

Physics II: 1702

Gravity, Electricity, & Magnetism

Professor Jasper Halekas

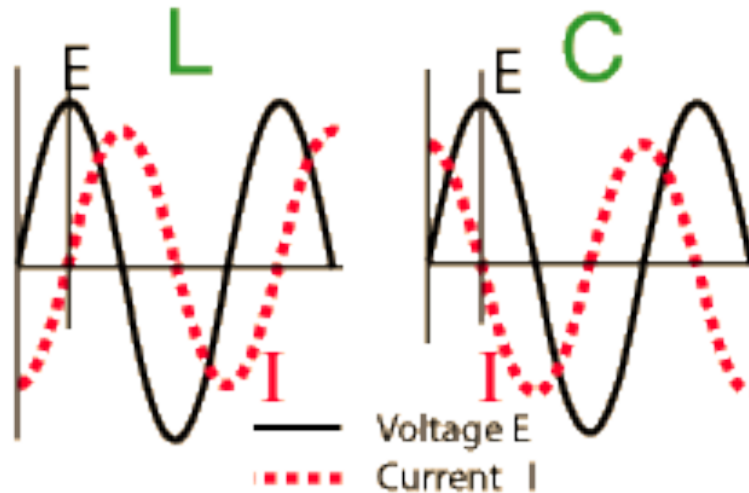
Van Allen 70 [Clicker Channel #18]

MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Mnemonic

A mnemonic for the phase relationships of current and voltage.

When a voltage is applied to an inductor, it resists the change in current. The current builds up more slowly than the voltage, lagging it in time and phase.



Since the voltage on a capacitor is directly proportional to the charge on it, the current must lead the voltage in time and phase to conduct charge to the capacitor plates and raise the voltage.

Voltage leads Current

E **L** **I**
in an inductor

Current leads Voltage

I **C** **E**
in a capacitor

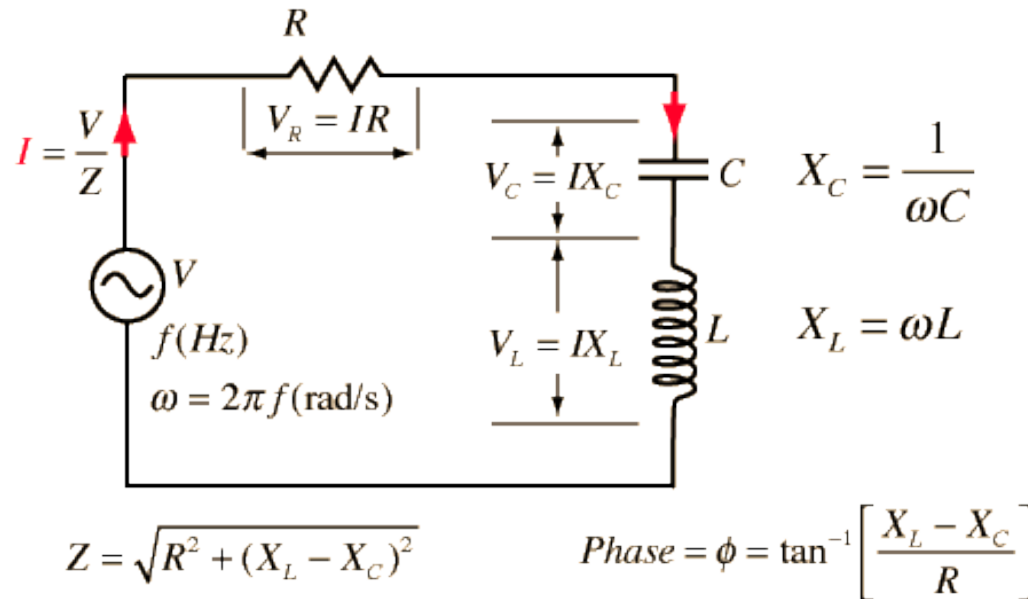
the

man, **RIP**

RLC Circuit Solution

Series resonant condition:

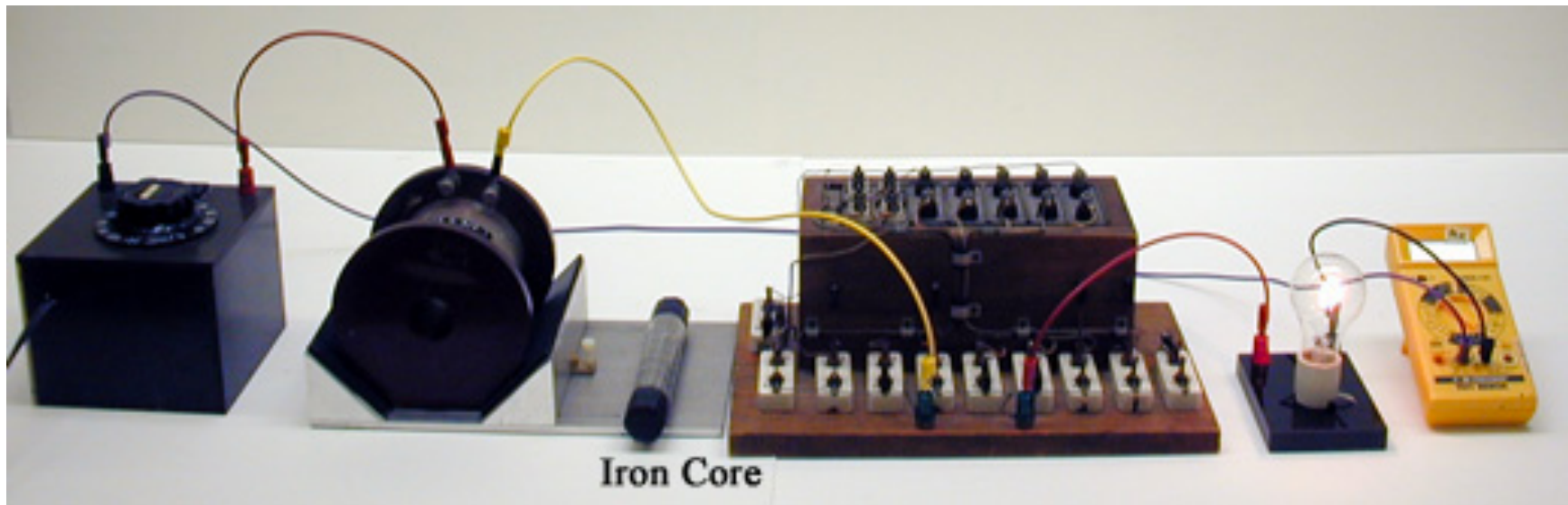
$$Z = R \quad \omega = \frac{1}{\sqrt{LC}}$$
$$X_C = X_L \quad \text{Phase} = \phi = 0$$



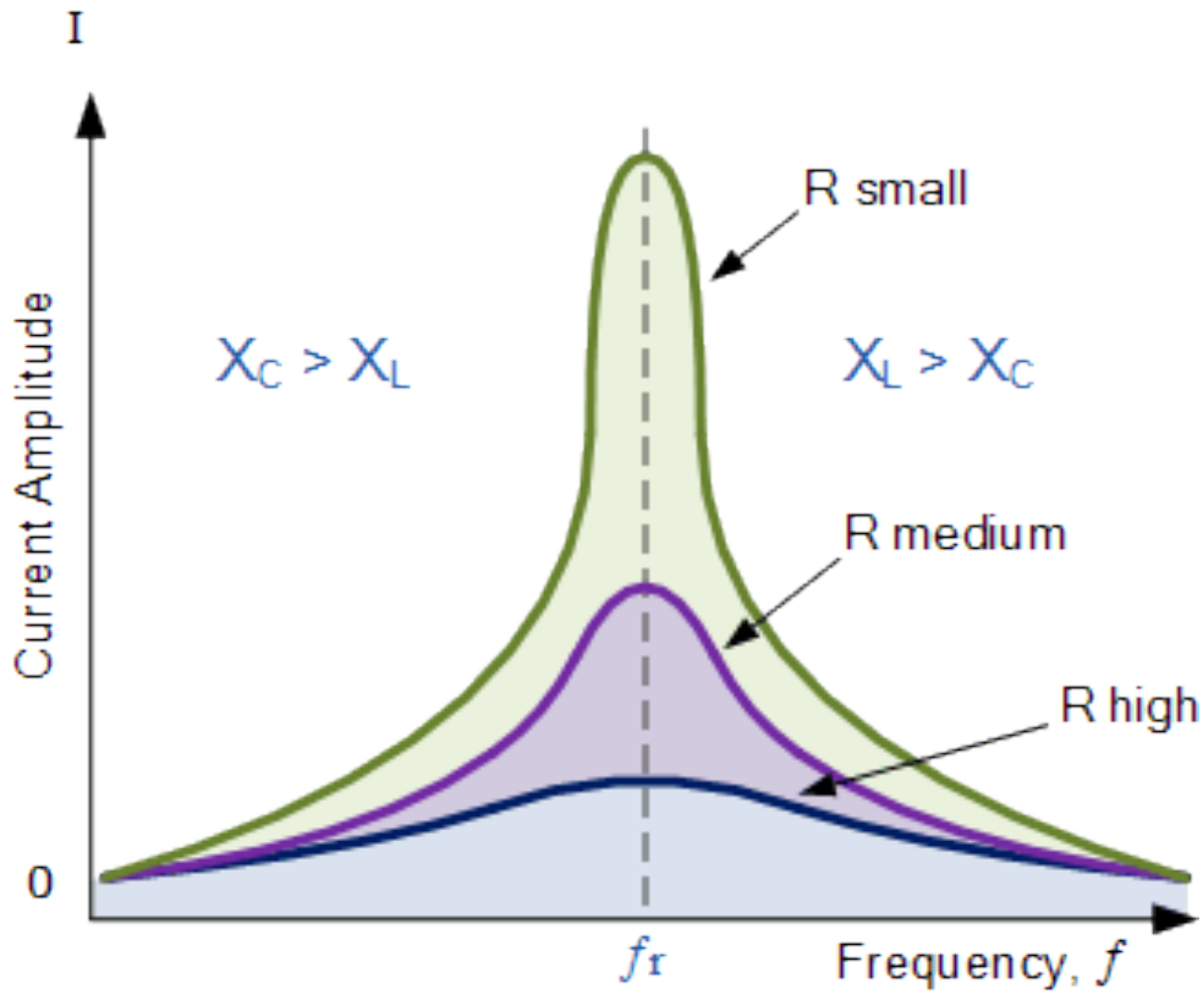
Concept Check

- Which changes could be made to increase the amplitude of the current in a driven RLC circuit where the voltage leads the current?
- Decreasing the resistance
- Decreasing the inductance
- Decreasing the capacitance
- Decreasing the driving frequency
- All of the above

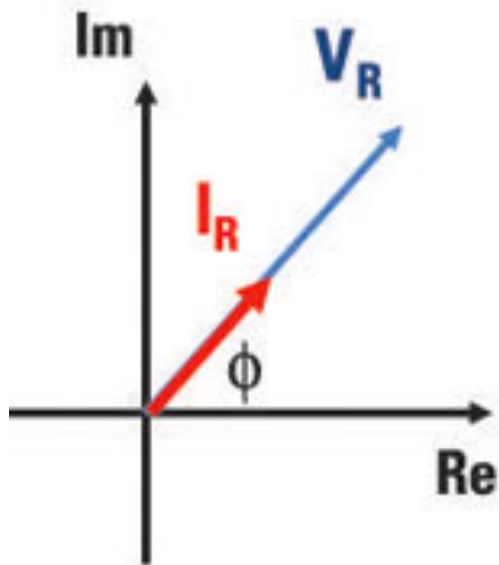
RLC Resonance Demo



RLC Circuit Resonance

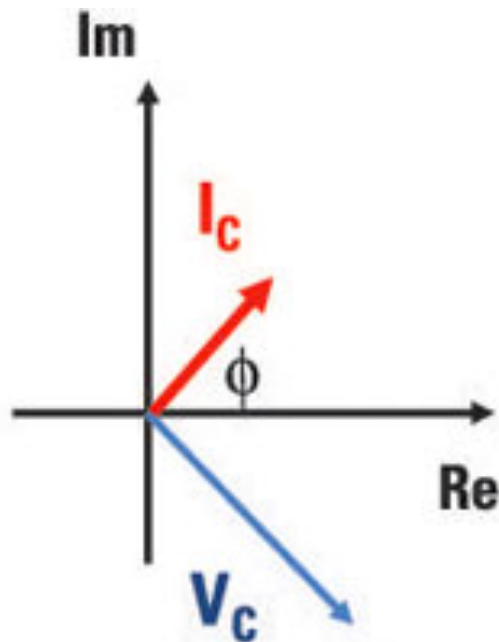


Phasor Representation



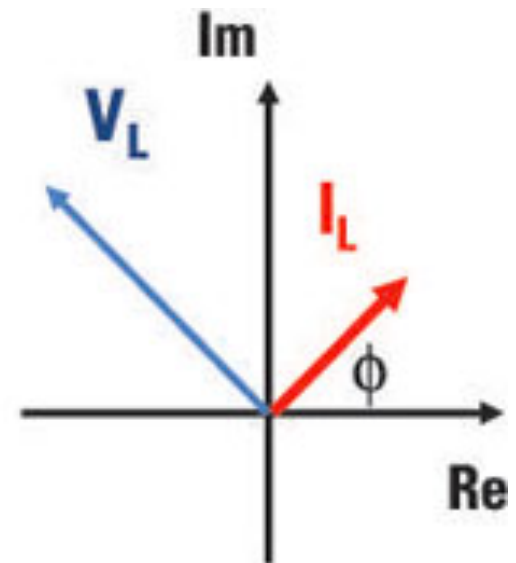
Resistor

Voltage in phase
with current



Capacitor

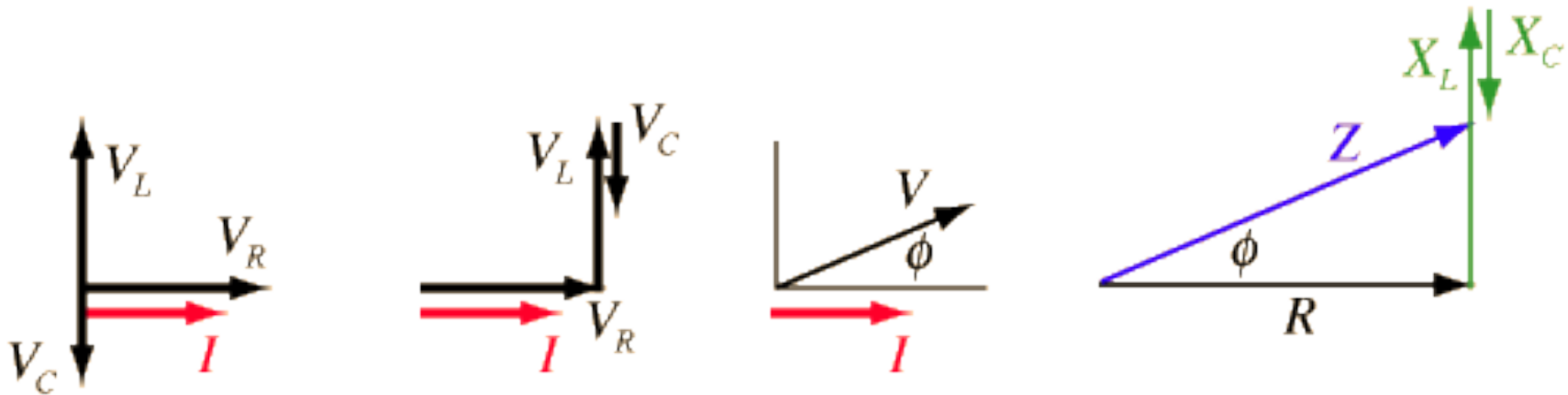
Voltage lags
current by 90°



Inductor

Voltage leads
current by 90°

Phasor Representation



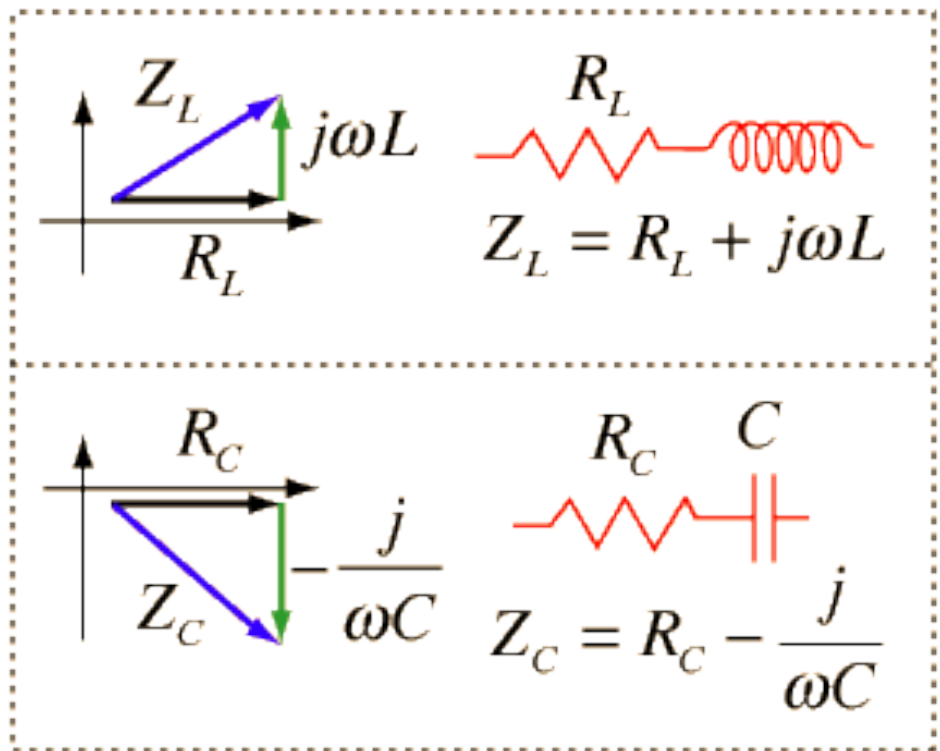
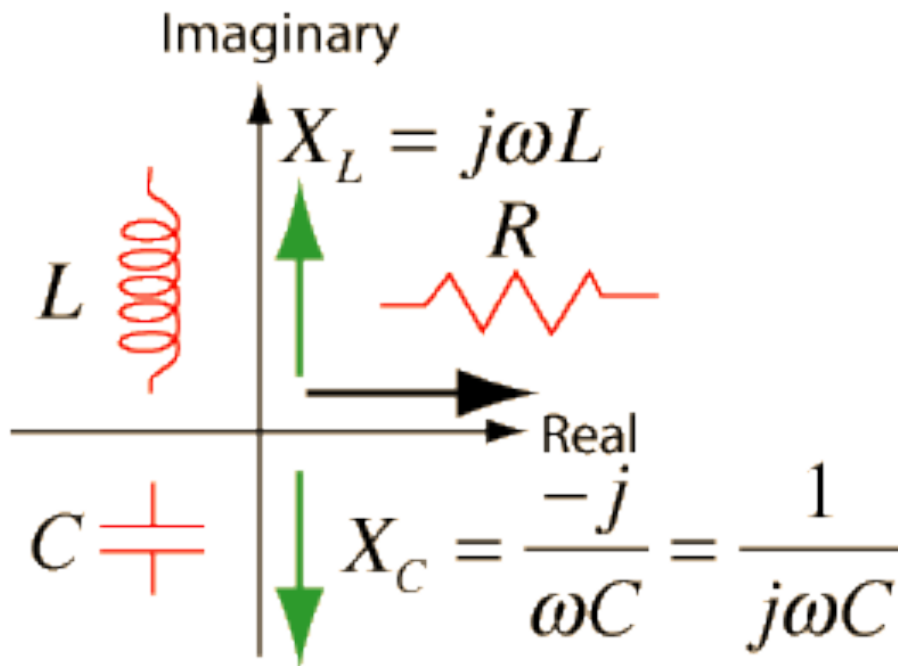
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1} \frac{V_L - V_C}{V_R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \frac{X_L - X_C}{R}$$

Complex Impedance



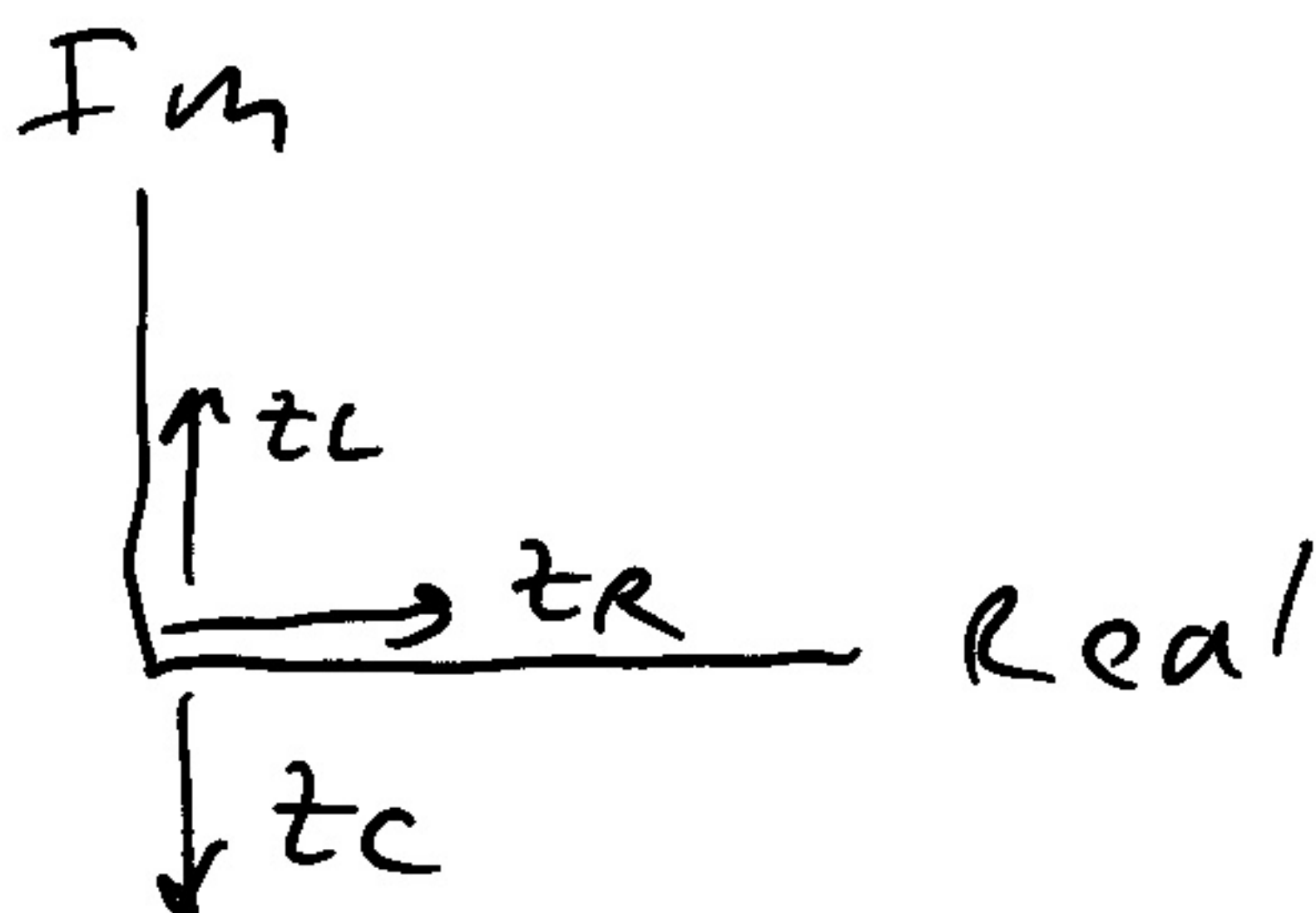
Complex Impedance

$$z_R = R$$

$$z_L = j\omega L \quad (j = \sqrt{-1})$$

$$z_C = -j/\omega C$$

— Just like phasor representation, but in complex plane



— If $\varepsilon = \varepsilon_m e^{j\omega t}$
— Look for $i = I e^{j(\omega t - \phi)}$

$$\begin{aligned}\varepsilon &= i z \\ &= i (z_R + z_L + z_C) \\ &= I e^{j(\omega t - \phi)} [R + j\omega L - j/\omega C] \\ &= \varepsilon_m e^{j\omega t} \\ \Rightarrow \varepsilon_m &= I e^{-j\phi} [R + j\omega L - j/\omega C]\end{aligned}$$

$$\text{or } \underline{z}_m / I = e^{-j\varphi} [R + j\omega L - j/\omega C]$$

$$= |Z| \quad (\text{real by definition})$$

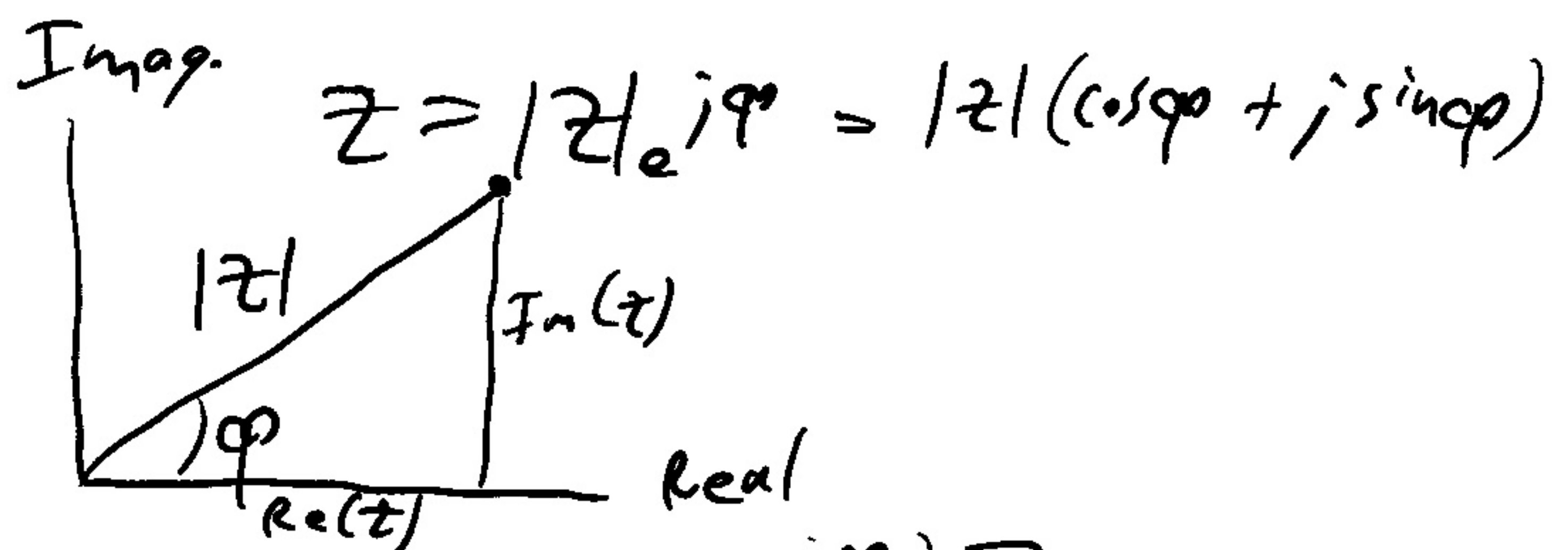
$$e^{-j\varphi} [R + j\omega L - j/\omega C] = |Z|$$

$$\Rightarrow R + j\omega L - j/\omega C = |Z| e^{j\varphi} = Z$$

impedance
magnitude

phase shift

In complex plane:



$$\varphi = \tan^{-1} \left[\frac{\text{Im}(e^{j\varphi})}{\text{Re}(e^{j\varphi})} \right]$$

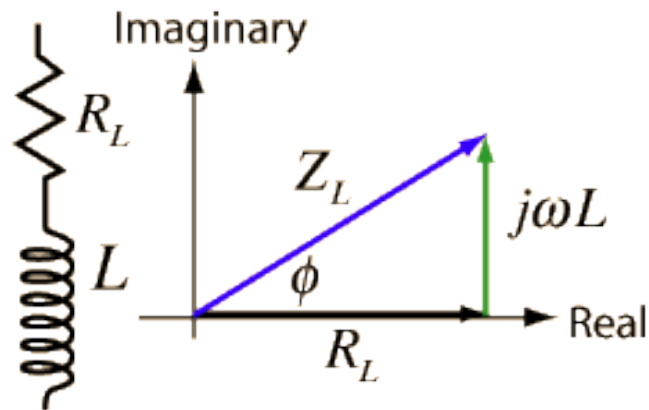
$$= \tan^{-1} \left[\frac{\omega L - 1/\omega C}{R} \right]$$

$$|Z| = \sqrt{\text{Re}(z)^2 + \text{Im}(z)^2} = \sqrt{R^2 + [\omega L - 1/\omega C]^2}$$

Same answer,
easier math!!



Complex Impedance

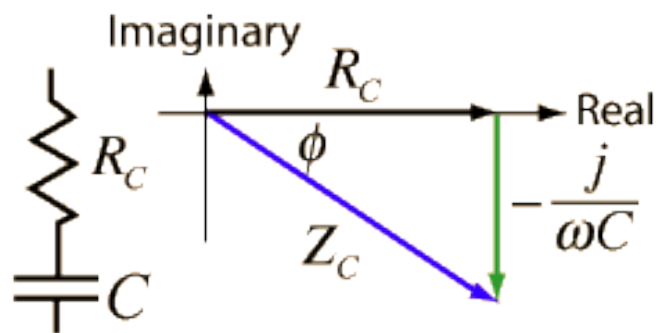


Cartesian form: $Z_L = R_L + j\omega L$

Polar form: $Z_L = |Z_L| e^{j\phi}$

where $|Z_L| = \sqrt{R_L^2 + \omega^2 L^2}$

$$\phi = \tan^{-1} \frac{\omega L}{R_L}$$



Cartesian form: $Z_C = R_C - \frac{j}{\omega C}$

Polar form: $Z_C = |Z_C| e^{j\phi}$

where $|Z_C| = \sqrt{R_C^2 + \frac{-1}{\omega C R_C}}$

$$\phi = \tan^{-1} \frac{-1}{\omega C R_C}$$