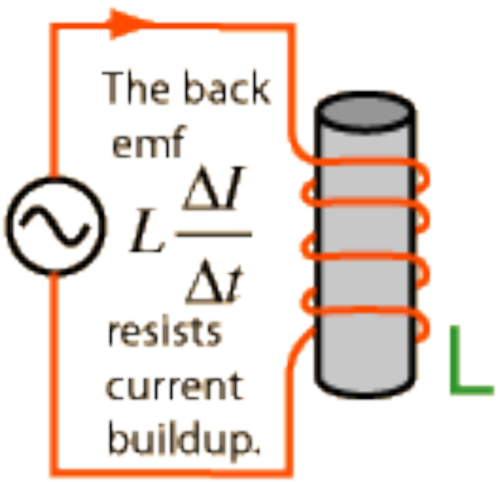
Physics II: 1702
Gravity, Electricity, & Magnetism

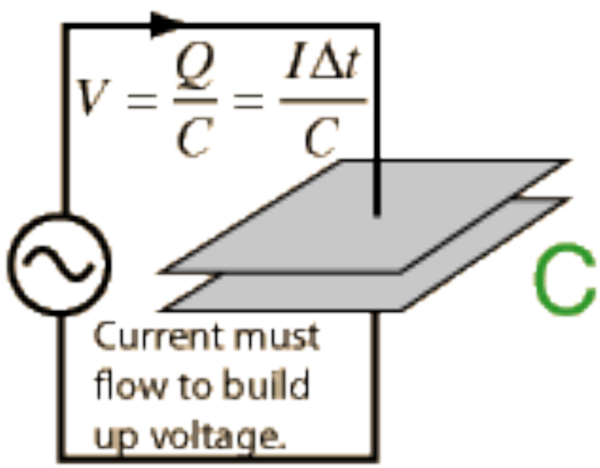
Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Conceptual View

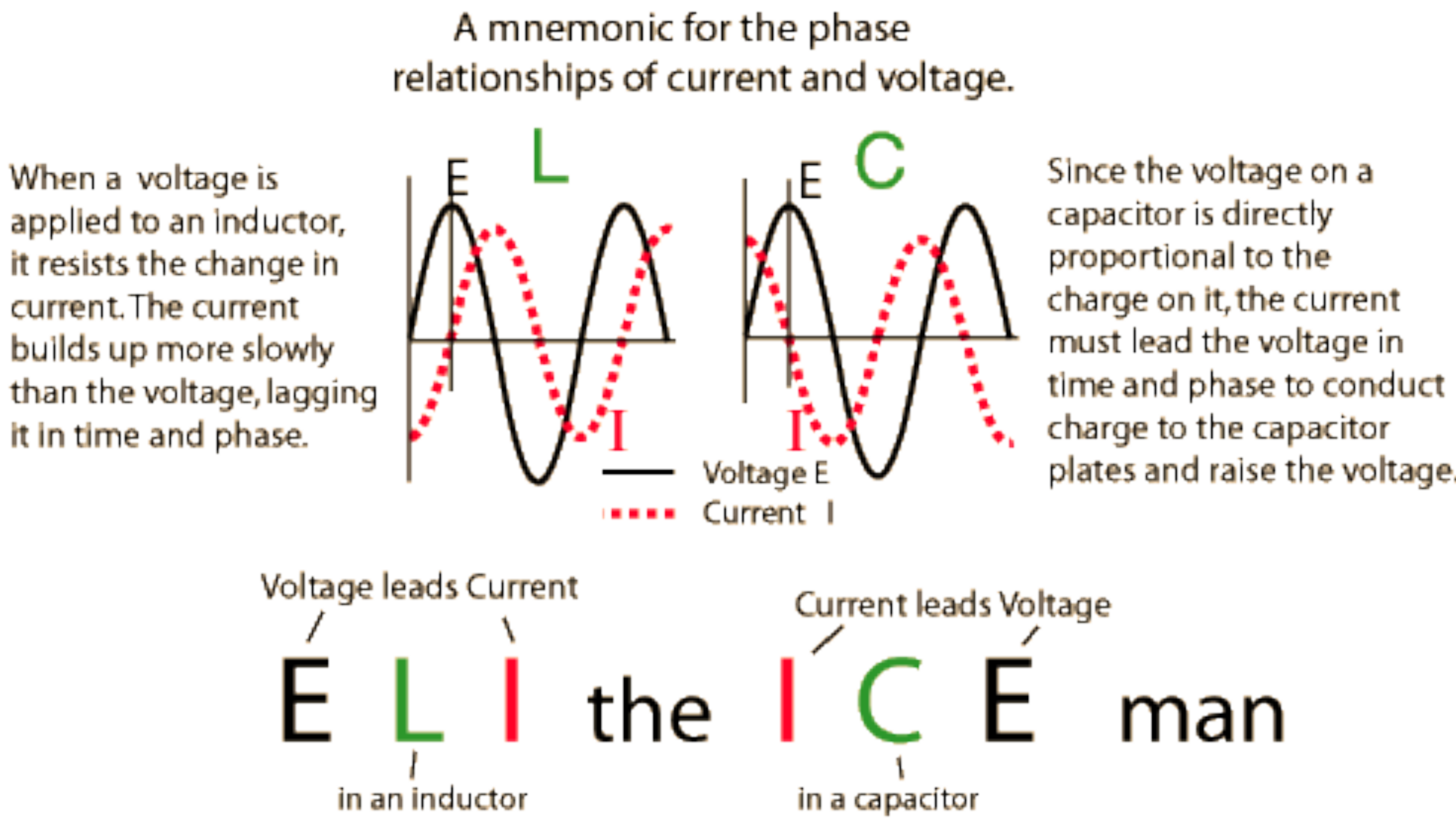
Inductance L



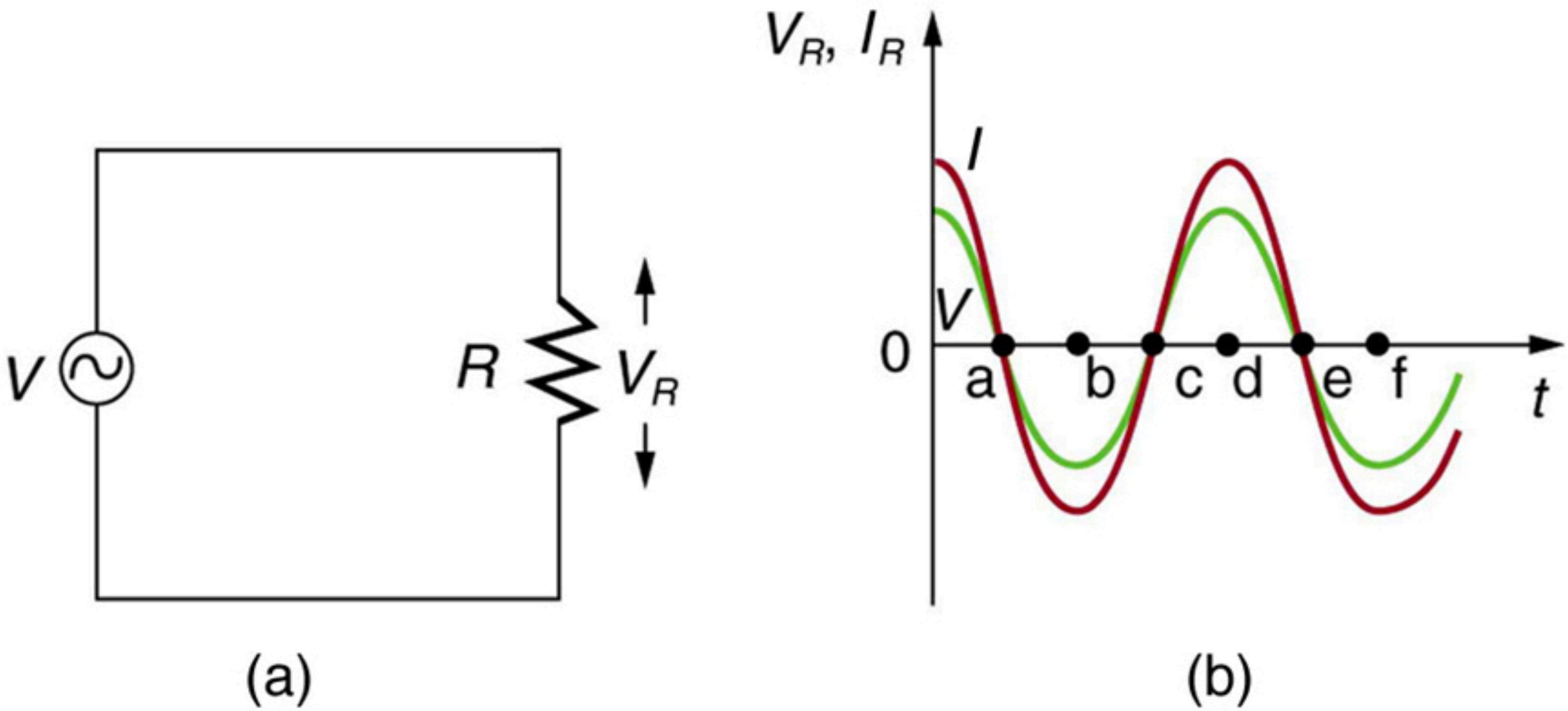
Capacitance C



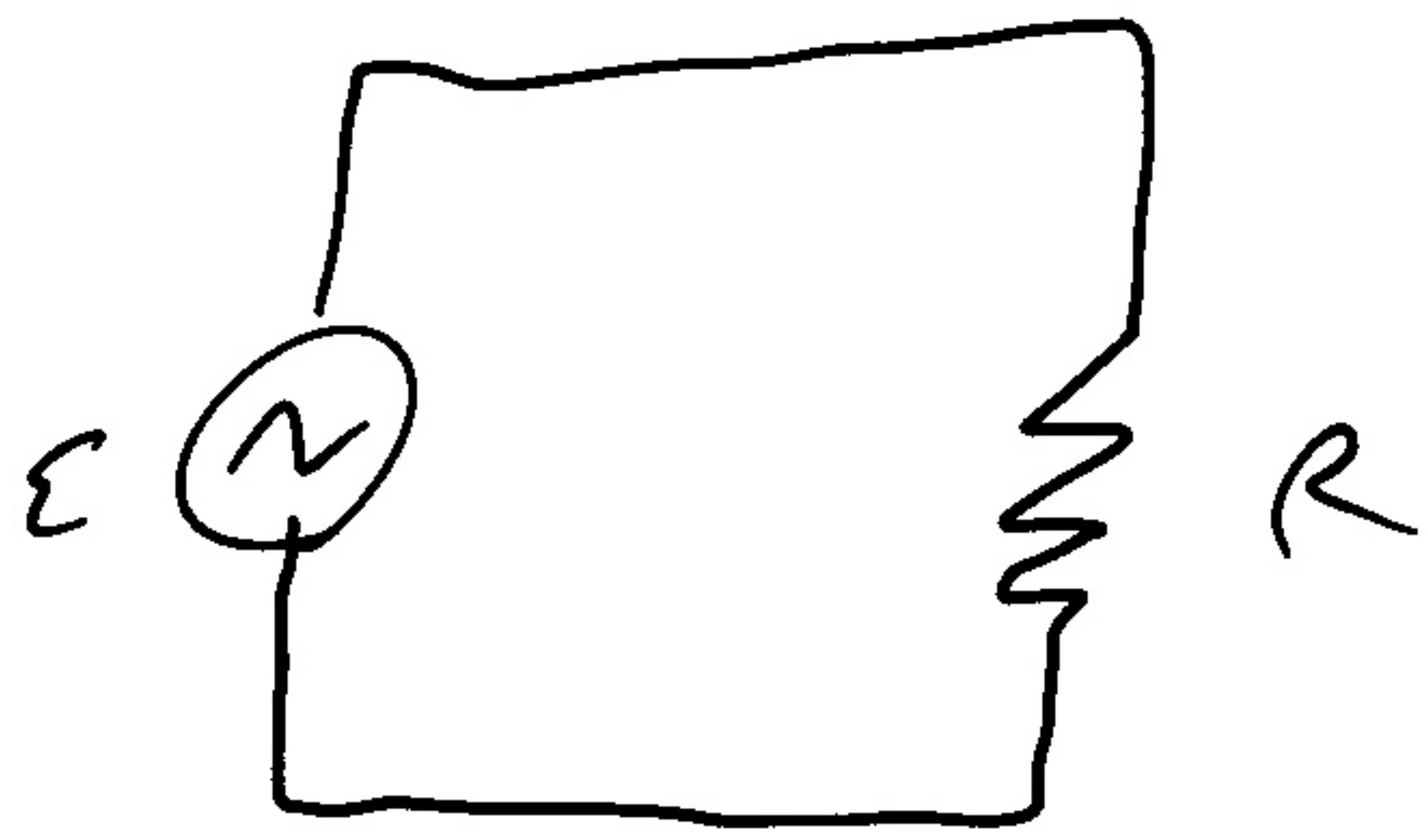
Mnemonic


A mnemonic for the phase
relationships of current and voltage.
When a voltage is applied to an inductor, it resists the change in current. The current builds up more slowly than the voltage, lagging it in time and phase.
E
L
I
E
C
I
Voltage E
Current I
Since the voltage on a capacitor is directly proportional to the charge on it, the current must lead the voltage in time and phase to conduct charge to the capacitor plates and raise the voltage.
Voltage leads Current
Current leads Voltage
E L I the I C E man
in an inductor
in a capacitor

Driven AC Circuits: Resistor



Driven AC Circuits



$$\varepsilon = \varepsilon_m \sin(\omega_d t)$$

AC EMF w/ driving angular frequency ω_d

Loop rule:

$$\varepsilon - v_R = 0 \quad \text{[oscillating quantities]}$$

$$\Rightarrow \quad v_R = \varepsilon_m \sin(\omega_d t)$$
$$= V_R \sin(\omega_d t)$$

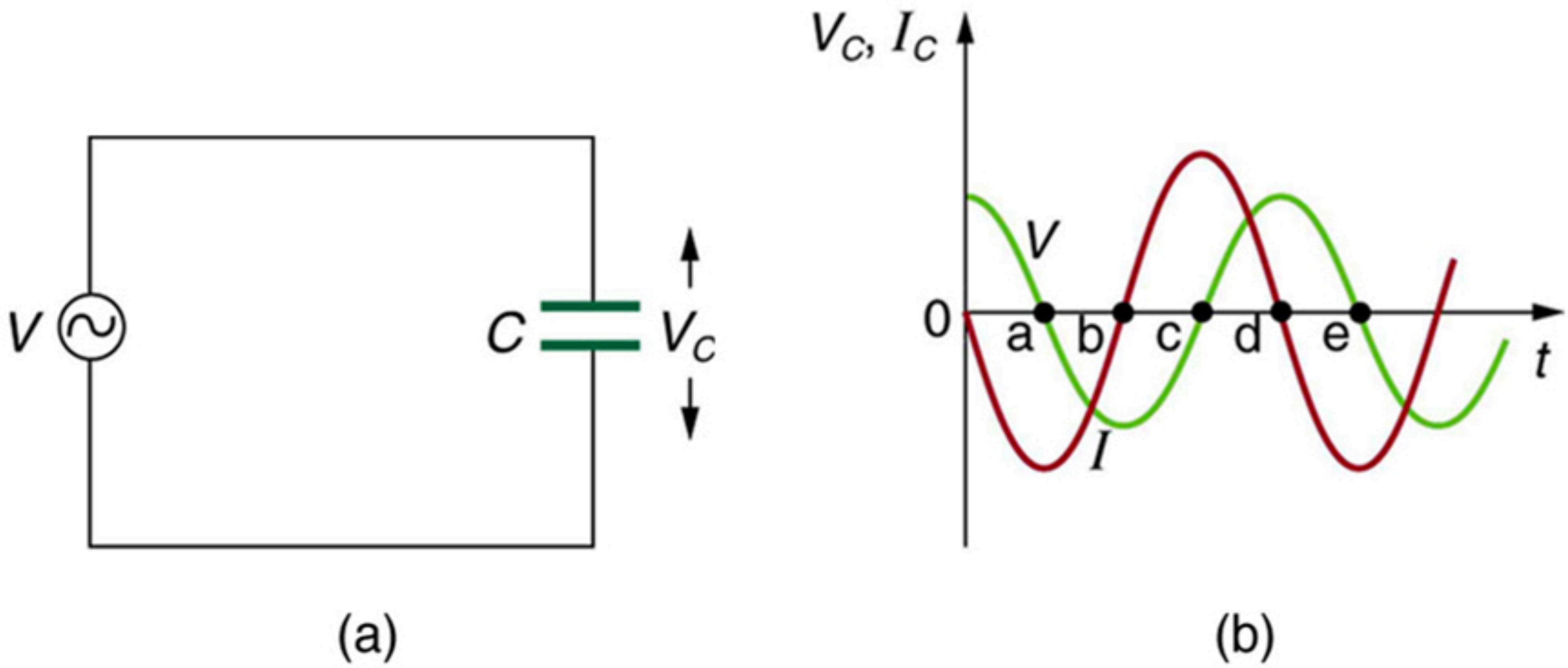
Ohm's Law $i = v_R / R$

$$\Rightarrow i = V_R/R \sin(\omega_d t)$$

$$= I_R \sin(\omega_d t)$$
$$= I_R \sin(\omega_d t - \varphi)$$
w/ $\varphi = 0$

$I_R = V_R/R$ relates the amplitudes

Driven AC Circuits: Capacitor



Current "Leads"

Concept Check

- What is the limit of the effective "resistance" to AC current flow for a capacitor at high frequency?

1. Infinite
2. zero

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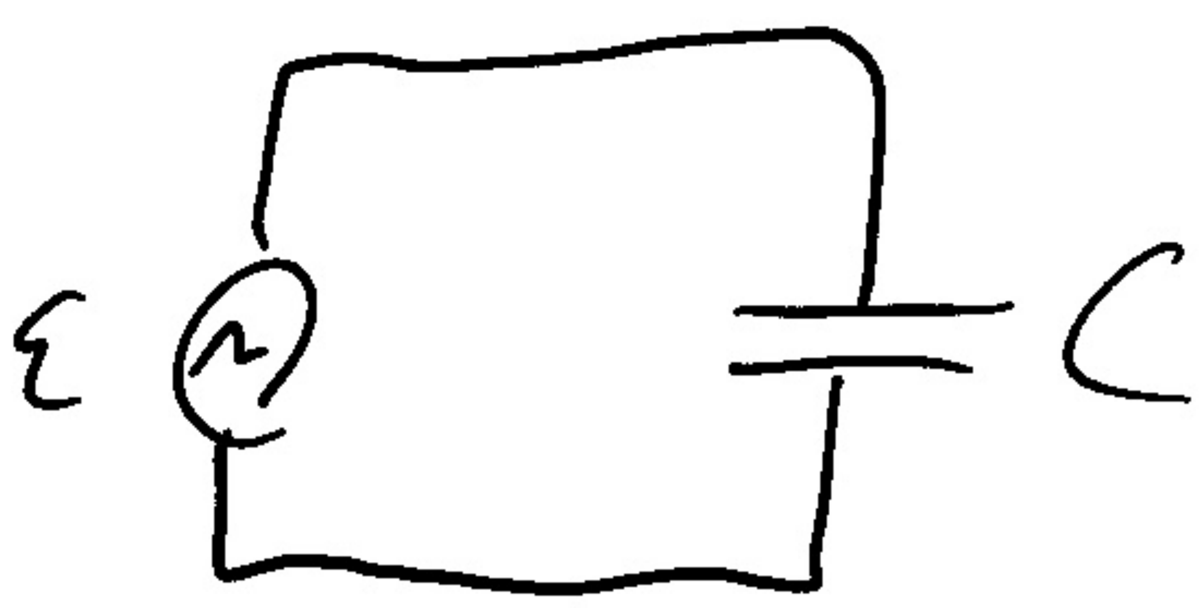


$$\mathcal{E} - v_C = 0$$

$$\Rightarrow v_C = \mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$$
$$= V_C \sin(\omega_d t)$$

$$v_C = q_C / C$$

$$\Rightarrow q_C = C v_C = C V_C \sin(\omega_d t)$$

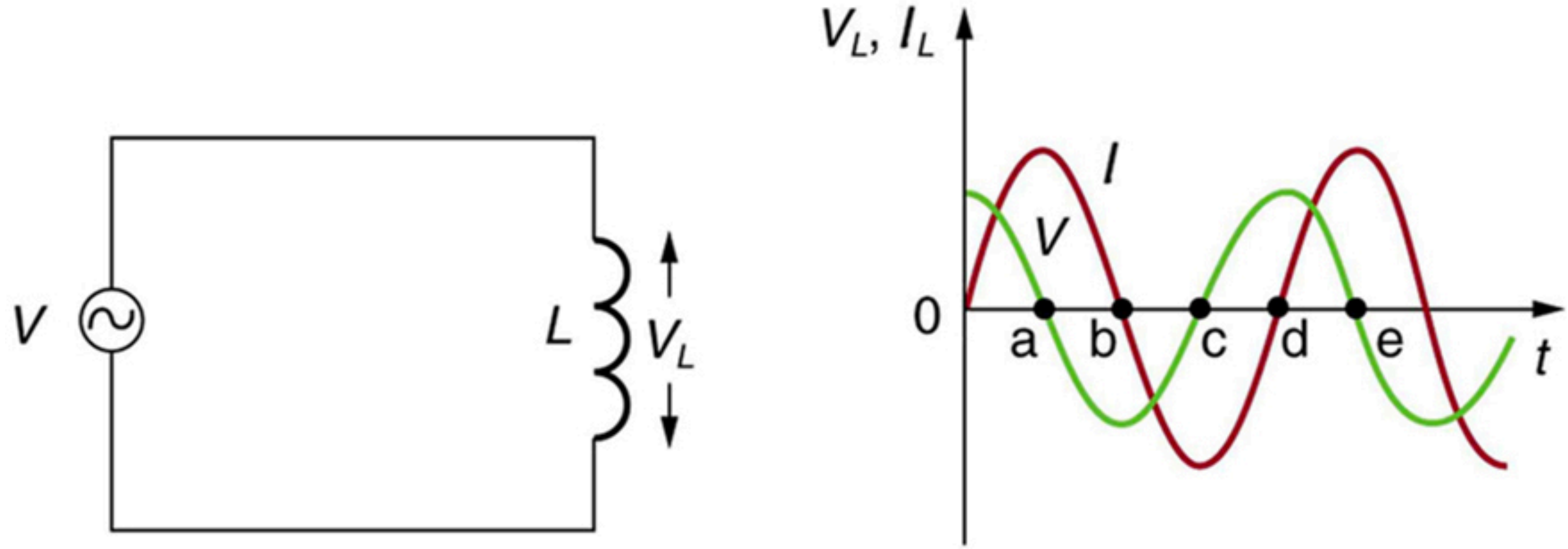
$$i_C = dq_C/dt = \omega_d C V_C \cos(\omega_d t)$$

$$= \omega_d C V_C \sin(\omega_d t + \pi/2)$$
$$= \frac{V_C}{X_C} \sin(\omega_d t - \varphi)$$
$$\text{w/ } \varphi = -\pi/2 \quad [-90°]$$

and $X_C = 1/(\omega_d C)$
= "Capacitive Reactance"

- $I_C = V_C / X_C$ relates amplitudes - like Ohm's law, but w/ X_C instead of R
- current leads voltage ($\varphi < 0$)

Driven AC Circuits: Inductor



(a) (b)

Current “Lags”

Concept Check

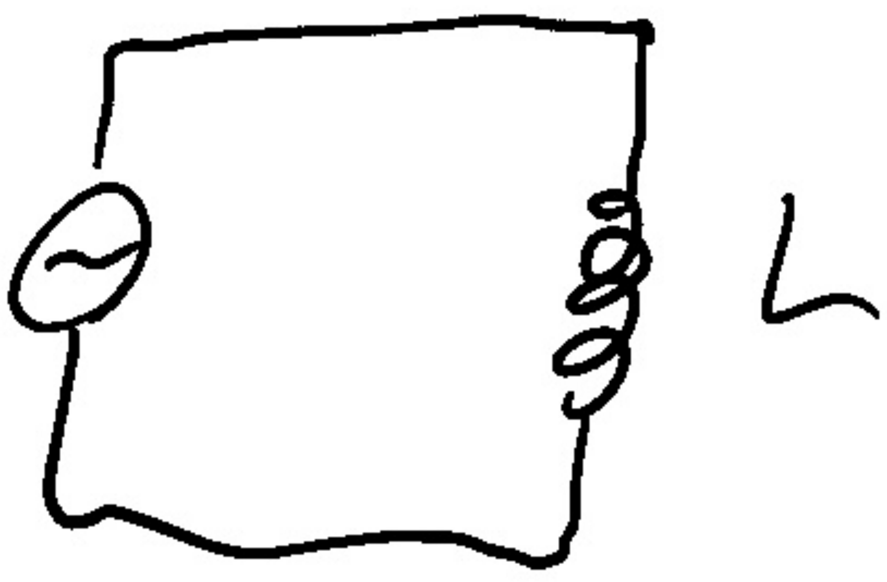
- What is the limit of the effective “resistance” to AC current flow for an inductor at high frequency?

1. Infinite
2. zero

Concept Check

- What is the limit of the effective “resistance” to AC current flow for an inductor at high frequency?

1. Infinite
2. zero



$$\mathcal{E} - V_L = 0$$

$$\Rightarrow V_L = \mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$$
$$= V_L \sin(\omega_d t)$$

$$di_L/dt = v_L/L = V_L/L \sin(\omega_d t)$$

$$\Rightarrow i_L = -V_L/\omega_d L \cos(\omega_d t)$$

$$= V_L/\omega_d L \sin(\omega_d t - \pi/2)$$

$$= V_L/X_L \sin(\omega_d t - \varphi)$$

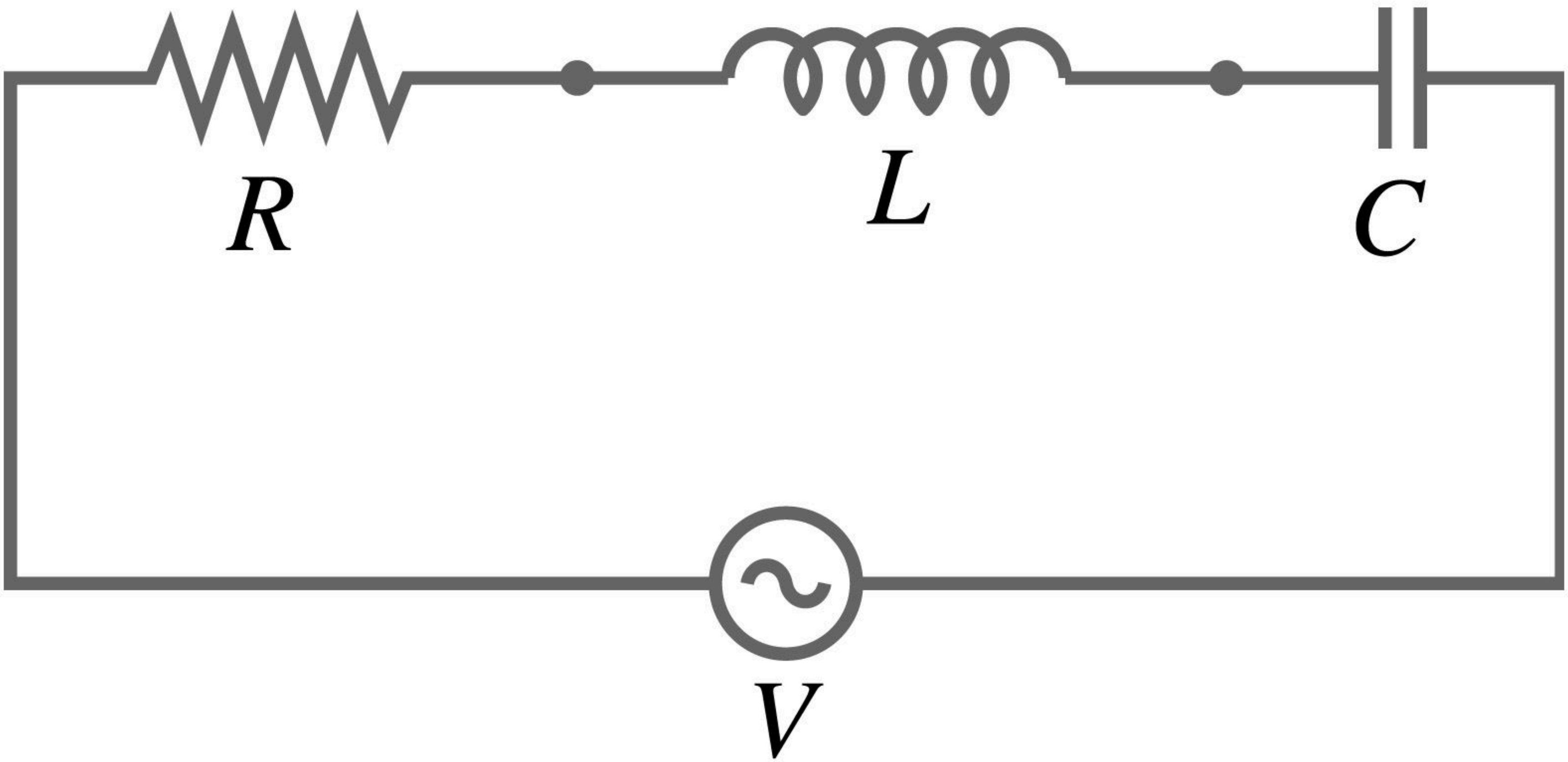
w/ $\varphi = \pi/2$ $(= 90°]$

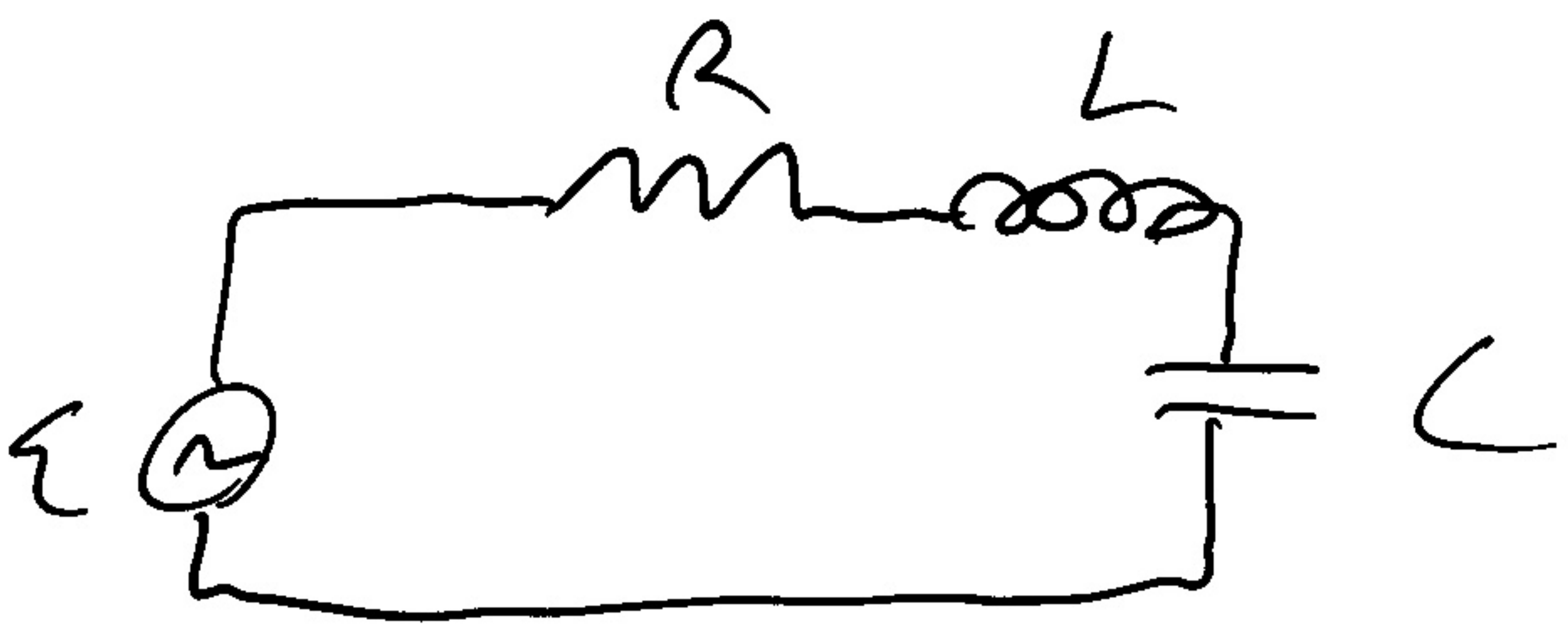
and $X_L = \omega_d L$ = "inductive reactance"

- $I_L = V_L/X_L$ relates amplitudes
- like Ohm's law, but X_L instead of R

- current lags voltage ($\varphi > 0$)

Driven RLC Circuit





$$\mathcal{E} - V_R - V_L - V_C = 0$$

- Current same throughout
- Look for solution

$$i = I \sin(\omega_d t - \varphi)$$

$$\mathcal{E}_m \sin(\omega_d t) - iR - L\,di/dt - q/C = 0$$

$$\mathcal{E}_m \sin(\omega_d t) - IR\sin(\omega_d t - \varphi)$$
$$- \omega_d L I \cos(\omega_d t - \varphi) + \frac{I}{\omega_d C} \cos(\omega_d t - \varphi) = 0$$

or
$$\mathcal{E}_m \sin(\omega_d t) - IR \sin(\omega_d t - \varphi)$$
$$- I[X_L - X_C] \cos(\omega_d t - \varphi) = 0$$

Expand trig functions:

$$\mathcal{E}_m \sin(\omega_d t) - IR\left[\sin\omega_d t \cos\varphi - \cos\omega_d t \sin\varphi\right]$$
$$- I[X_L - X_C]\left[\cos(\omega_d t)\cos\varphi + \sin\omega_d t \sin\varphi\right] = 0$$

collect terms:

$$\mathcal{E}_m - IR\cos\varphi - I[X_L - X_C]\sin\varphi = 0$$

$$IR\sin\varphi - I[X_L - X_C]\cos\varphi = 0$$

$$\Rightarrow \boxed{\tan\varphi = \frac{X_L - X_C}{R}}$$

$$\rightarrow \mathcal{E}_m / I = R\cos\varphi + [X_L - X_C]\sin\varphi$$

$$= R\left[\cos\varphi + \frac{X_L - X_C}{R}\sin\varphi\right]$$

$$= R\left(\cos\varphi + \sin\varphi\tan\varphi\right]$$

$$= R\left[\cos\varphi + \sin^2\varphi / \cos\varphi\right]$$

$$= R\left[\cos^2\varphi + \sin^2\varphi\right] / \cos\varphi$$

$$= \boxed{R / \cos\varphi}$$

$$= R\sqrt{1 + \tan^2\varphi}$$

$$= R\sqrt{1 + (X_L - X_C)^2 / R^2}$$

$$= \boxed{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\mathcal{E}_m / I = \sqrt{R^2 + [X_L - X_C]^2}$$
$$= Z$$
$$= R / \cos\varphi$$

Full AC Ohm's Law

$$\boxed{\mathcal{E}_m = I Z}$$

$$\boxed{\varphi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]}$$

$$Z = \sqrt{R^2 + [X_L - X_C]^2}$$
$$= R / \cos\varphi$$

or $\cos\varphi = R/Z$

if $X_L = X_C$

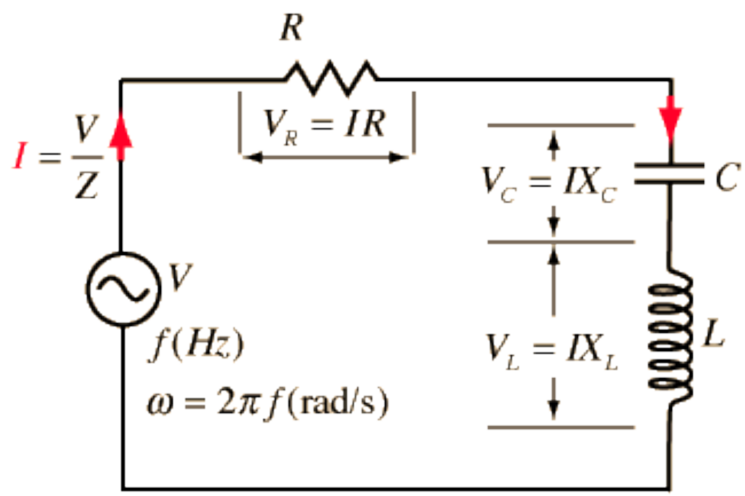
$Z = R$ and $\varphi = 0$

RLC Circuit Solution

Series resonant condition:

$$Z = R \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$X_C = X_L \qquad Phase = \phi = 0$$



$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad Phase = \phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$