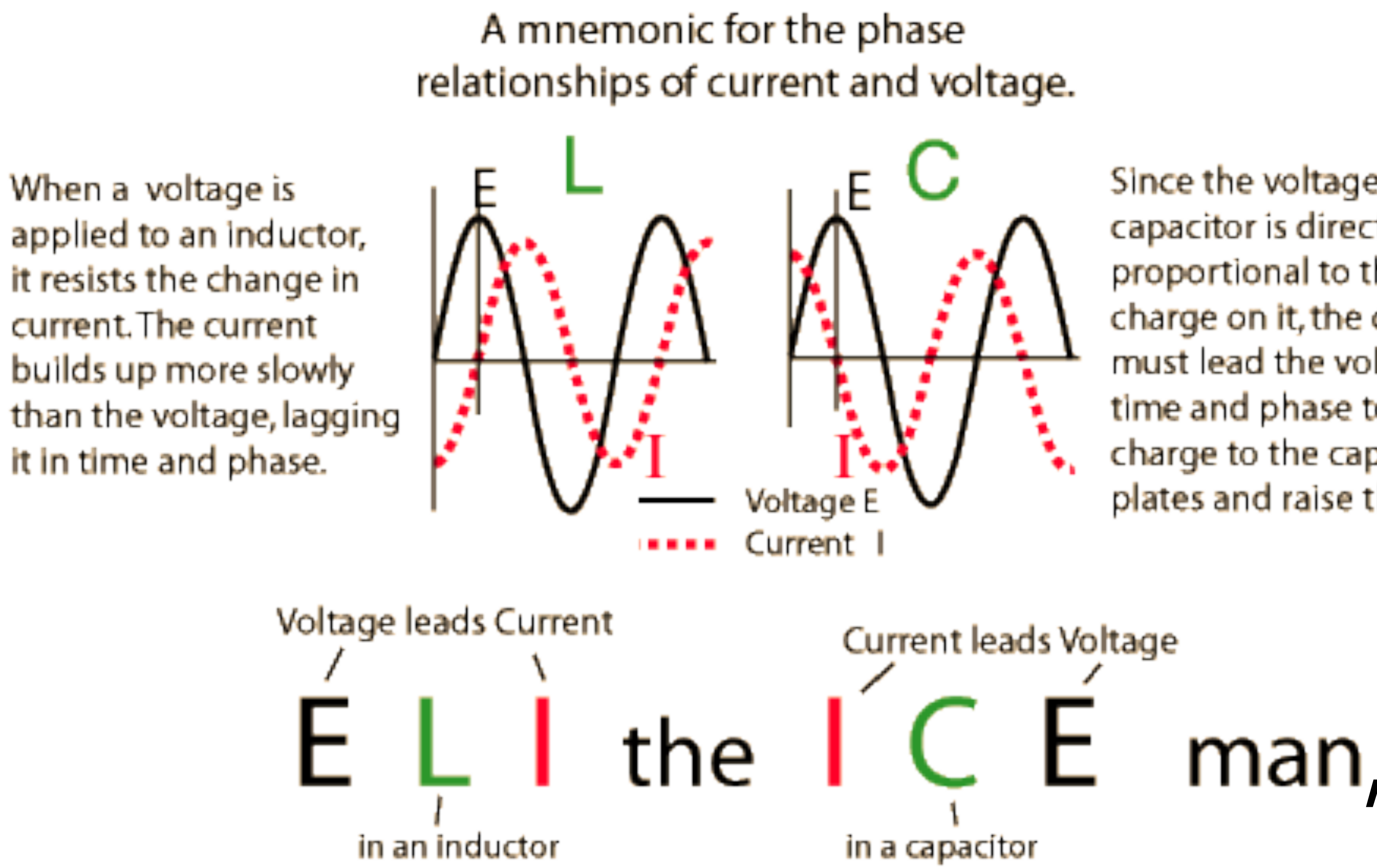
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Mnemonic



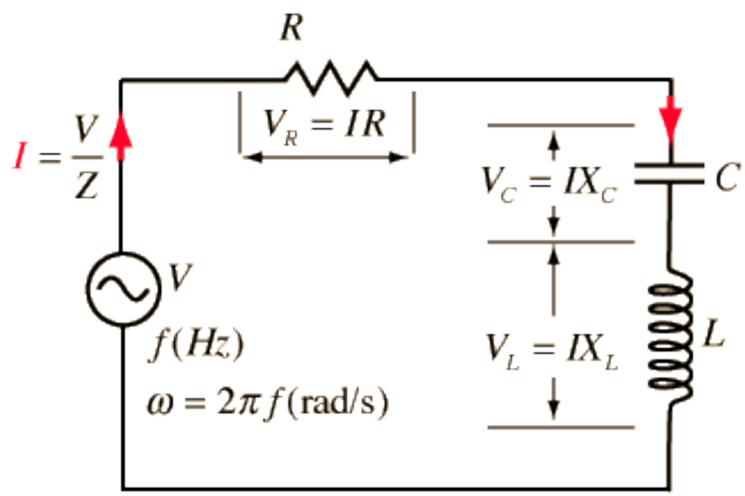
RIP

RLC Circuit Solution

Series resonant condition:

$$Z = R \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$X_C = X_L \qquad Phase = \phi = 0$$



$$X_C = \frac{1}{\omega C}$$

$$X_L = \omega L$$

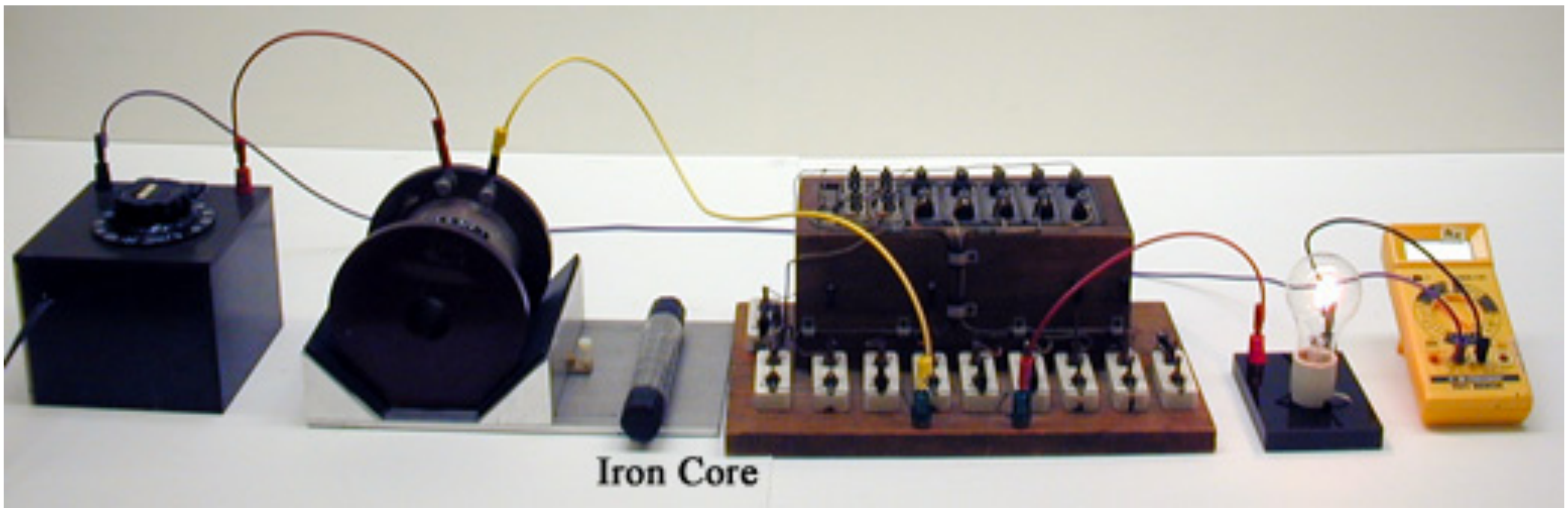
$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad Phase = \phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$

Concept Check

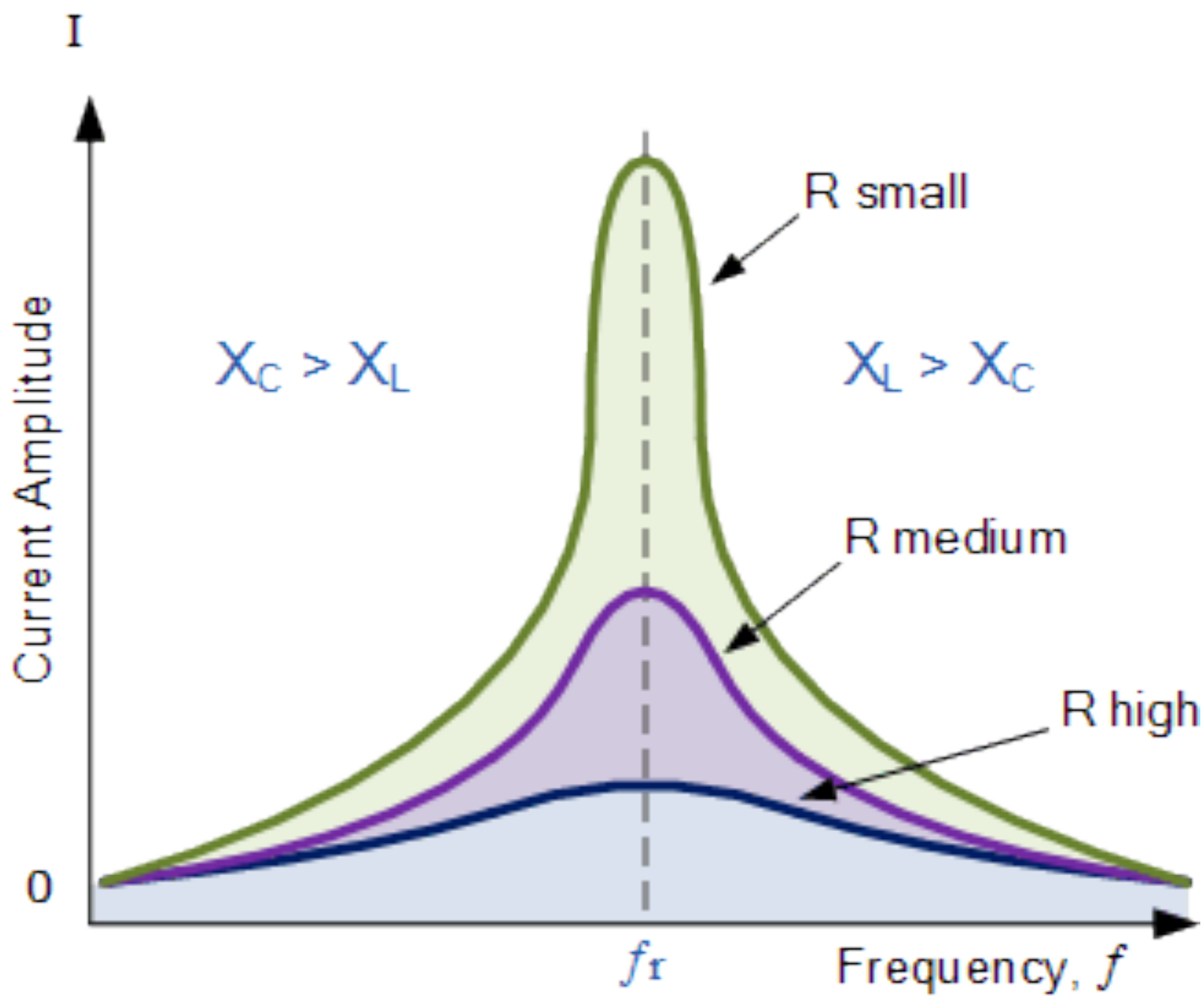
- Which changes could be made to increase the amplitude of the current in a driven RLC circuit where the voltage leads the current?

- Decreasing the resistance
- Decreasing the inductance
- Decreasing the capacitance
- Decreasing the driving frequency
- All of the above

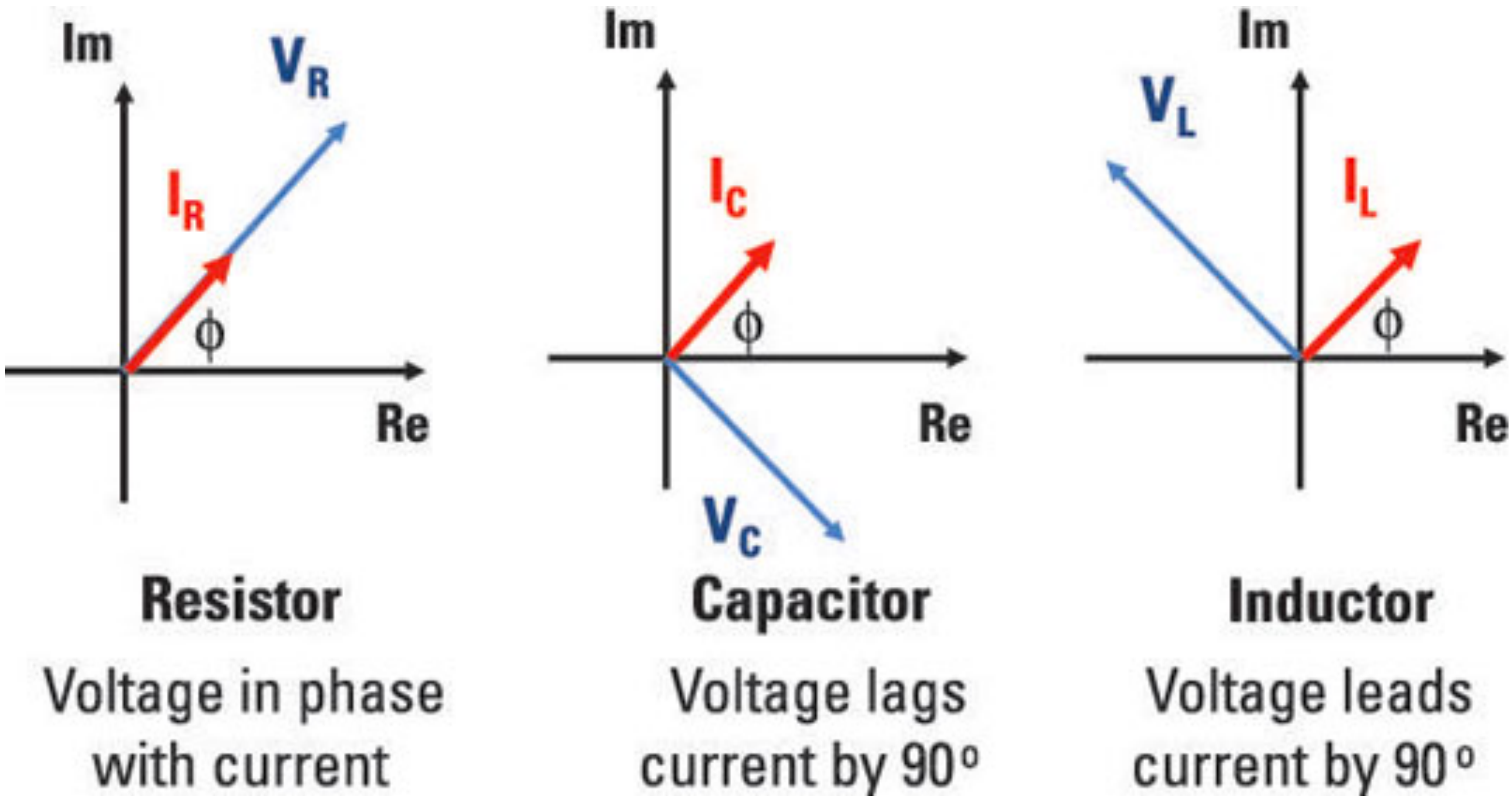
RLC Resonance Demo



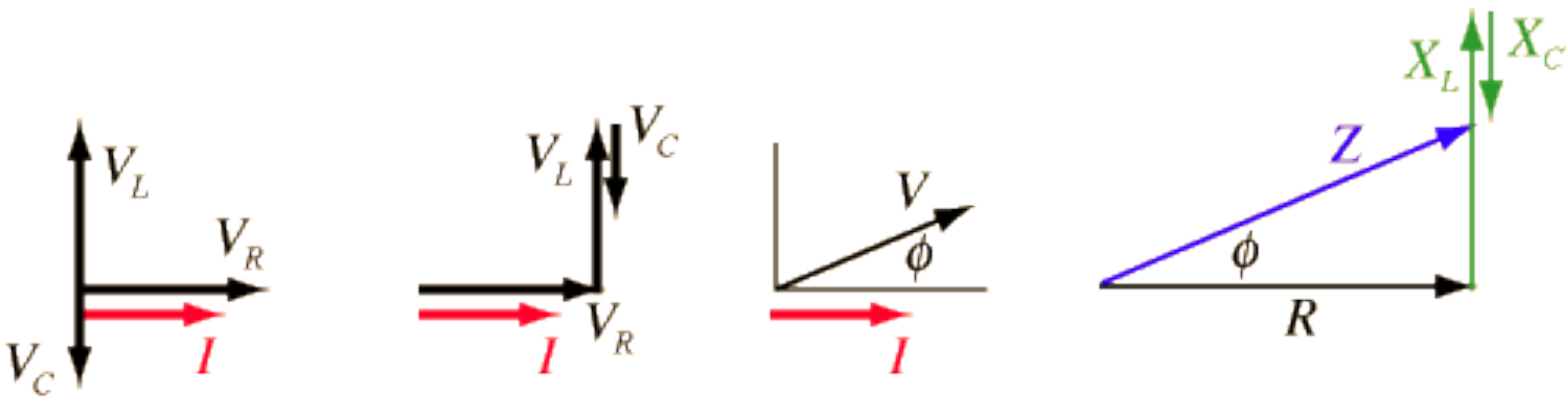
RLC Circuit Resonance



Phasor Representation



Phasor Representation



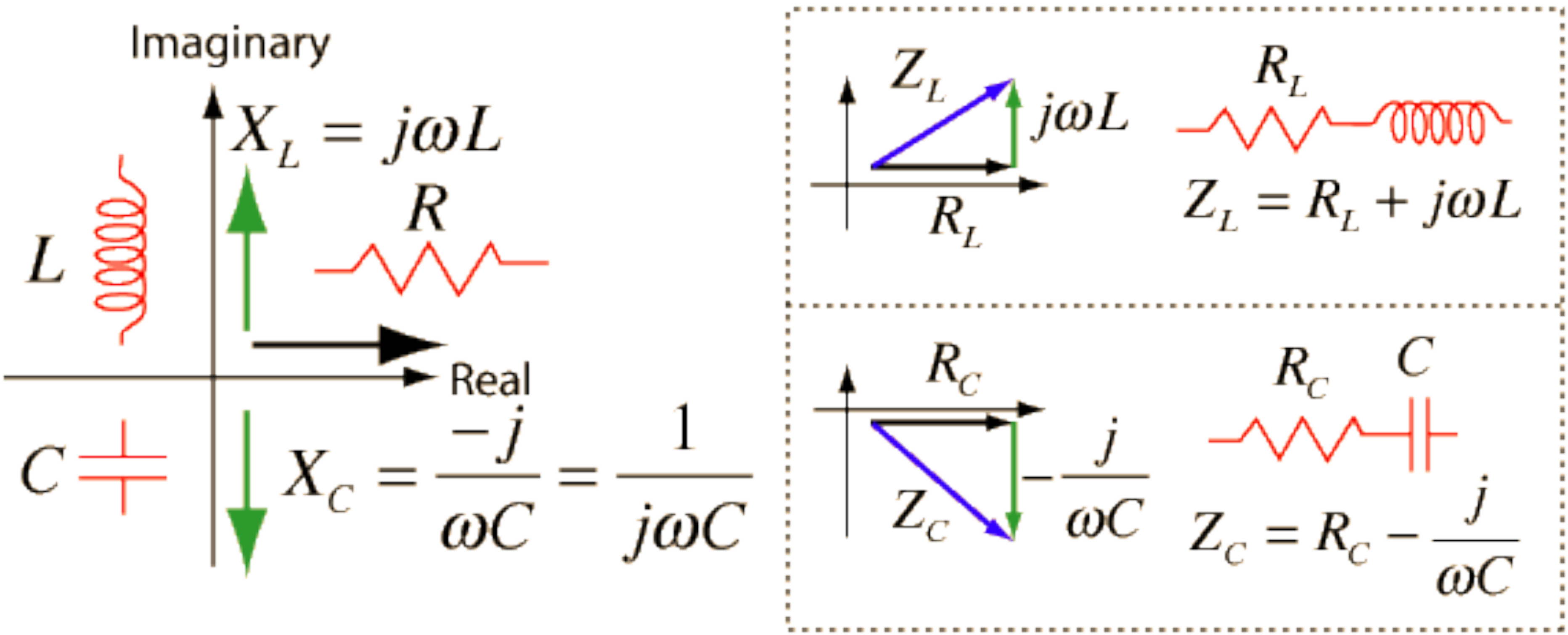
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1} \frac{V_L - V_C}{V_R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \frac{X_L - X_C}{R}$$

Complex Impedance



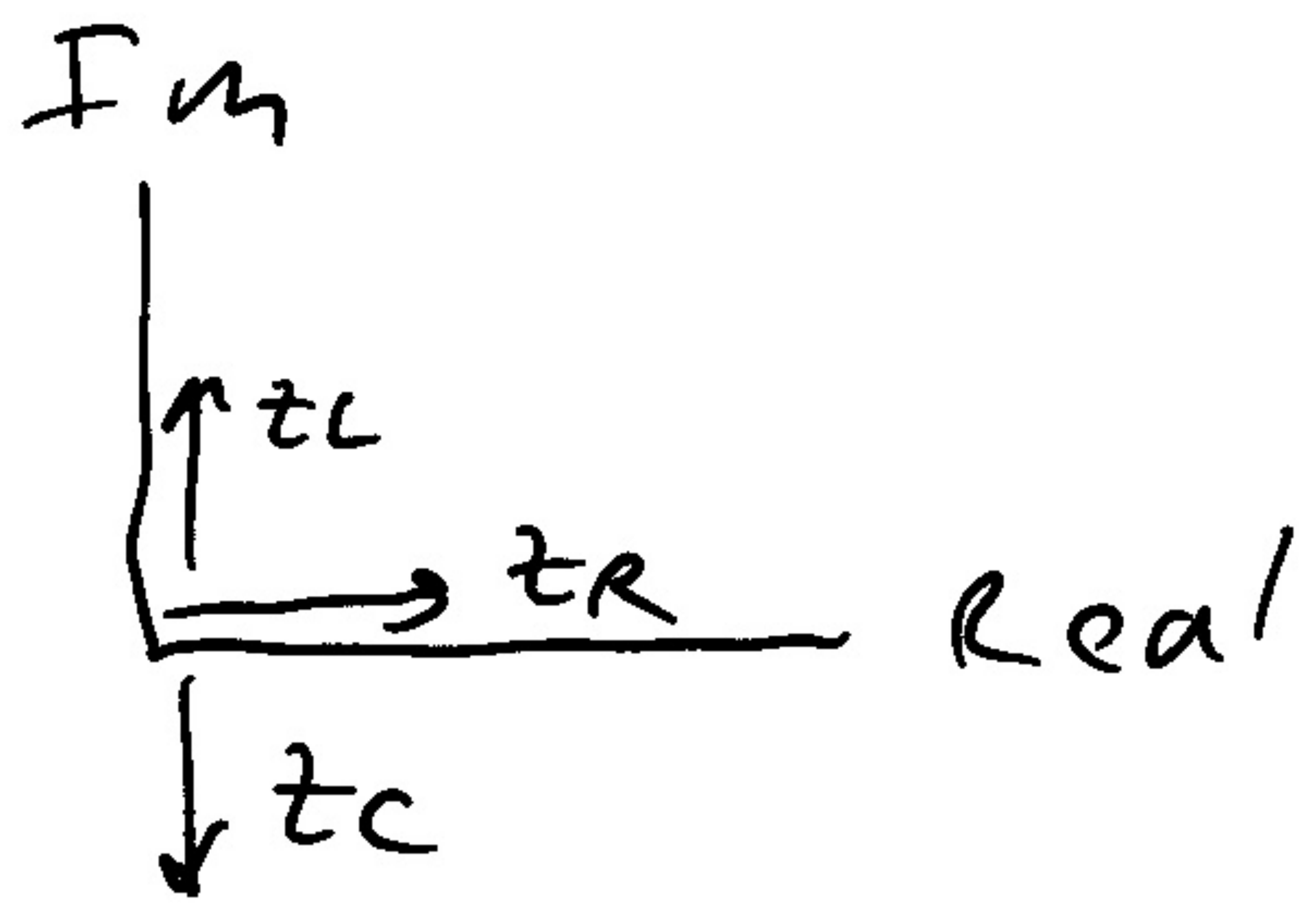
Complex Impedance

$$Z_R = R$$

$$Z_L = j\omega L \quad (j = \sqrt{-1})$$

$$Z_C = -j/\omega C$$

- Just like phasor representation, but in complex plane



- If $\quad \mathcal{E} = \mathcal{E}_m e^{j\omega t}$
- Look for $\quad i = I e^{j(\omega t - \varphi)}$

$$\begin{aligned}
\mathcal{E} &= i Z \\
&= i (Z_R + Z_L + Z_C) \\
&= I e^{j(\omega t - \varphi)} [R + j\omega L - j/\omega C] \\
&= \mathcal{E}_m e^{j\omega t}
\end{aligned}$$

$$\Rightarrow \mathcal{E}_m = I e^{-j\varphi} [R + j\omega L - j/\omega C]$$

$$\text{or} \quad \mathcal{E}_m/I = e^{-j\varphi}\left[R + j\omega L - j/\omega C\right]$$

$$= |Z| \quad (\text{real by definition})$$

$$e^{-j\varphi}\left[R + j\omega L - j/\omega C\right] = |Z|$$

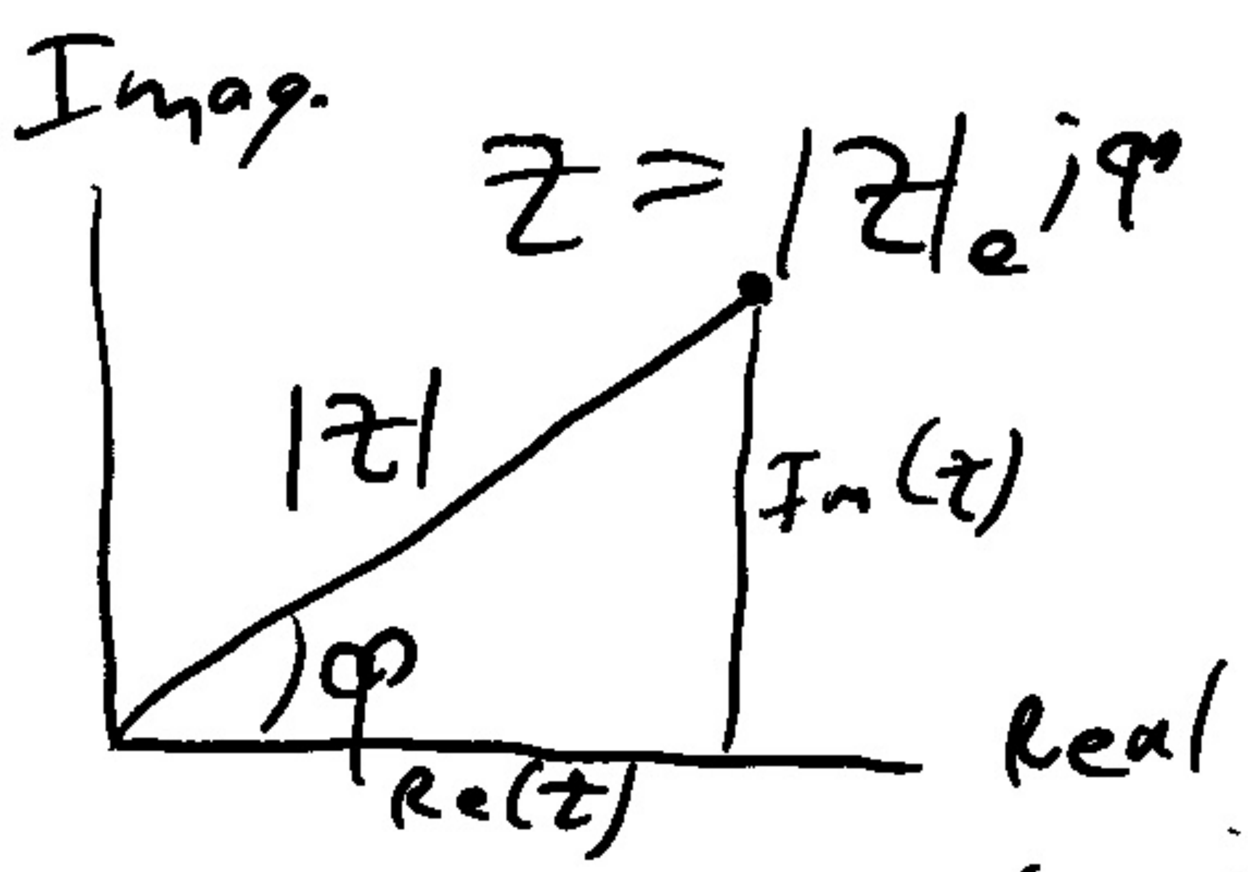
$$\Rightarrow \quad R + j\omega L - j/\omega C = |Z| e^{j\varphi} = Z$$

impedance magnitude → $|Z|$

phase shift → $e^{j\varphi}$

In complex plane:

$$Z = |Z| e^{j\varphi} = |Z|(\cos\varphi + j\sin\varphi)$$



$$\varphi = \tan^{-1}\left[\frac{\text{Im}(e^{j\varphi})}{\text{Re}(e^{j\varphi})}\right]$$

$$= \tan^{-1}\left[\frac{\omega L - 1/\omega C}{R}\right]$$

$$|Z| = \sqrt{\text{Re}(Z)^2 + \text{Im}(Z)^2} = \sqrt{R^2 + \left[\omega L - 1/\omega C\right]^2}$$

Same answer
easier math!!

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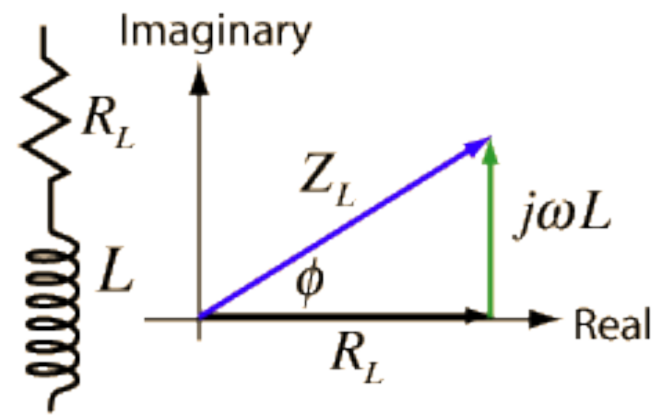
Complex Impedance

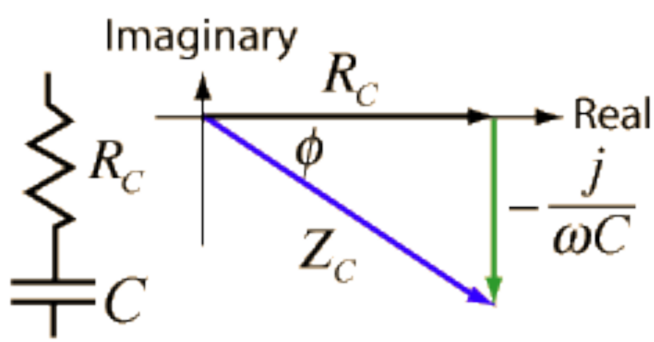


Cartesian form: $Z_L = R_L + j\omega L$

Polar form: $Z_L = |Z_L| e^{j\phi}$

where $|Z_L| = \sqrt{R_L^2 + \omega^2 L^2}$

$$\phi = \tan^{-1}\frac{\omega L}{R_L}$$



Cartesian form: $Z_C = R_C - \frac{j}{\omega C}$

Polar form: $Z_C = |Z_C| e^{j\phi}$

where $|Z_C| = \sqrt{R_C^2 + \frac{-1}{\omega C R_C}}$

$$\phi = \tan^{-1}\frac{-1}{\omega C R_C}$$