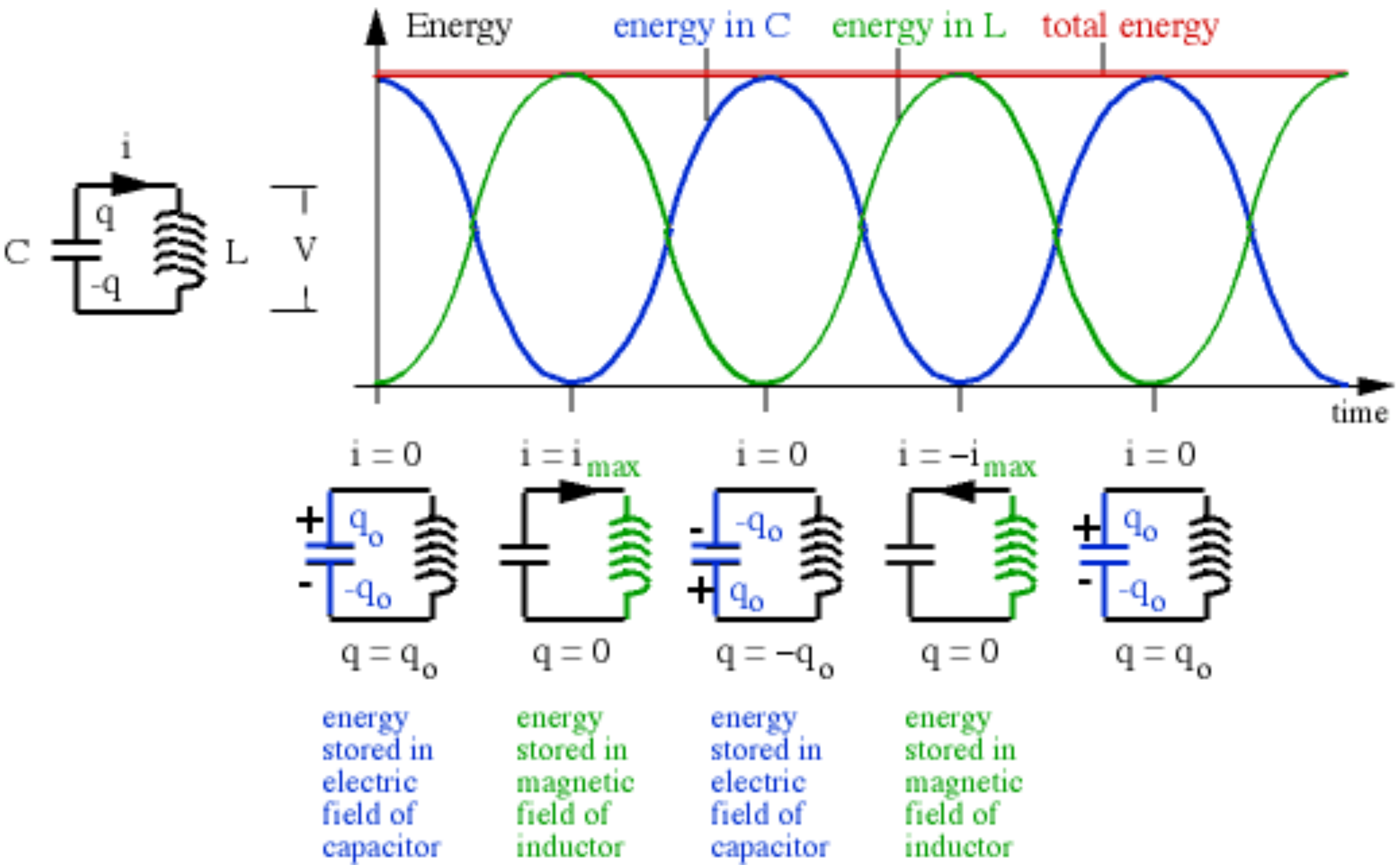
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Announcements

- I will be on travel Tuesday and Wednesday
- All Tues/Wed office hours are canceled
  - Appointments available Thursday/Friday if needed
- Guest Lecturer Wednesday: Prof. Howes

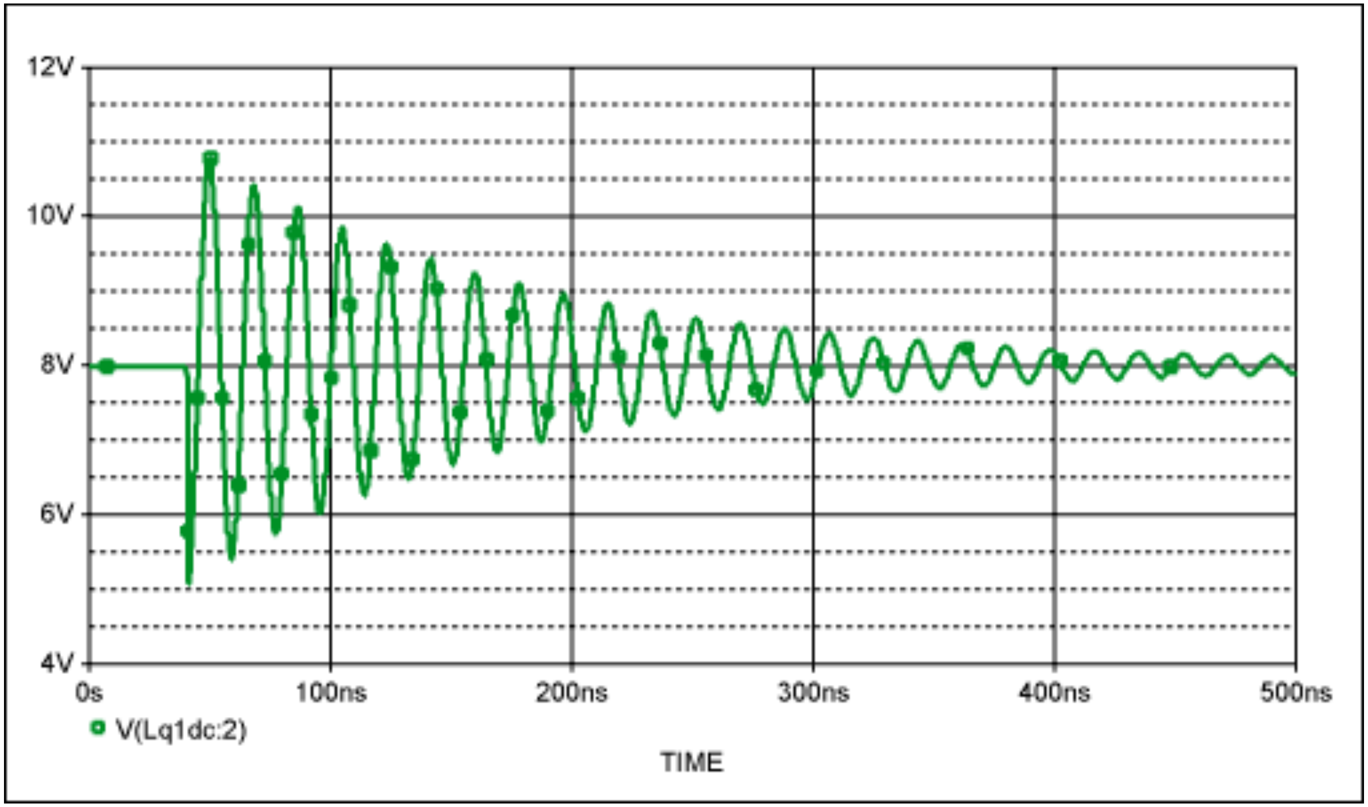
LC Circuit



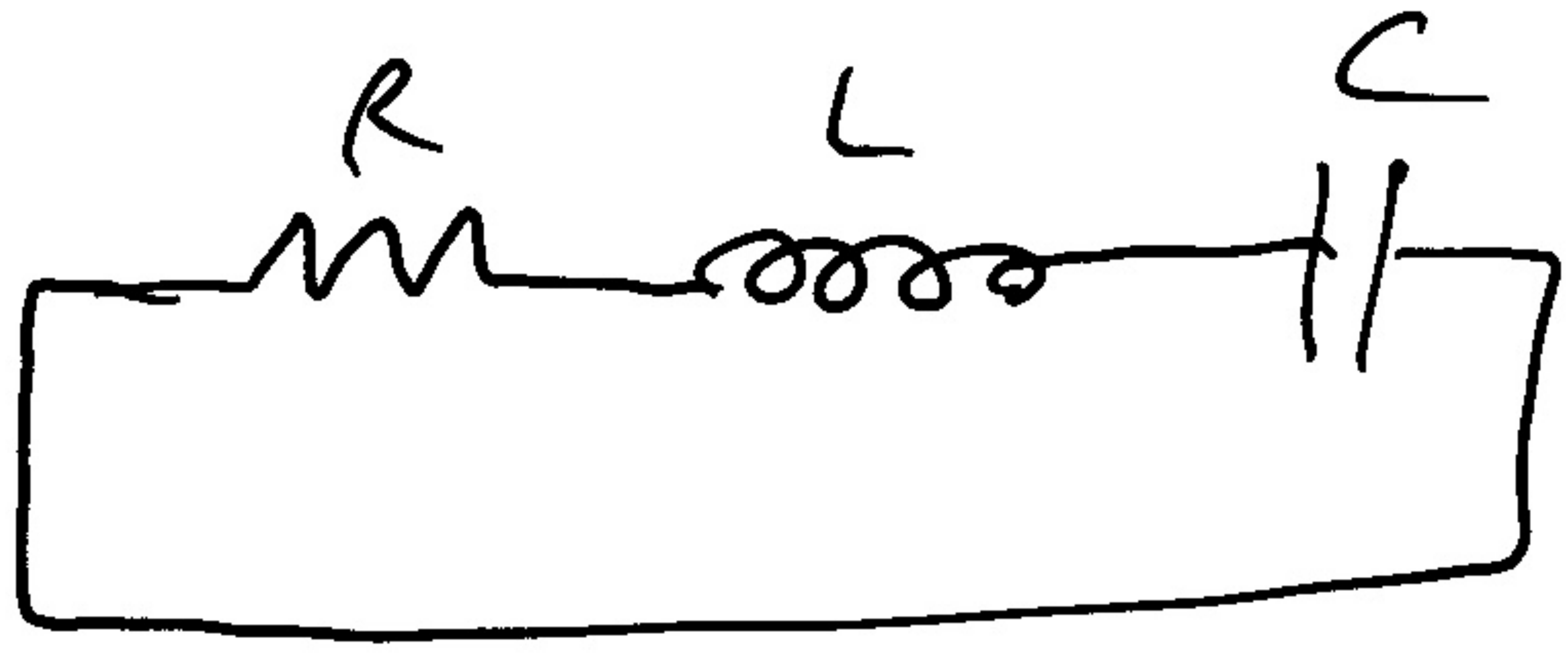
Energy
energy in C
energy in L
total energy
time
i
q
-q
C
L
V
i = 0
i = i max
i = 0
i = −i max
i = 0
q₀
-q₀
q = q₀
q = 0
q = −q₀
q = 0
q = q₀
energy stored in electric field of capacitor
energy stored in magnetic field of inductor
energy stored in electric field of capacitor
energy stored in magnetic field of inductor

RLC Circuit

- Any real circuit has some resistance…



RLC circuit



$$IR + L\,dI/dt + Q/C = 0$$

or $LQ'' + RQ' + Q/C = 0$

$\Rightarrow Q'' + R/L\,Q' + Q/LC = 0$

$Q = Ae^{i\omega t}$ works for LC
so let's try it again

$\Rightarrow -\omega^2 Q + R/L \cdot i\omega Q + 1/LC\,Q = 0$

$-\omega^2 + R\omega/L\, i + 1/LC = 0$

or $\omega^2 - iR\omega/L - 1/LC = 0$

use quadratic formula

$\Rightarrow \omega = \left(iR/L \pm \sqrt{-R^2/L^2 + 4/LC}\right)/2$

$= iR/2L \pm \sqrt{1/LC - (R/2L)^2}$

$= iR/2L \pm \omega'$

w/ $\omega' = \sqrt{1/LC - (R/2L)^2}$

$$\Rightarrow Q(t) = A e^{i(iR/2L + \omega')t} + B e^{i(iR/2L - \omega')t}$$

$$= e^{-R/2L\, t} \left[A e^{i\omega' t} + B e^{-i\omega' t} \right]$$

could also write in terms of sines and cosines

$$Q(t) = e^{-R/2L\, t} \left[C \cos(\omega' t) + D \sin(\omega' t) \right]$$

- Oscillations change frequency from pure LC case and damp as a function of time

- This is a damped harmonic oscillator

AC Circuits

AC Circuits

The Voltage in your wall sockets at home is AC.

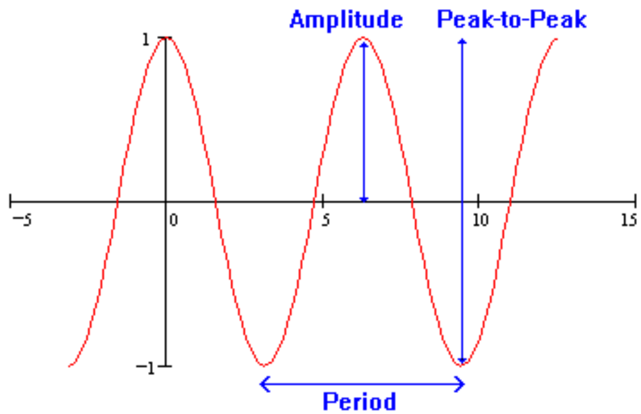
AC stands for Alternating Current, but would perhaps more appropriately be called Alternating Voltage.

Alternating = Sinusoidal with time

Sinusoidal Oscillations

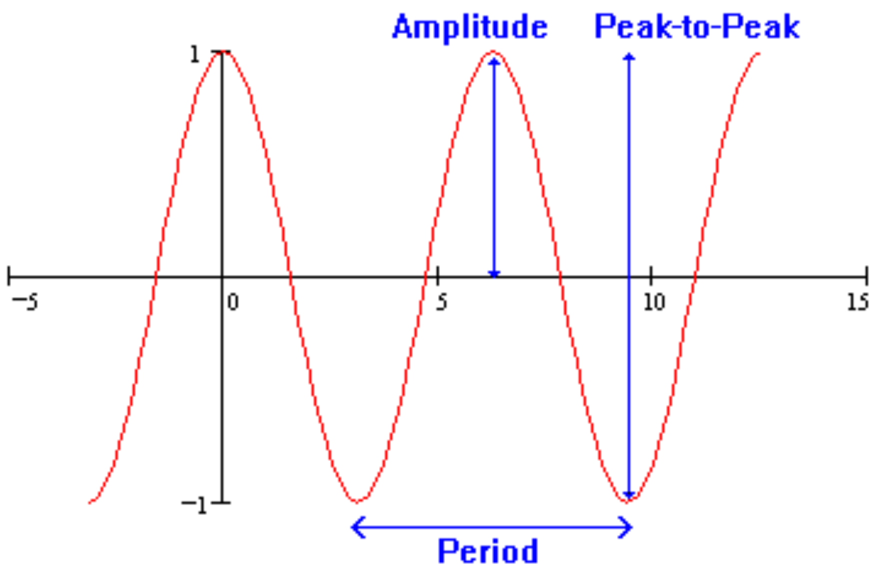
$$V(t) = V_{peak}\sin(\omega t)$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$



ω is the **angular frequency** (radians / second)
f is the **frequency** in (cycles / second = Hertz)
T is the **period** in (seconds), i.e. time for one cycle

Standard US Power



In the United States of America

f = 60 cycles/second or Hertz
T = (1 / 60) seconds
V_{peak} = 170 Volts

One might be interested in something like the average Voltage.
But, the average V(t) = 0.

AC Voltage Levels

$$V_{rms} = V\,(\text{root mean square}) = \sqrt{\langle V^2 \rangle}$$

Time Average of Voltage squared.

$$V_{rms} = V_p \sqrt{\langle \sin^2(\omega t) \rangle} = V_p \sqrt{1/2} = V_p / \sqrt{2}$$

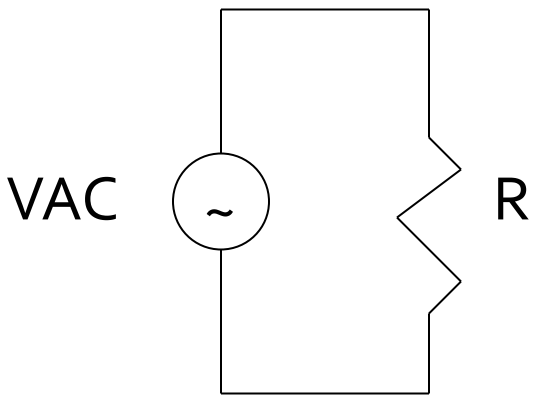
Time Average of sin² or cos² over many cycles = 1/2

$$V_{rms} = 170V / \sqrt{2} = 120\,Volts$$

This is how we refer to the US Standard Voltage.

AC Power

Example: AC Voltage across a resistor



$$V(t) = V_p \sin(\omega t)$$

$$i(t) = \frac{V}{R} = \frac{V_p \sin(\omega t)}{R} = i_{peak} \sin(\omega t)$$

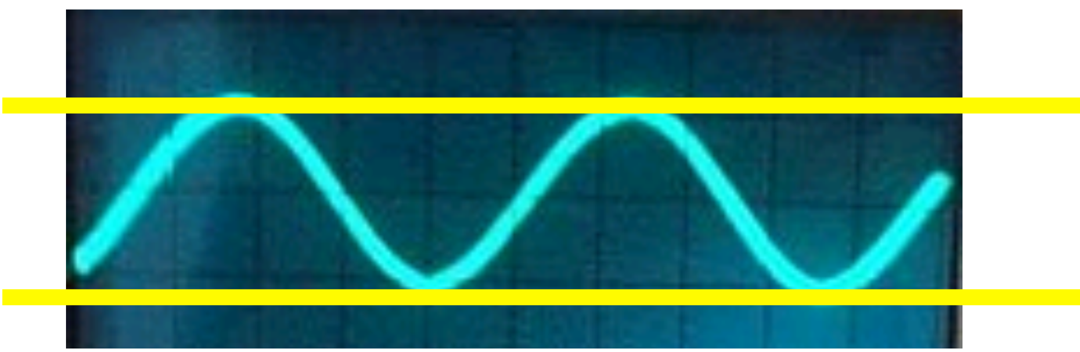
$$P(t) = iV = i_{peak} V_p \sin^2(\omega t)$$

Instantaneous Power is oscillating from zero to maximum.

AC Power

$$\int \sin^2(\omega t)\,dt = \int [1/2 - 1/2\cos(2\omega t)]\,dt$$

$$\int \sin^2(\omega t)\,dt = \frac{1}{2}$$ When integrated over a complete cycle.



Power = maximum

Power = 0

AC Power

$$P(t) = iV = i_{peak} V_{peak} \sin^2(\omega t)$$

$$< P(t) >= i_{peak} V_{peak} < \sin^2(\omega t) >= \frac{1}{2} i_{peak} V_{peak}$$

$$< P(t) >= \left(\frac{1}{\sqrt{2}} i_{peak} \right) \left(\frac{1}{\sqrt{2}} V_{peak} \right)$$

$$\boxed{P_{ave} = i_{rms} V_{rms}}$$

Same form as before, but now RMS values.

Why AC Voltage?

Why do we use AC Voltage in the United States and most of the world?

1. One reason is the ease of generating from a power generator. A rotating coil in a magnetic field creates an induced current, but it is sinusoidally alternating.

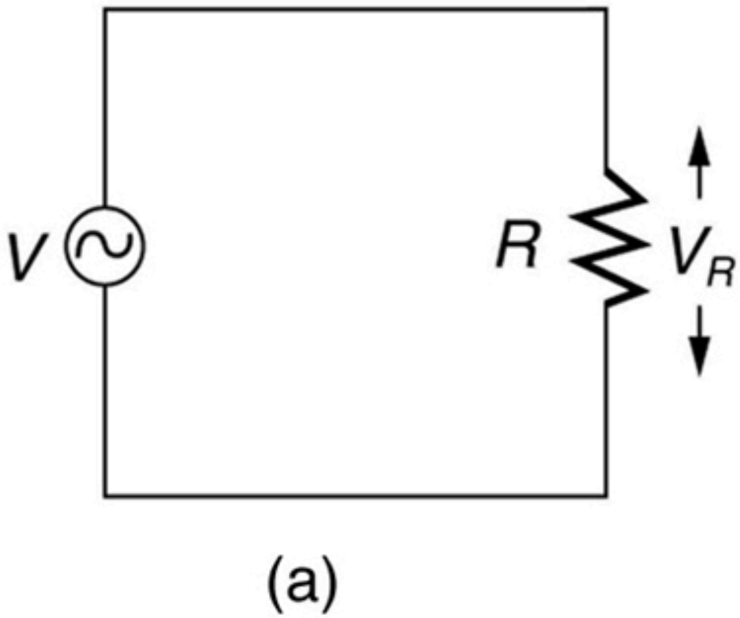
2. AC turns out to be easy to change from one peak Voltage level to another, using a **Transformer**.

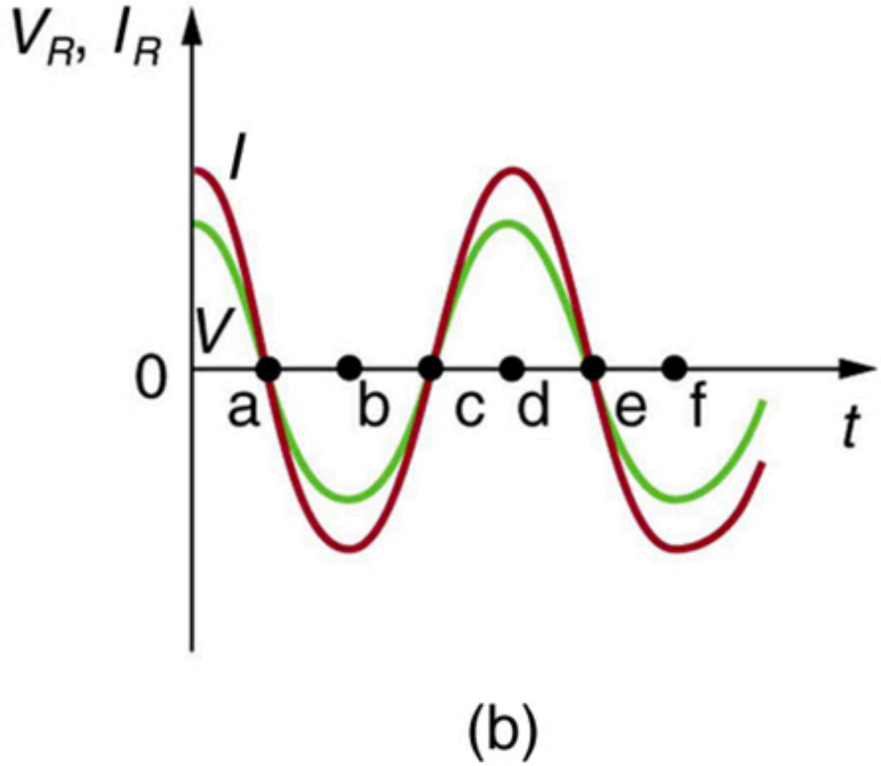
Concept Check

Q17) The voltage across and the current through a single circuit element connected to an ac generator are shown in the graph. Which one of the following statements concerning this circuit element is true?

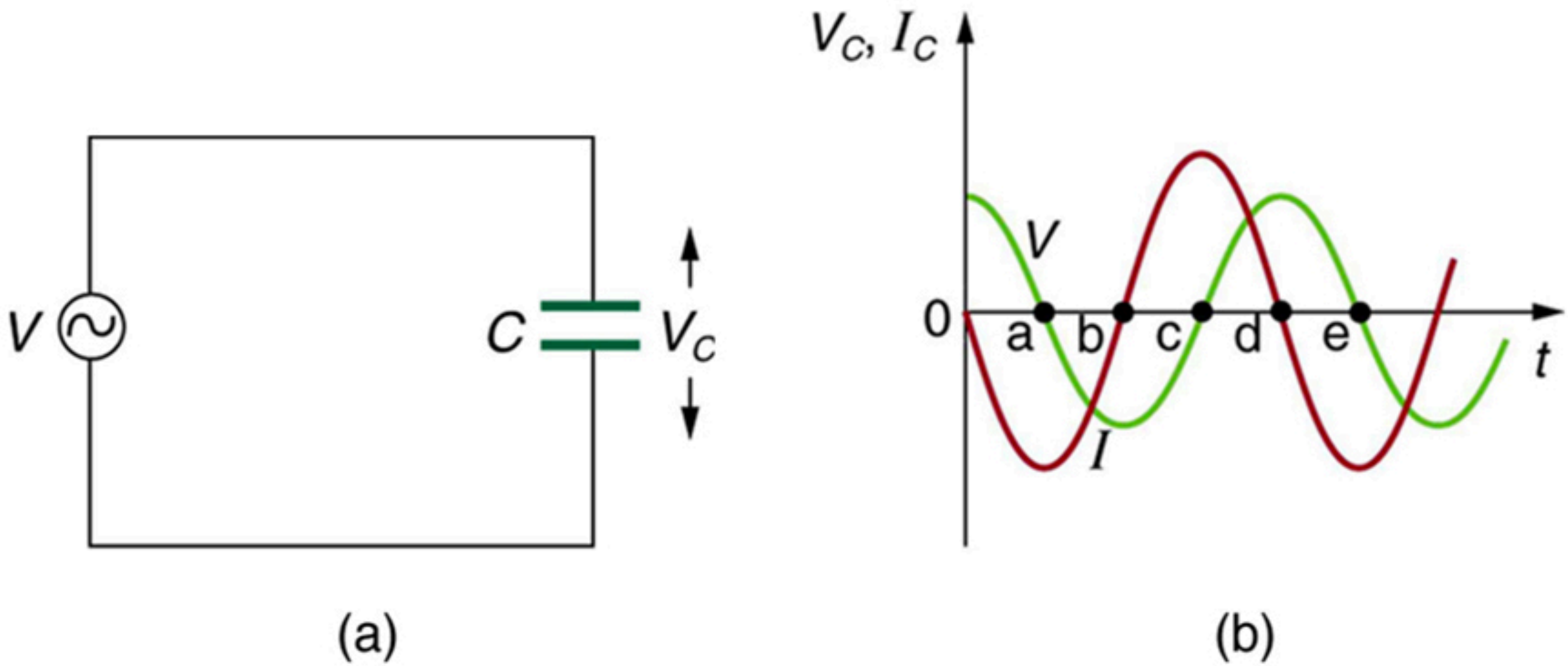
1) The element is a resistor.
2) The element is a capacitor.
3) The element is an inductor.
4) The element could be a resistor or an inductor.
5) The element could be an inductor or a capacitor.

Driven AC Circuits: Resistor



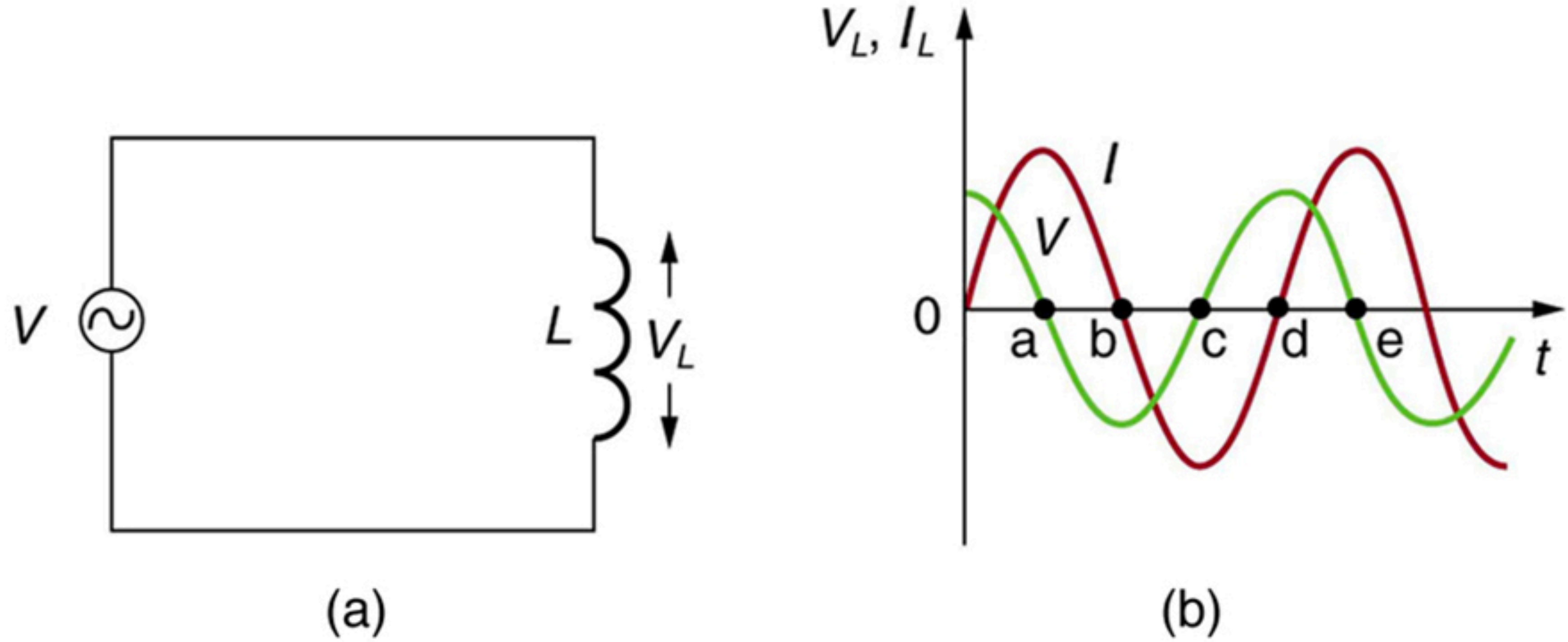


Driven AC Circuits: Capacitor



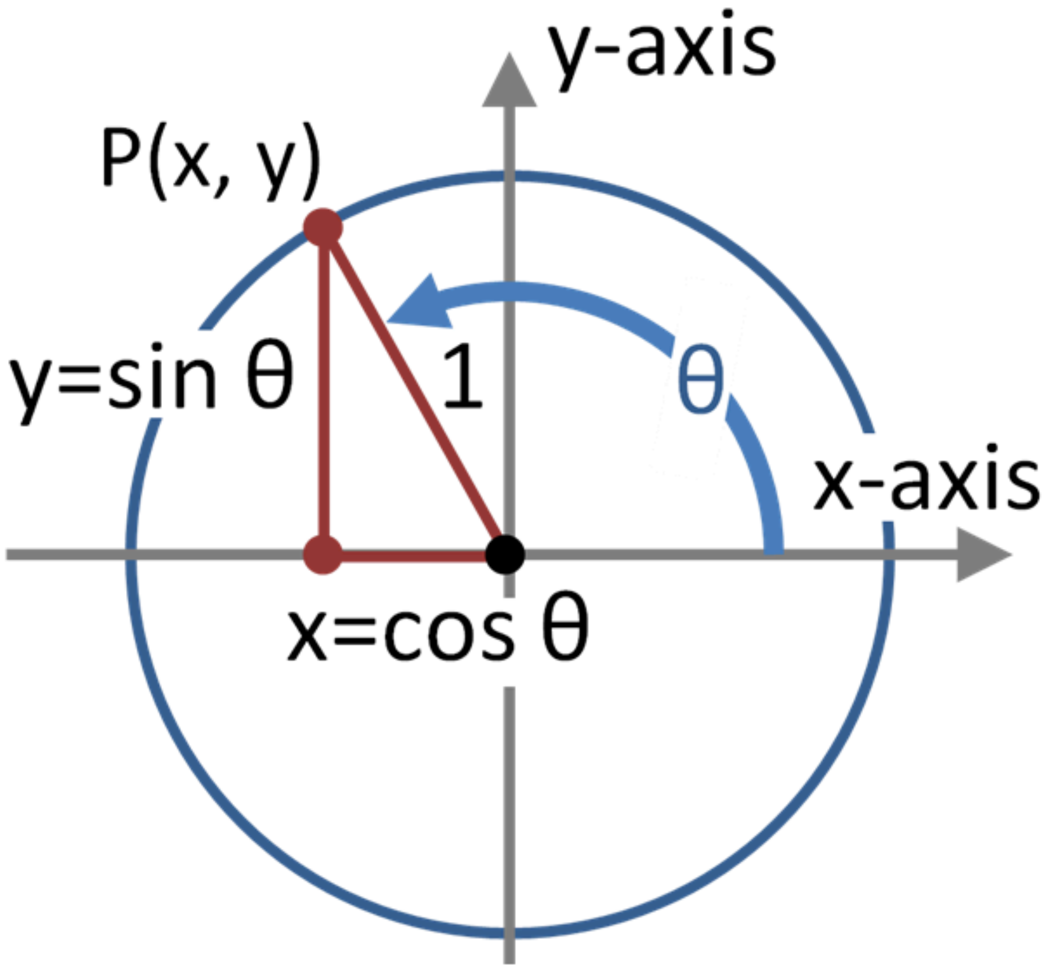
Current “Leads”

Driven AC Circuits: Inductor



Current “Lags”

The Unit Circle in Trigonometry



Phasor Representation