Physics II: 1702
Gravity, Electricity, & Magnetism

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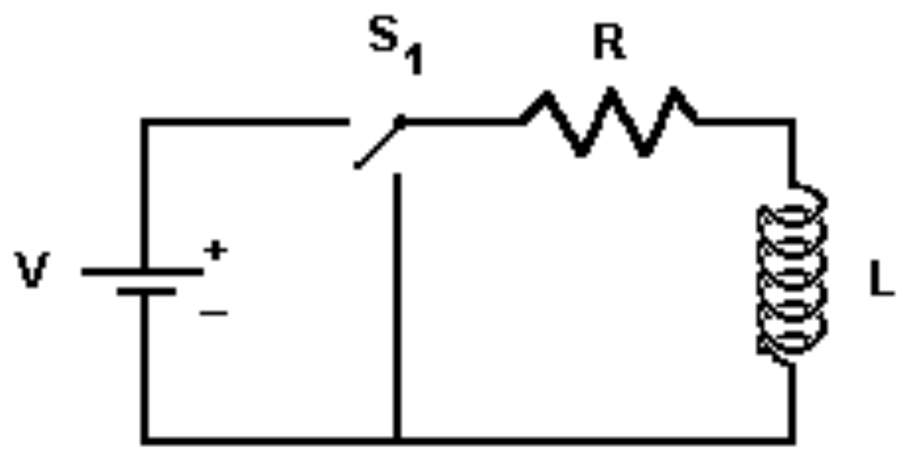
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

RL Circuits

RL Series Circuit

$$\tau_{RL} = \frac{L}{R}$$

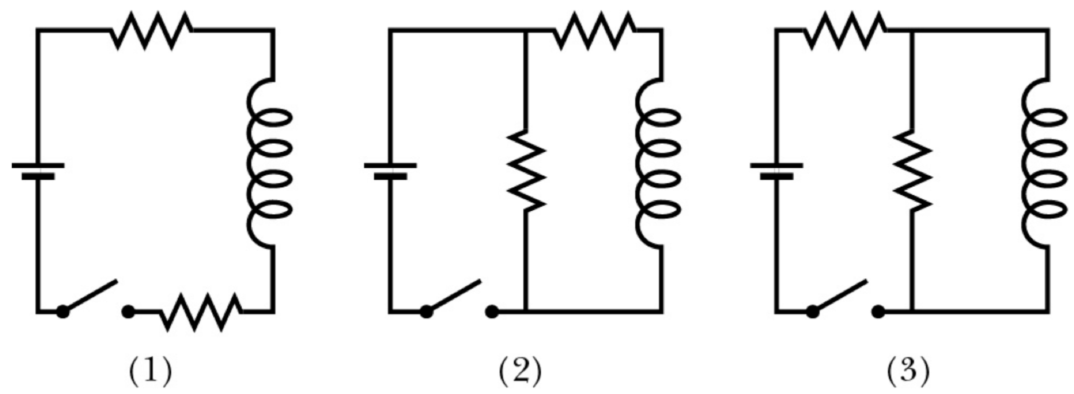
$$i(t) = I_0\left(1 - e^{-tR/L}\right)$$



Concept Check

Q49) The figure below shows three circuits with identical batteries, inductors, and resistors. Rank the circuits according to the current through the battery (a) immediately after the switch is closed and (b) a long time later, greatest first.

1) (a) all tie (b) 2, 3, 1
2) (a) all tie (b) 2 = 3, 1
3) (a) 2, 3, 1 (b) 2, 3, 1
4) (a) 2, 3, 1 (b) 2 = 3, 1
5) (a) 2 = 3, 1 (b) all tie



Electric Field Energy

Electric Field Energy Density

Recall that for a capacitor C, there is stored potential energy in the electric field.

$$U = \frac{1}{2} C V^2$$

The energy is stored in the electric field and the density is:

$$u_E = \frac{U}{Volume} = \frac{1}{2} \varepsilon_0 \, |\vec{E}|^2$$

Magnetic Field Energy

For an inductor L, with current i, there is stored energy in the magnetic field.

$$U = \frac{1}{2} L i^2$$

The energy density in the magnetic field is:

$$u_B = \frac{U}{Volume} = \frac{1}{2}\frac{1}{\mu_0} |\vec{B}|^2$$

Magnetic Energy

$$V = L\, dI/dt + IR$$

Multiply by I

$$VI = IL\, dI/dt + I^2 R$$

- VI: work done by battery
- $IL\, dI/dt$: rate of energy storage in inductor
- I^2R: power dissipation in resistor

$$IL\, dI/dt = d/dt \left(\tfrac{1}{2} L I^2\right)$$

$$\Rightarrow U_B = \tfrac{1}{2} L I^2$$

= energy stored in inductor

Solenoid: $L = \mu_0 n^2 \ell A$

$$\Rightarrow U_B = \tfrac{1}{2} \mu_0 n^2 \ell A \cdot I^2$$

$$u_B = U_B / \text{volume}$$

$$= U_B / (\ell A) = \tfrac{1}{2} \mu_0 n^2 I^2$$

recall $B = \mu_0 n I$ for solenoid

$$\Rightarrow u_B = \tfrac{1}{2} B^2 / \mu_0$$

Electromagnetic Energy

$$u_{EM} = \frac{1}{2}\left[\varepsilon_0 E^2 + B^2/\mu_0\right]$$

- true anywhere in space

- For materials

$$u_{em} = \frac{1}{2}\left[\varepsilon E^2 + B^2/\mu\right]$$

LC circuit:

$$U = \frac{1}{2} Q^2/C + \frac{1}{2} L I^2$$

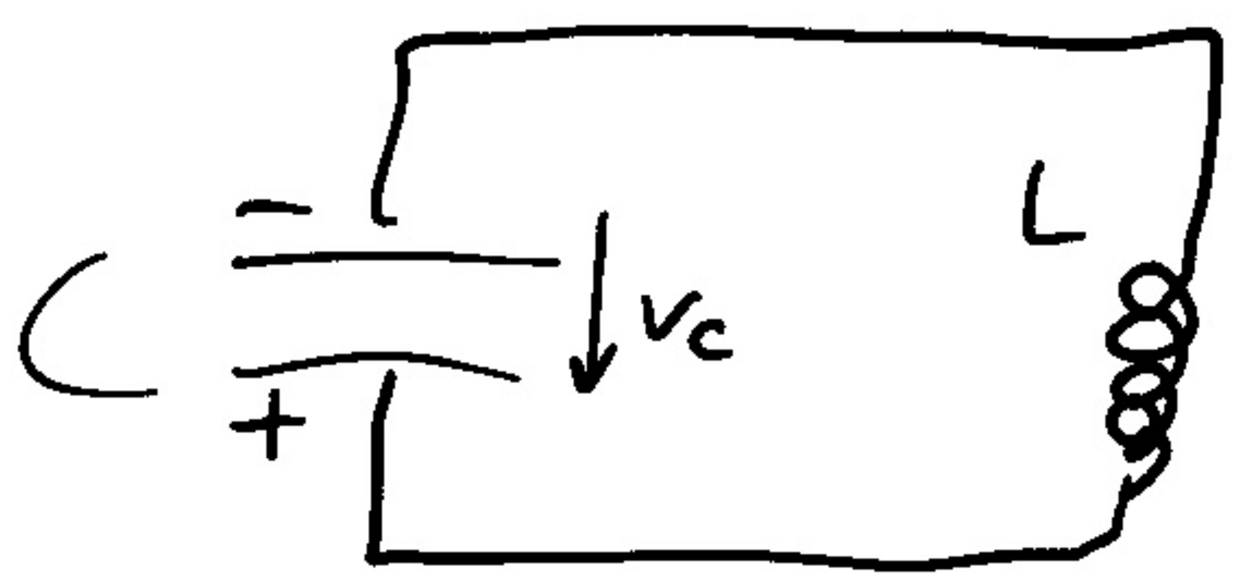
$$dU/dt = \frac{1}{C} Q \, dQ/dt + L I \, dI/dt$$
$$= I\left(Q/C + L \, dI/dt\right)$$
$$= 0$$

$$\Rightarrow \quad Q/C + L \, dI/dt = 0$$

or $Q/C + L \, d^2Q/dt^2 = 0$

or $Q'' = -\frac{1}{LC} \cdot Q$

circuit analysis



$\downarrow I$, $dI/dt \uparrow$, $\mathcal{E} \downarrow$

$\Sigma \Delta V = 0$

$$V_c - \mathcal{E} = 0$$
$$Q/C + L\, dI/dt = 0$$

write as $Q'' = -\omega^2 Q$
w/ $\omega = 1/\sqrt{LC}$

Analogues:
- harmonic oscillator
- gyrating charged particle in B
- block and spring
- wave

solution $Q(t) = A\cos\omega t + B\sin\omega t$

or $Q(t) = Ce^{i\omega t} + De^{-i\omega t}$

or $Q(t) = E\cos(\omega t + \varphi)$

<u>All equivalent</u>

Use boundary conditions;

say $Q(0) = 0$
$I(0) = I_0$

$$Q(t) = I_0/\omega \sin(\omega t)$$

$$I(t) = dQ/dt = I_0 \cos(\omega t)$$

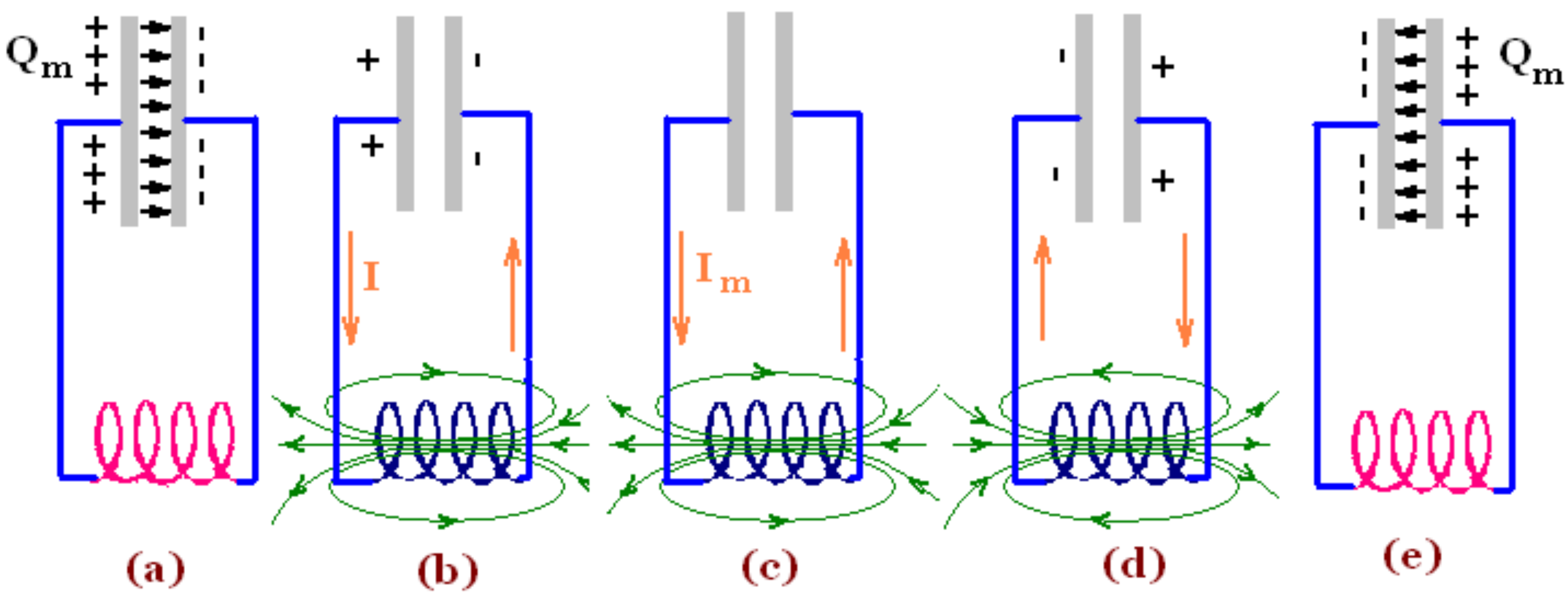
or
$$Q(t) = -I_0/\omega \cos(\omega t + \pi/2)$$
$$= I_0/\omega \sin(\omega t)$$
$$I(t) = I_0 \sin(\omega t + \pi/2)$$
$$= I_0 \cos(\omega t)$$

or
$$Q(t) = I_0/2i\omega \left[e^{i\omega t} - e^{-i\omega t}\right]$$
$$= I_0/\omega \sin(\omega t)$$
$$I(t) = I_0/2 \left[e^{i\omega t} + e^{-i\omega t}\right]$$
$$= I_0 \cos(\omega t)$$

Amplitude of $I(t) = I_0$
Amplitude of $Q(t) = Q_0 = I_0/\omega$

$$I_0 = Q_0 \omega = Q_0/\sqrt{LC}$$
$$I_0^2 = Q_0^2/LC$$
$$L I_0^2 = Q_0^2/C$$
$$\tfrac{1}{2} L I_0^2 = \tfrac{1}{2} Q_0^2/C$$
$$U_{B\max} = U_{E\max}$$

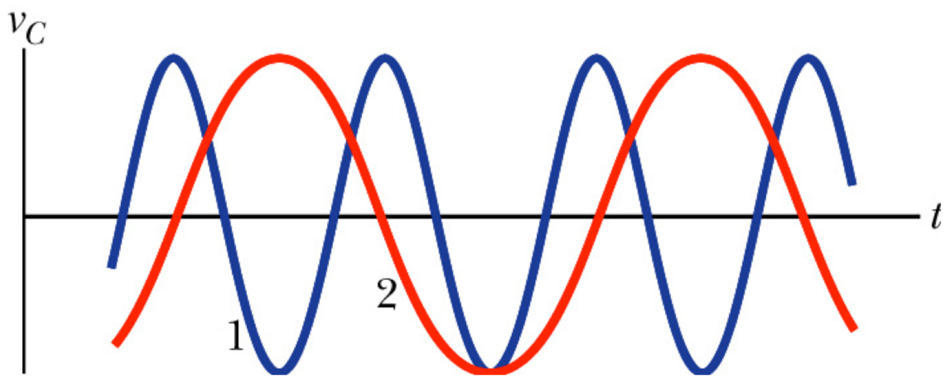
LC Circuits



Concept Check

Q7) The figure below shows graphs of the capacitor voltage v_C for LC circuits 1 and 2, which contain identical capacitances and have the same maximum charge Q. Are (a) the inductance and (b) the maximum current I in circuit 1 greater than, less than, or the same as those in circuit 2?

1) (a) greater than (b) greater than
2) (a) greater than (b) less than
3) (a) less than (b) greater than
4) (a) less than (b) less than



Concept Check

In an oscillating LC circuit, the total stored energy is U and the maximum charge on the capacitor is Q. When the charge on the capacitor is Q/2, the energy stored in the inductor is:

1) U/2
2) U/4
3) (4/3)U
4) 3U/2
5) 3U/4

LC Circuit Energy