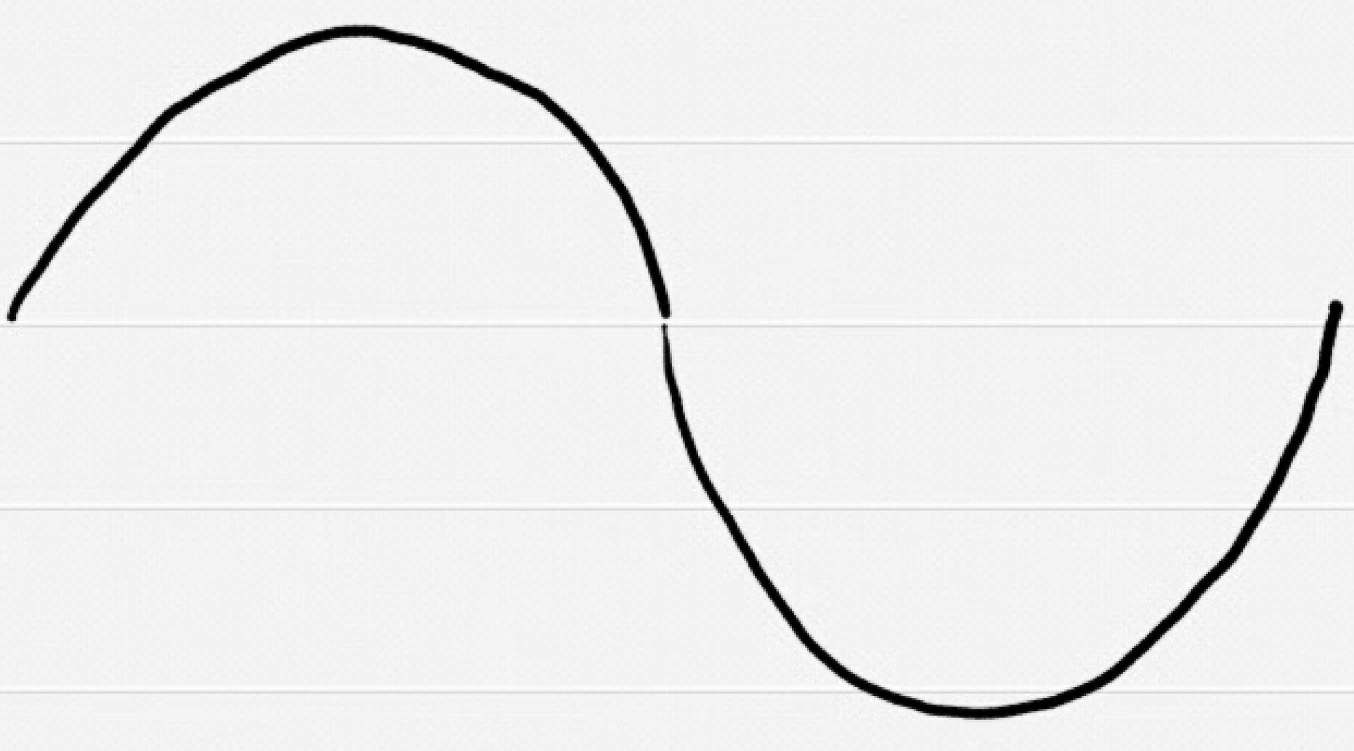
$$B = B_0 \hat{k}$$

$$\Phi_B = \vec{B} \cdot \vec{A}$$
$$= BA \cos\theta$$
$$= BA \cos\omega t$$

$$\mathcal{E} = -d\Phi_B/dt$$
$$= -d/dt \; BA \cos\omega t$$
$$= \omega BA \sin\omega t$$

Φ

$\mathcal{E}$



Lenz’s Law

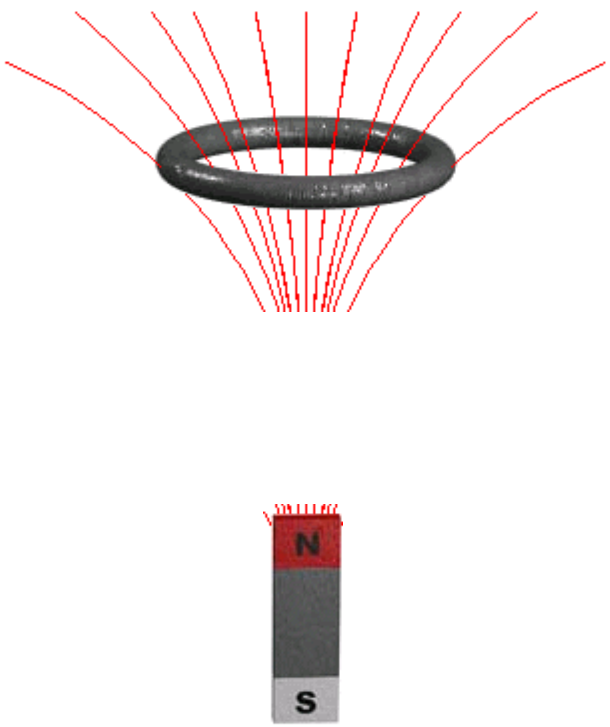
The (-) negative sign in Faraday’s Law is an important reminder.

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Lenz’s Law: The induced EMF tends to induce a current in the direction which opposes the changes in Magnetic Flux.

Lenz's Law Applied

If we move the magnet closer to the loop, what is the direction of the induced EMF and thus the direction of the current in the loop?



1. The Magnetic Flux is increasing.

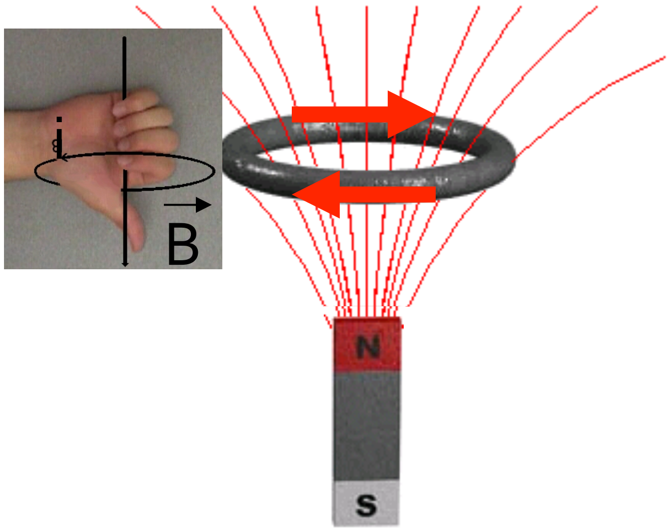
$$\Phi_B = \int_{surf} \vec{B} \cdot d\vec{A}$$

2. Therefore

$$\frac{d\Phi_B}{dt} > 0$$

3. Induced current must create B-field that fights the change!

Lenz's Law Applied



$$\frac{d\Phi_B}{dt} > 0$$

Induced current must create B-field that fights the change!

An induced B-field down through the loop will create a slight decrease in Φ_B.

Thus, we have determined the direction of the **induced current** and EMF.