Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Maxwell's Equations Are Complete!

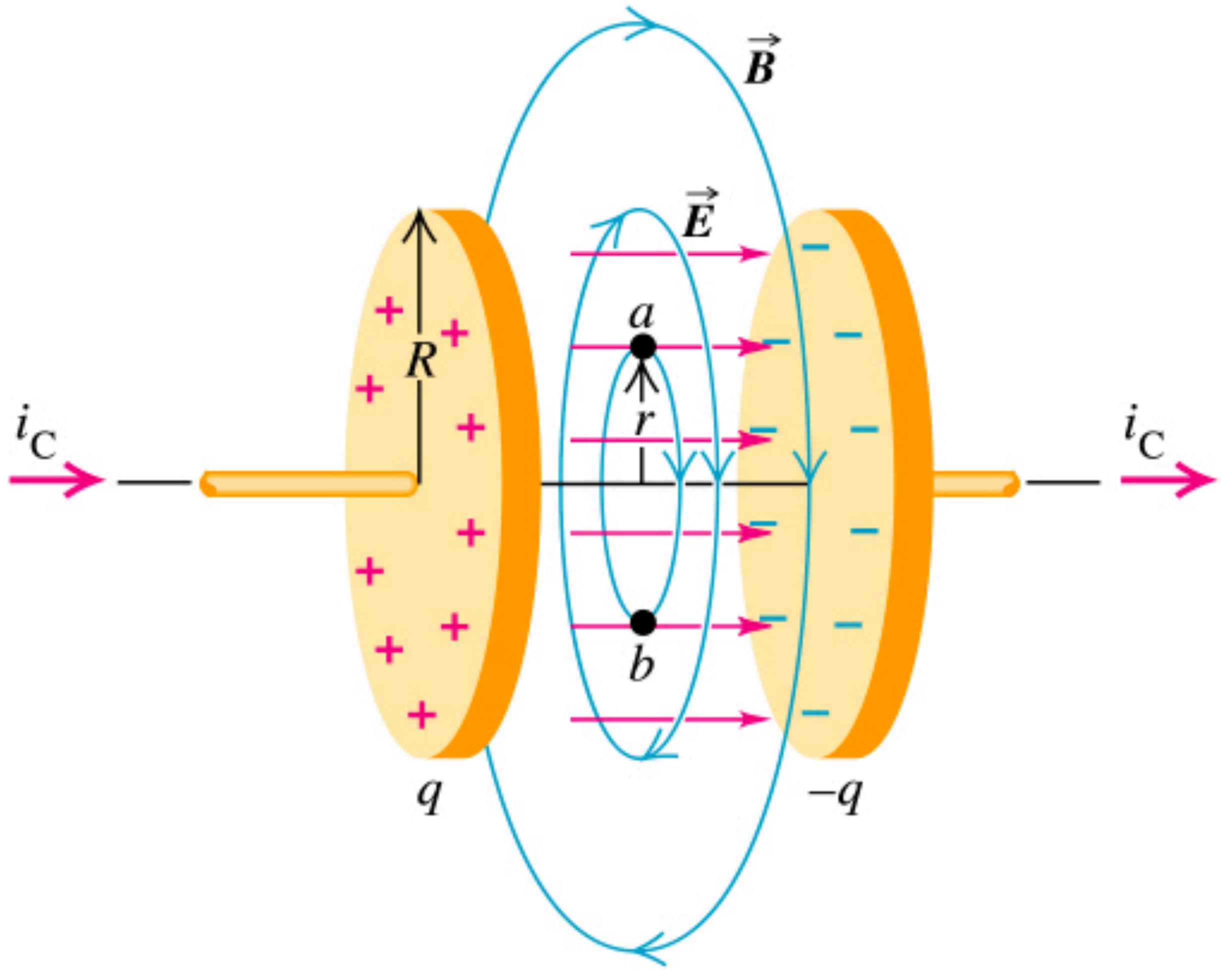
$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0} \quad ✔$$

$$\oint \mathbf{B}\cdot d\mathbf{A} = 0 \quad ✔$$

$$\oint \mathbf{E}\cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \quad ✔$$

$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc} \quad ✔$$

Finite Capacitor



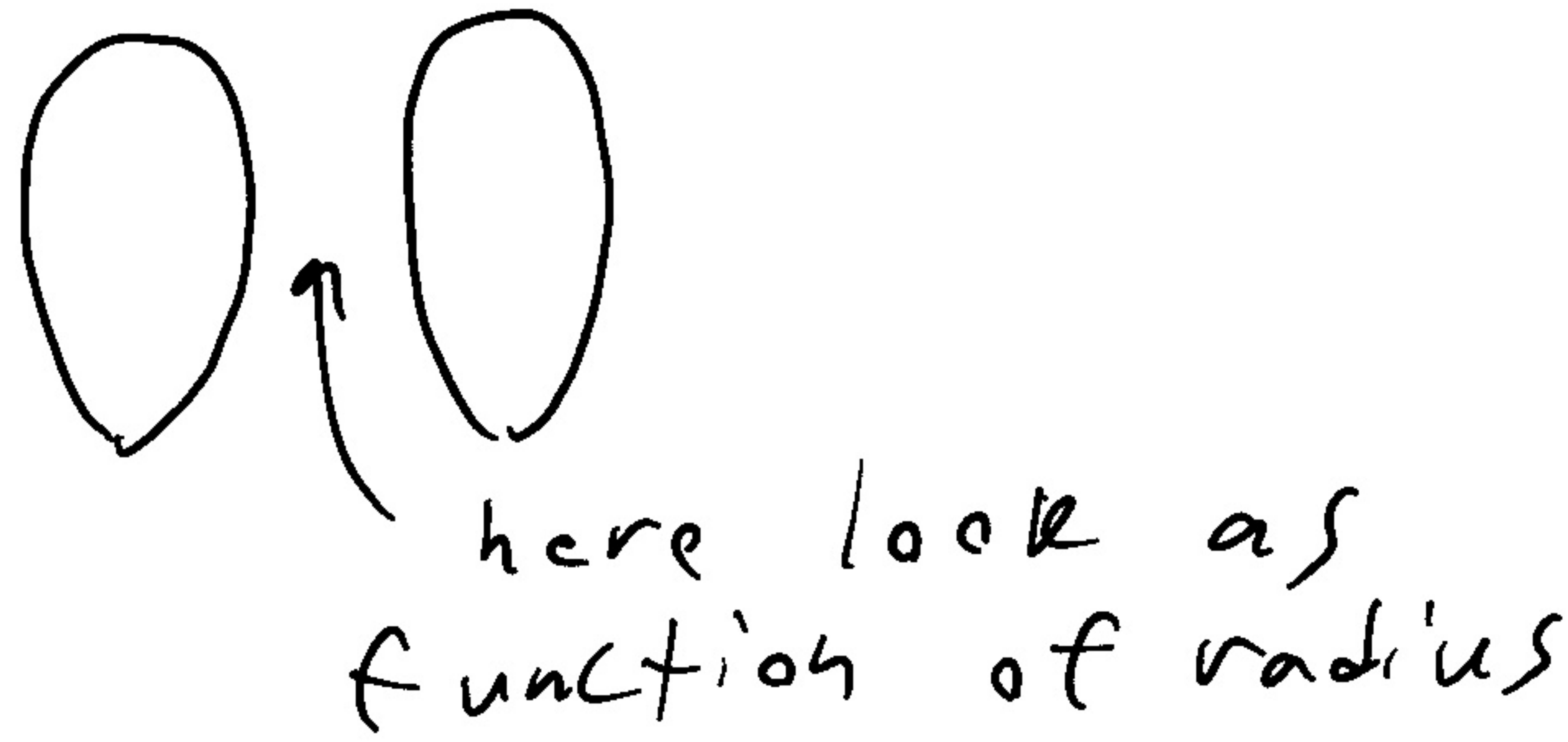
- Magnetic Field around charging capacitor

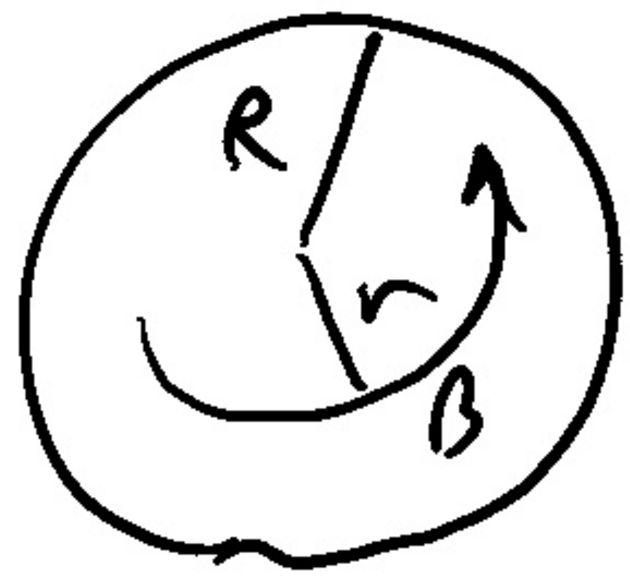
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enc} + \mu_0 i_{d\,enc}$$

- can approximate as B of wire

$$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r = \mu_0 i$$

$$\Rightarrow B = \mu_0 i / 2\pi r$$





$\odot\ i_d = \varepsilon_0 \, d\varphi_E / dt$

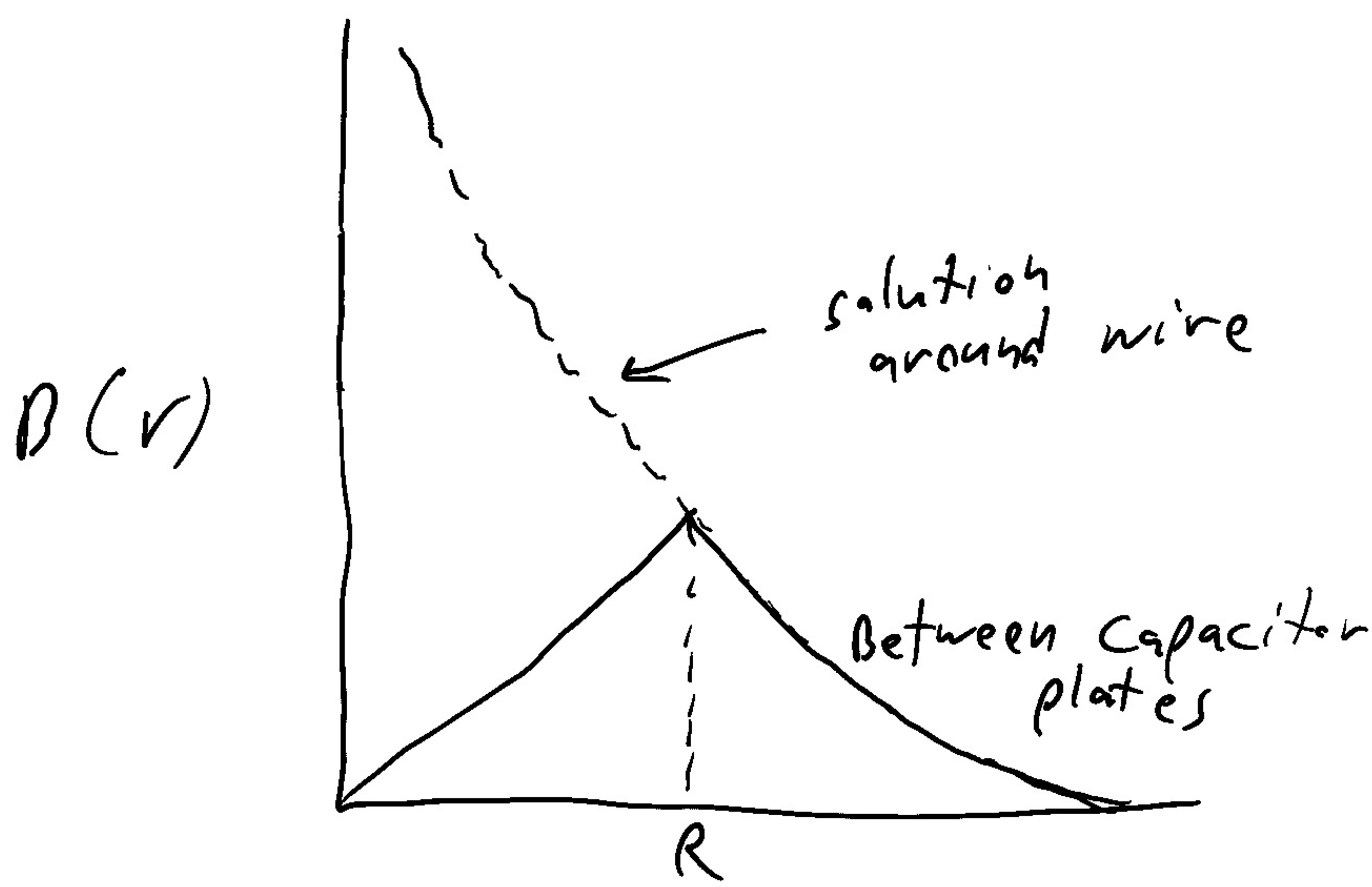
Look @ fraction of i_d within r

$$i_{d\,enc} = i_d \cdot \pi r^2 / \pi R^2$$
$$= i \cdot r^2 / R^2$$

$$\oint \vec{B} \cdot d\vec{\ell} = B \cdot 2\pi r = \mu_0 i \, r^2 / R^2$$

$$\Rightarrow B = \frac{\mu_0 i r}{2\pi R^2} \quad r < R$$

$$B = \mu_0 i / 2\pi r \quad r > R$$



What We Knew Way Back When

Speed of Light Finite
Ole Romer: 1676

Speed of Light = $3x10^8$ m/s
Jean Delambre: 1809

What We Knew Way Back When

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 \iint_S \mathbf{J} \cdot \mathrm{d}\mathbf{S} = \mu_0 I_{\mathrm{enc}}$$

Ampere's Law: 1826

Library of Congress

What We Knew Way Back When

$$\oint_{\partial\Sigma} \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \int_{\Sigma} \mathbf{B} \cdot d\mathbf{A}.$$

Faraday’s Law: 1832

What We Knew Way Back When

$$\oiint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \Phi_E = \frac{Q}{\varepsilon_0}$$

Gauss's Law: 1835

Full Version of Ampere's Equation

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \iint_S \left(\mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \mathbf{E} \right) \cdot \mathrm{d}\mathbf{S}$$

Maxwell's Equations:
1861

Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

EM Waves: Qualitative

- What happens if I wiggle the magnetic field?
- By Faraday's Law, an electric field is induced, proportional to the derivative of magnetic field
- By Ampere's law, this (changing) electric field induces a magnetic field, proportional to the derivative of the electric field
- We have discovered perpetual motion!

EM Waves: Dimensionally

- $E*L = -B*L^2/t$
- $B*L = \mu_o\varepsilon_o E*L^2/t$
- L/t = characteristic velocity c
- $E/B = c = 1/\sqrt{\mu_o\varepsilon_o}$ *a.k.a. the speed of light*

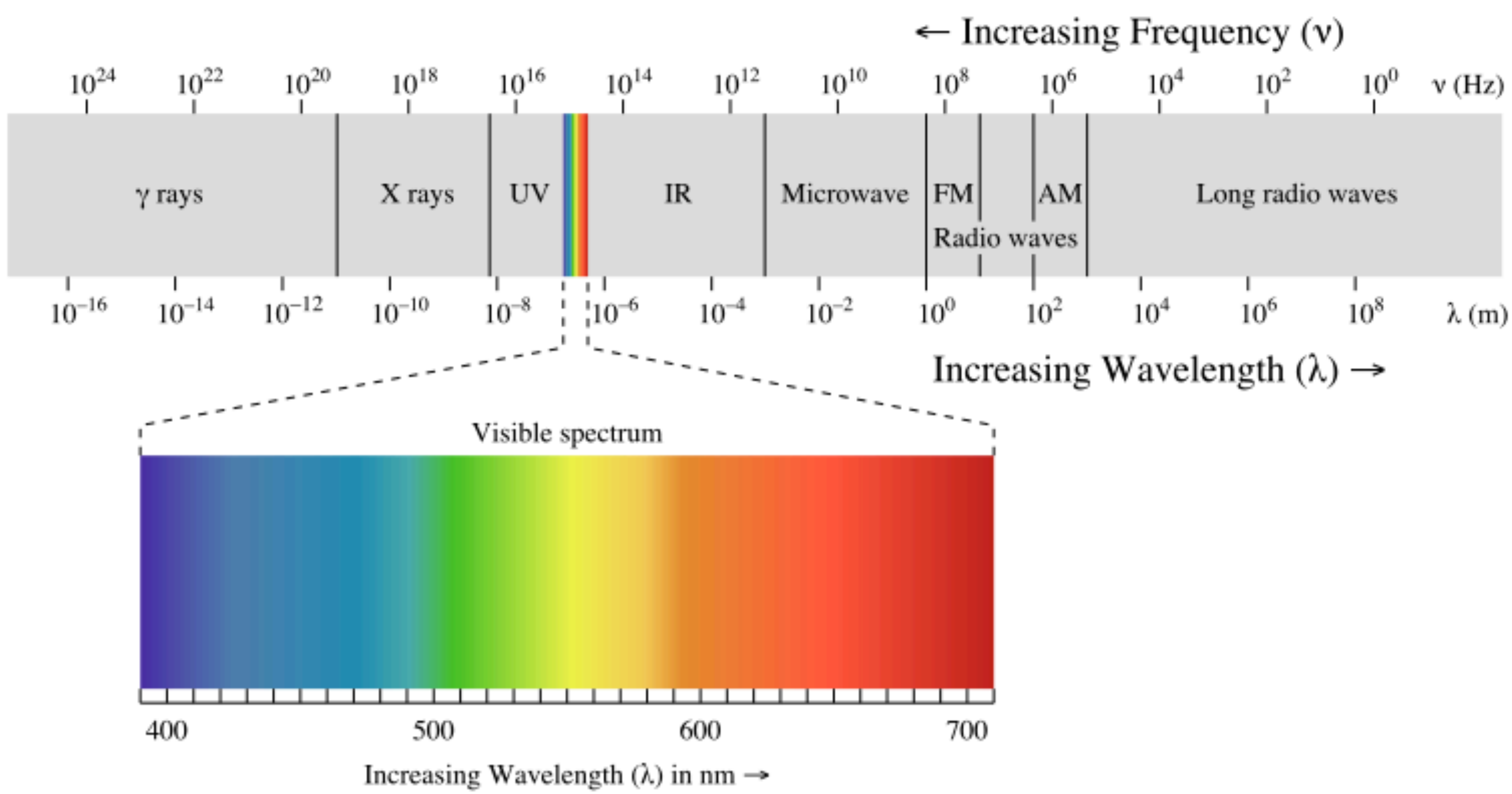
Whoa!

- $c = 1/\sqrt{(\mu_o \varepsilon_o)}$

- Light is an electromagnetic wave!!!

- Maxwell (a really smart dude) figured this out in 1862 (a year after figuring out all the equations)

EM Spectrum

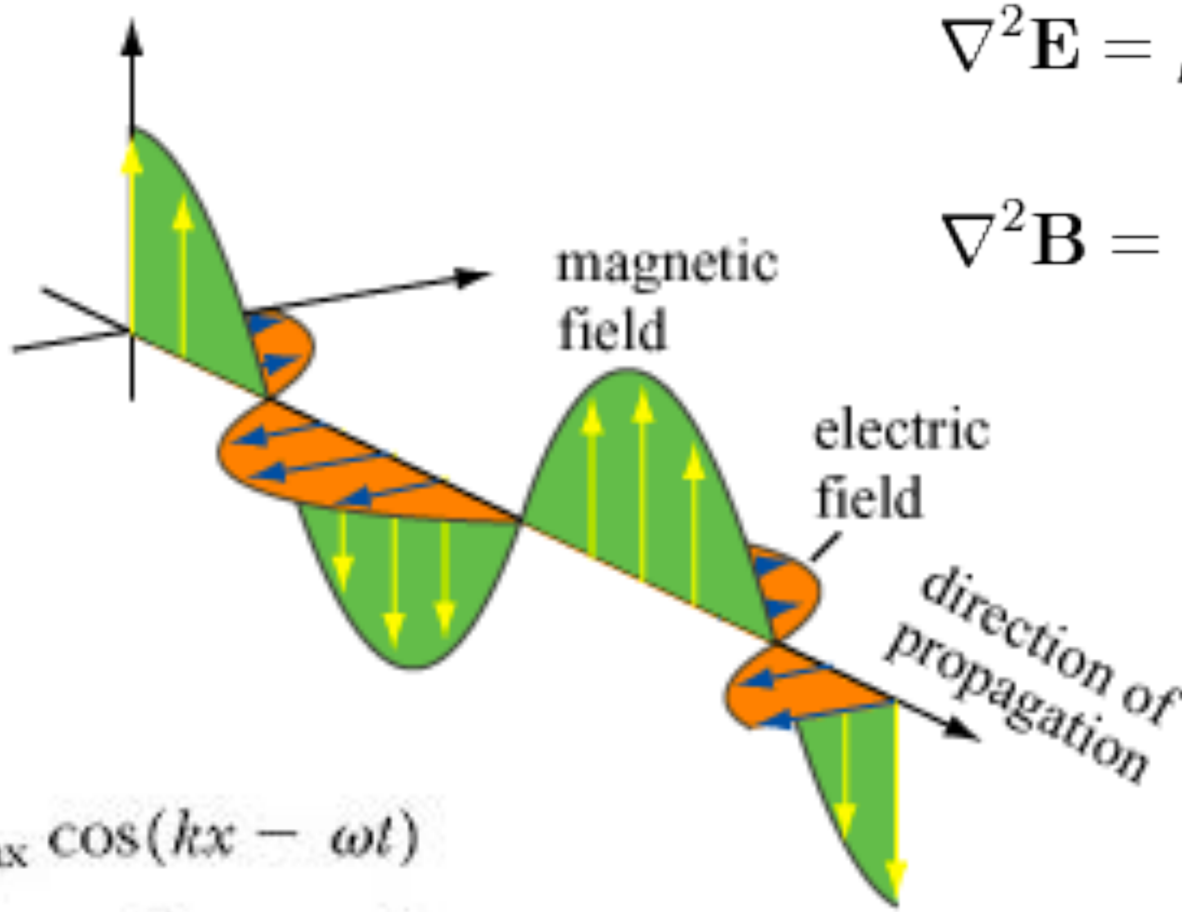


← Increasing Frequency (ν)
10^{24} 10^{22} 10^{20} 10^{18} 10^{16} 10^{14} 10^{12} 10^{10} 10^{8} 10^{6} 10^{4} 10^{2} 10^{0} ν (Hz)
γ rays
X rays
UV
IR
Microwave
FM
AM
Radio waves
Long radio waves
10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4} 10^{6} 10^{8} λ (m)
Increasing Wavelength (λ) →
Visible spectrum
400
500
600
700
Increasing Wavelength (λ) in nm →

Electromagnetic Wave

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

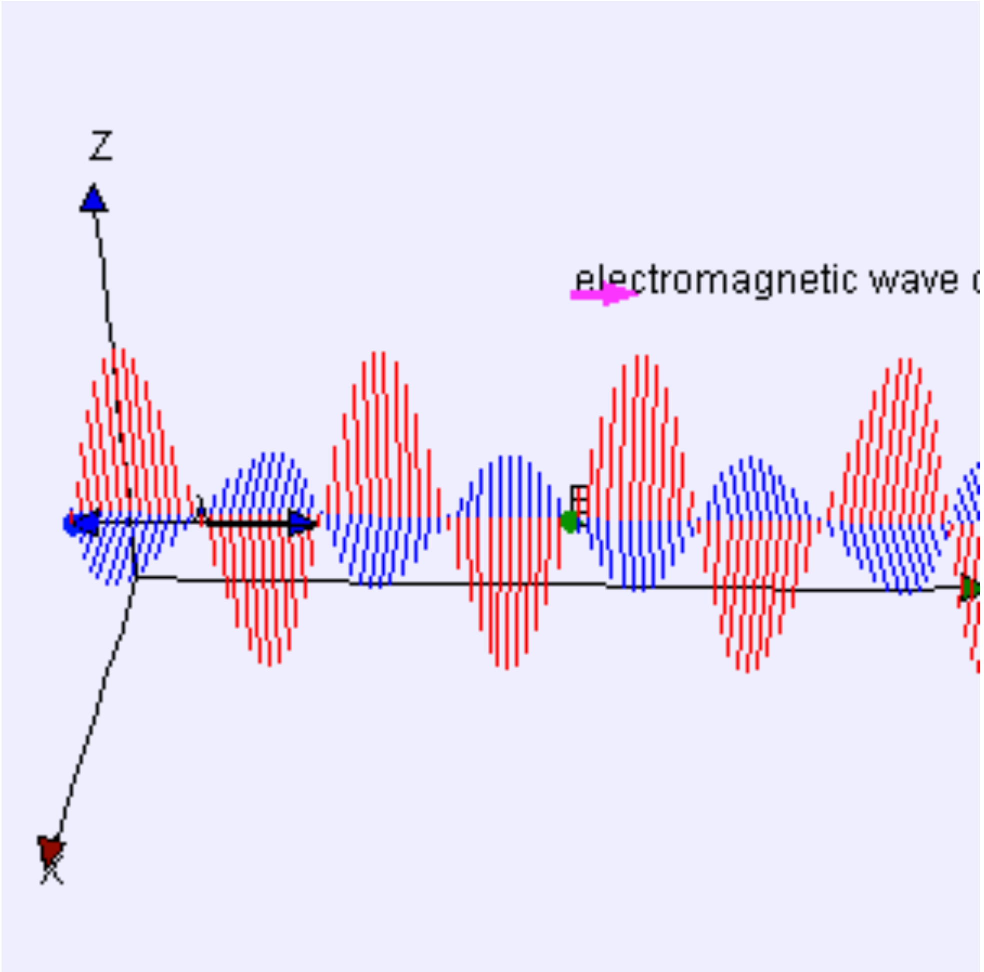
$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}.$$



$$E = E_{\max} \cos(kx - \omega t)$$

$$B = B_{\max} \cos(kx - \omega t)$$

What Travels in a Light Wave?



EM Waves

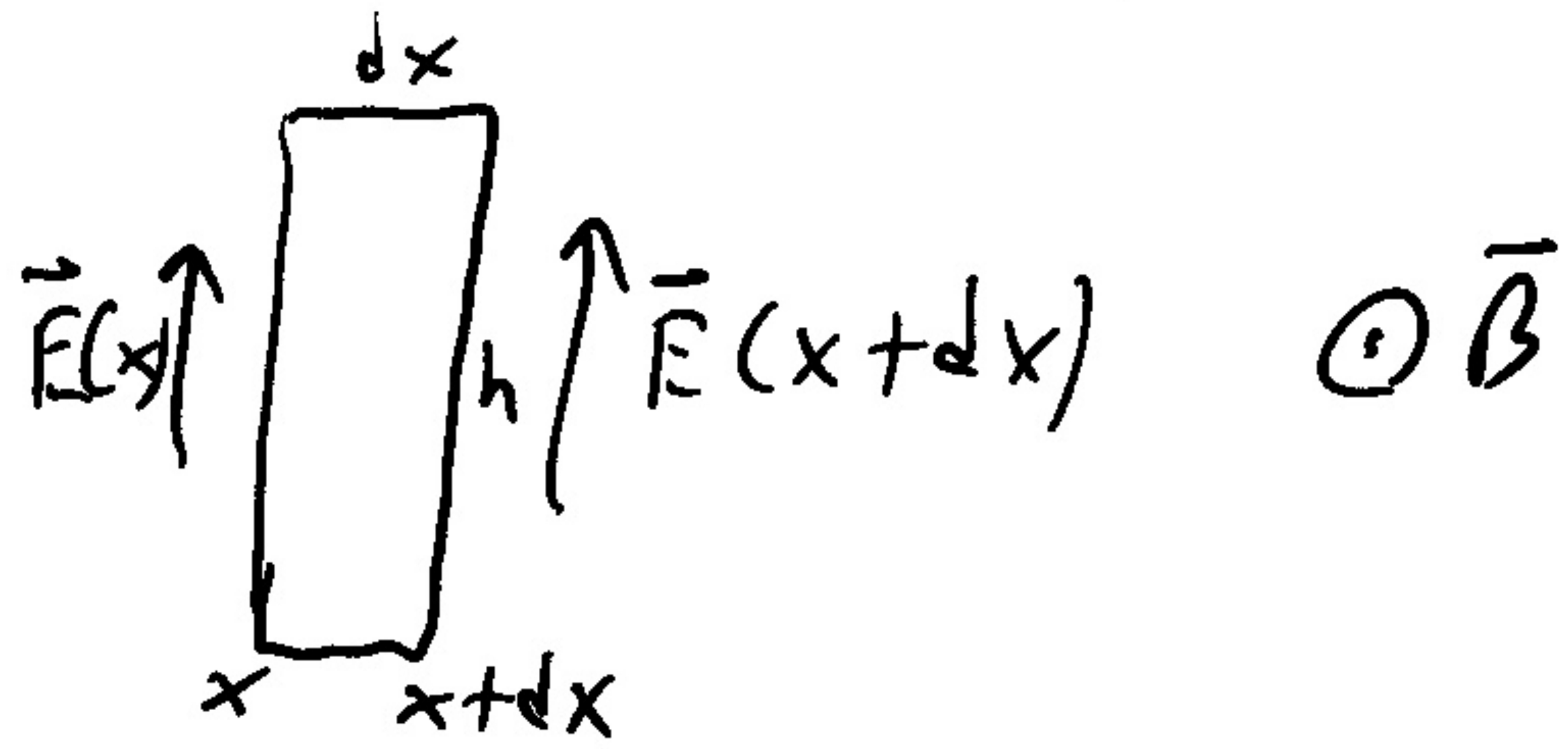
Write $\vec{E} = E(x,t)\,\hat{j}$

$\vec{B} = B(x,t)\,\hat{k}$

Faraday's Law

$$\oint \vec{E}\cdot d\vec{l} = -d\varphi_B/dt$$

Look @ loop in x-y plane



$$\oint \vec{E}\cdot d\vec{l} = h\,E(x+dx) - h\,E(x)$$
$$= -d/dt\,[h\,dx\,B]$$

$$\Rightarrow \frac{E(x+dx) - E(x)}{dx} = -d/dt\,B$$

$$\Rightarrow \partial E/\partial x = -\partial B/\partial t$$