Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_{\mathrm{B}}}{d\mathrm{t}}$$

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 \varepsilon_0 \frac{d\Phi_{\mathrm{E}}}{d\mathrm{t}} + \mu_0 i_{enc}$$

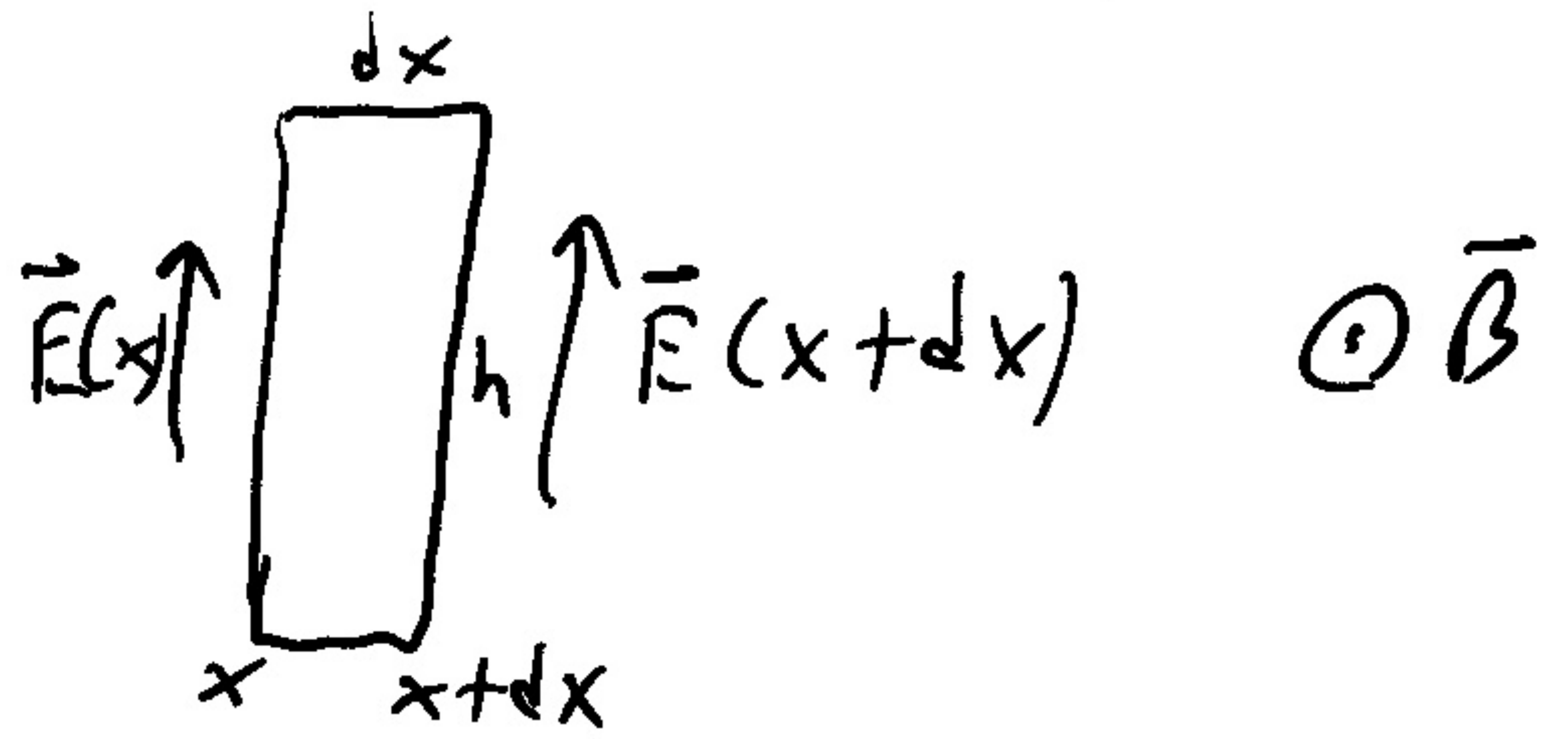
EM Waves

Write $\vec{E} = E(x,t)\,\hat{j}$
$\vec{B} = B(x,t)\,\hat{k}$

Faraday's Law

$$\oint \vec{E}\cdot d\vec{l} = -d\phi_B/dt$$

Look @ loop in x-y plane



$$\oint \vec{E}\cdot d\vec{l} = h\,E(x+dx) - h\,E(x)$$
$$= -\frac{d}{dt}\left[h\,dx\,B\right]$$

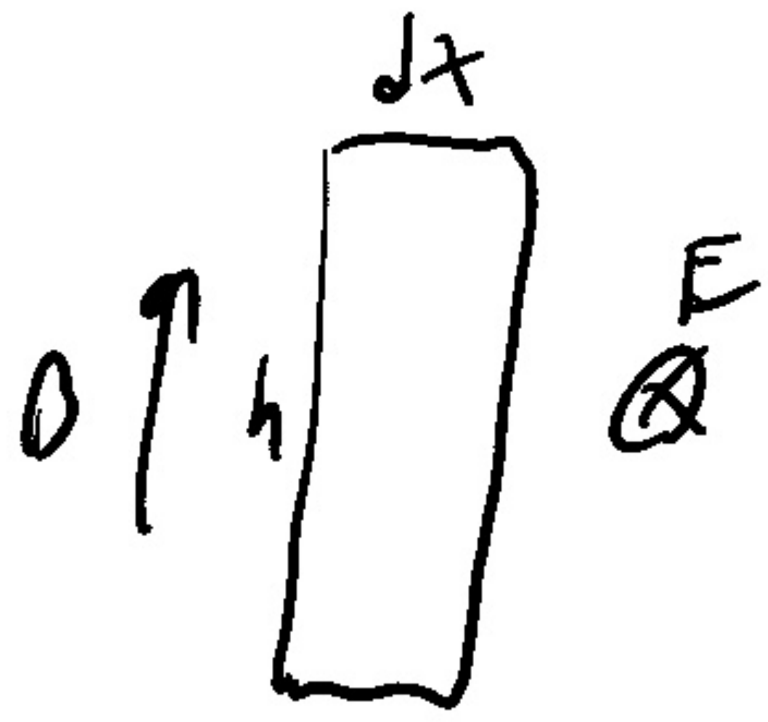
$$\Rightarrow \frac{E(x+dx)-E(x)}{dx} = -\frac{d}{dt}B$$

$$\Rightarrow \partial E/\partial x = -\partial B/\partial t$$

- Ampere's Law (no current)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \, d\Phi_E / dt$$

- Take loop in x-z plane



$$h B(x+dx) - h B(x) = \mu_0 \varepsilon_0 \, d/dt \, [-E h \, dx]$$

$$\Rightarrow \partial B / \partial x = -\mu_0 \varepsilon_0 \, \partial E / \partial t$$

$$\partial E / \partial x = -\partial B / \partial t$$

$$\partial B / \partial x = -\mu_0 \varepsilon_0 \, \partial E / \partial t$$

$$\Rightarrow \partial^2 E / \partial x^2 = -\partial^2 B / \partial x \partial t$$

$$\partial^2 B / \partial x \partial t = -\mu_0 \varepsilon_0 \, \partial^2 E / \partial t^2$$

$$\Rightarrow \partial^2 E / \partial x^2 = \mu_0 \varepsilon_0 \, \partial^2 E / \partial t^2 = 1/c^2 \, \partial^2 E / \partial t^2$$

similarly $\partial^2 B / \partial x^2 = 1/c^2 \, \partial^2 B / \partial t^2$

Solutions:

$$E = E_m \sin(kx - \omega t - \varphi_0)$$
$$B = B_m \sin(kx - \omega t - \varphi_0)$$

$k = 2\pi/\lambda$ = wave number

λ = wavelength

$\omega = 2\pi f$ = angular frequency

$\omega/k = c$

– Check $\partial^2 E/\partial x^2 = -k^2 E$

$\partial^2 E/\partial t^2 = -\omega^2 E$

so $-k^2 E = 1/c^2 \cdot -\omega^2 E$

if $\omega/k = c$

– $\partial E/\partial x = k E_m \cos(kx - \omega t - \varphi)$

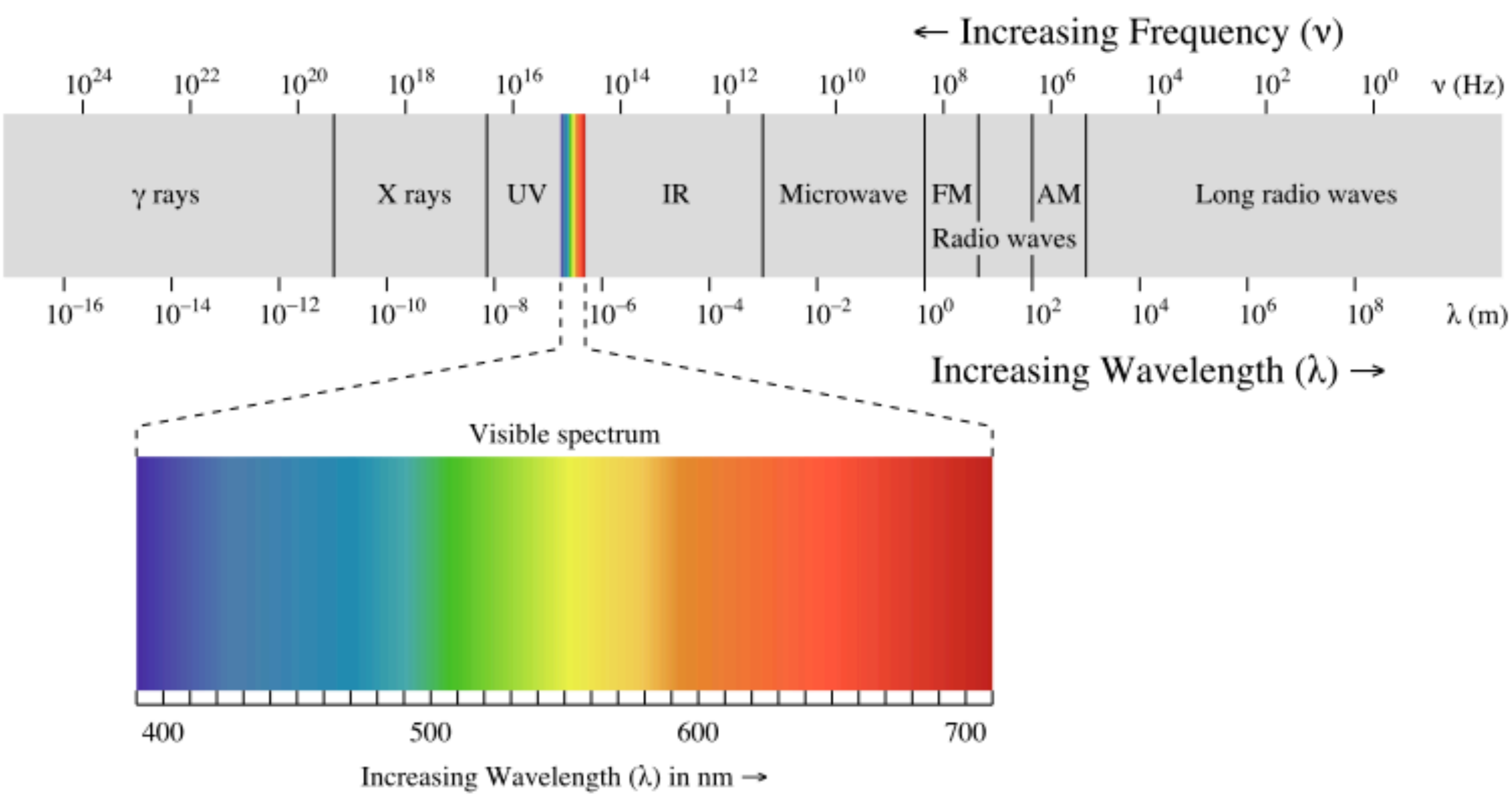
$= -\partial B/\partial t$

$= \omega B_m \cos(kx - \omega t - \varphi)$

$\Rightarrow k E_m = \omega B_m$

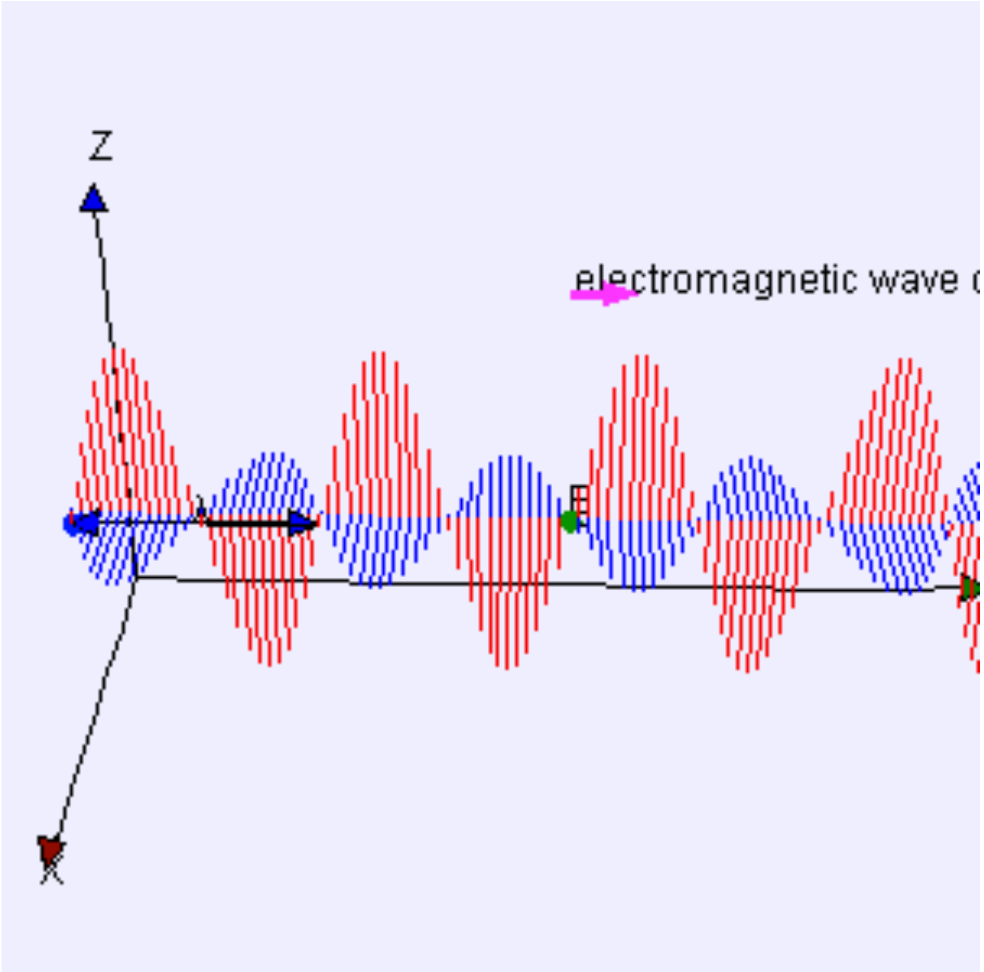
or $E_m/B_m = \omega/k = c$

EM Spectrum



← Increasing Frequency (ν)
10^24
10^22
10^20
10^18
10^16
10^14
10^12
10^10
10^8
10^6
10^4
10^2
10^0
ν (Hz)
γ rays
X rays
UV
IR
Microwave
FM
AM
Radio waves
Long radio waves
10^−16
10^−14
10^−12
10^−10
10^−8
10^−6
10^−4
10^−2
10^0
10^2
10^4
10^6
10^8
λ (m)
Increasing Wavelength (λ) →
Visible spectrum
400
500
600
700
Increasing Wavelength (λ) in nm →

What Travels in a Light Wave?



Wavelength and Frequency

- Frequency: $f = 2\pi\omega$
- Wavelength: $\lambda = 2\pi/k$

- $\omega/k = c$

- $f*\lambda = c$
  - (number of wavelengths per second = velocity!)

Imagine Traveling With the Wave

- $x = c*t$

  $= \omega/k*t$

- $\sin(kx - \omega t + \phi)$

  $= \sin(k*\omega/k*t - \omega t + \phi)$

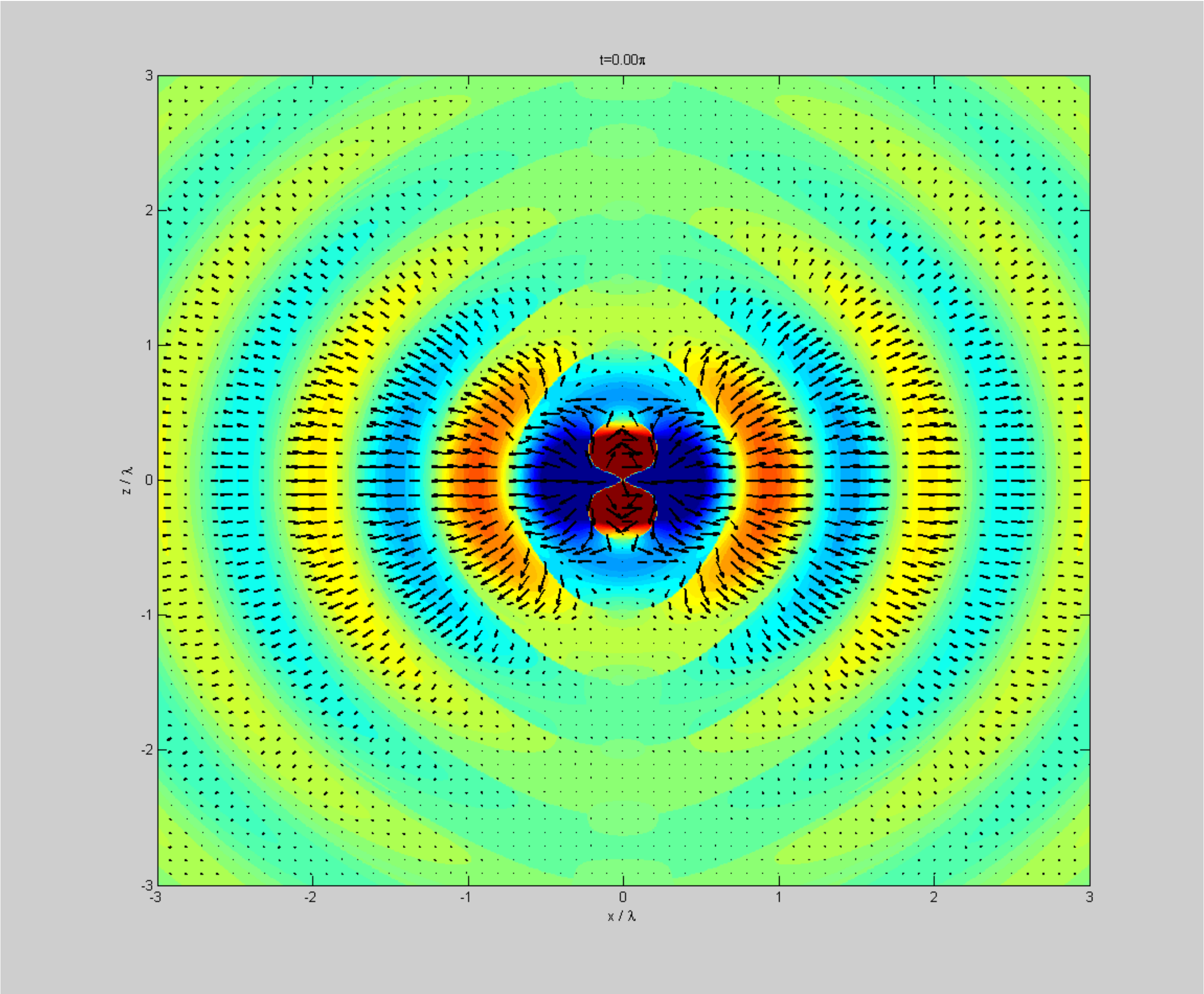
  $= \sin(\phi)$

Concept Check

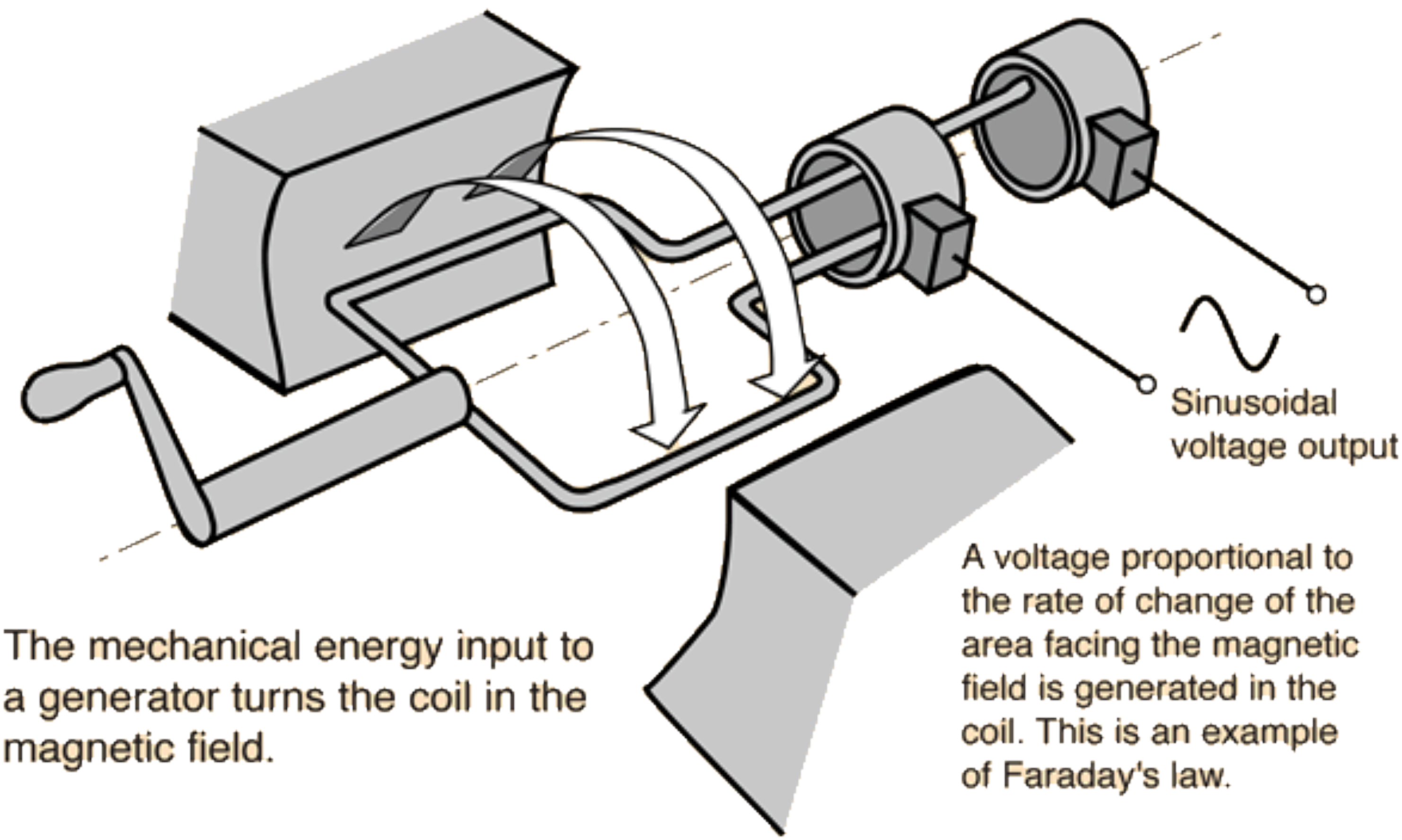
- Are charged particles required to generate or transmit an electromagnetic wave?

A. Only to generate
B. Only to transmit
C. No charged particles needed for either
D. Charged particles needed for both

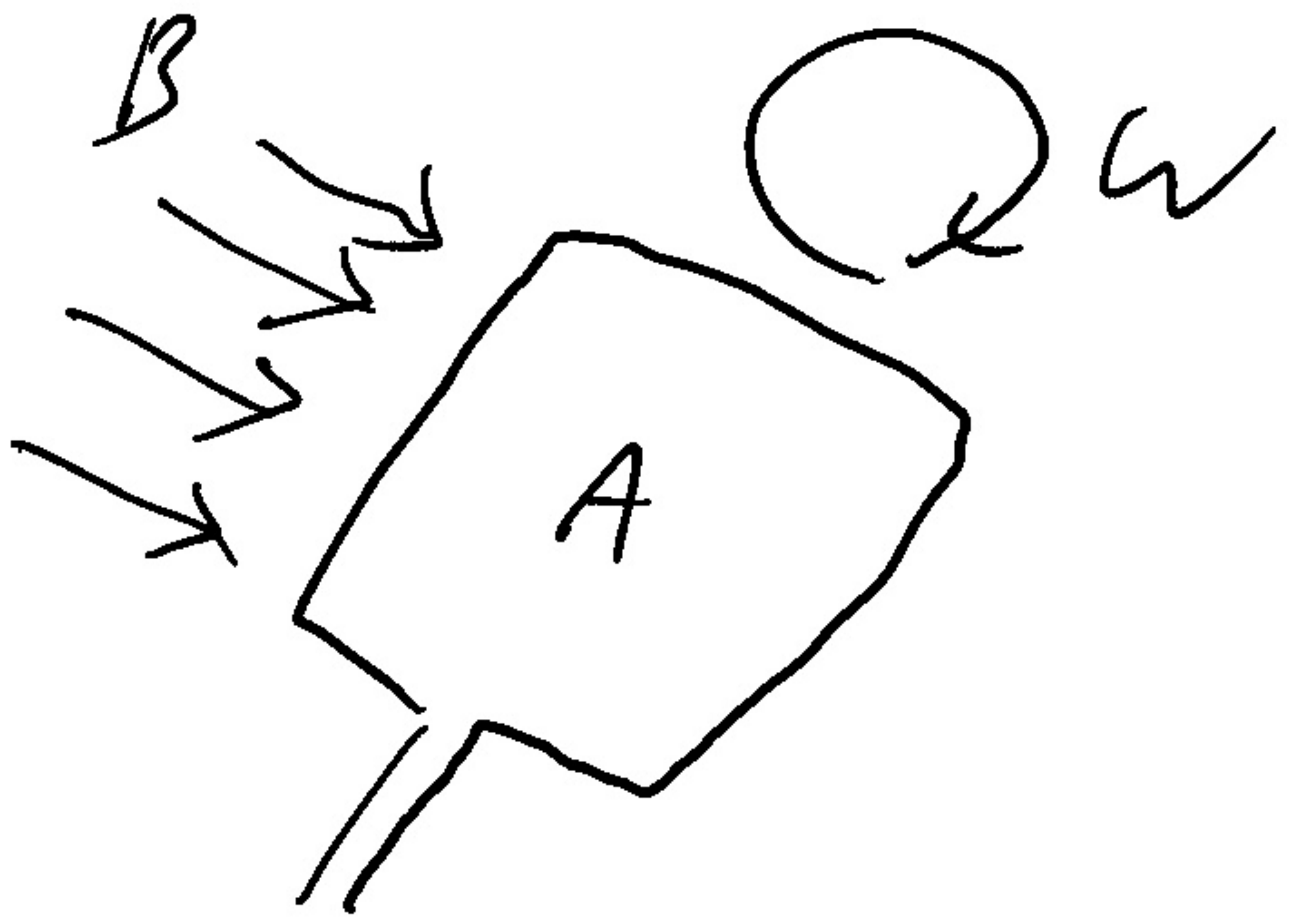
How to Generate an EM Wave



It’s All Energy: AC Power Generator



Generator



- spin using mechanical impulse at angular rate ω

$$\varepsilon = -d\phi_B/dt$$

$$= -d/dt\,(\vec{B}\cdot\vec{A})$$

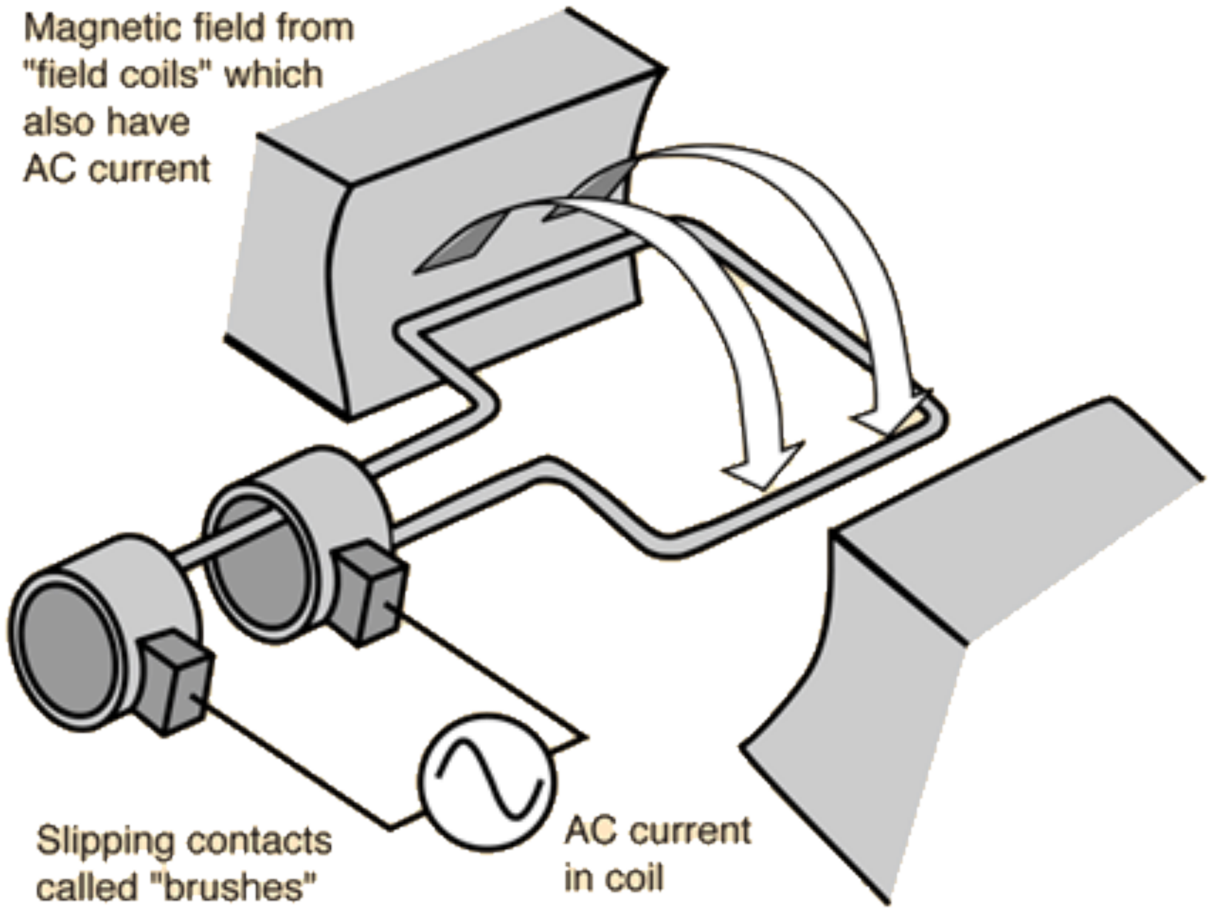
$$= -d/dt\,(BA\cos\theta)$$

$$= -d/dt\,(BA\cos(\omega t))$$

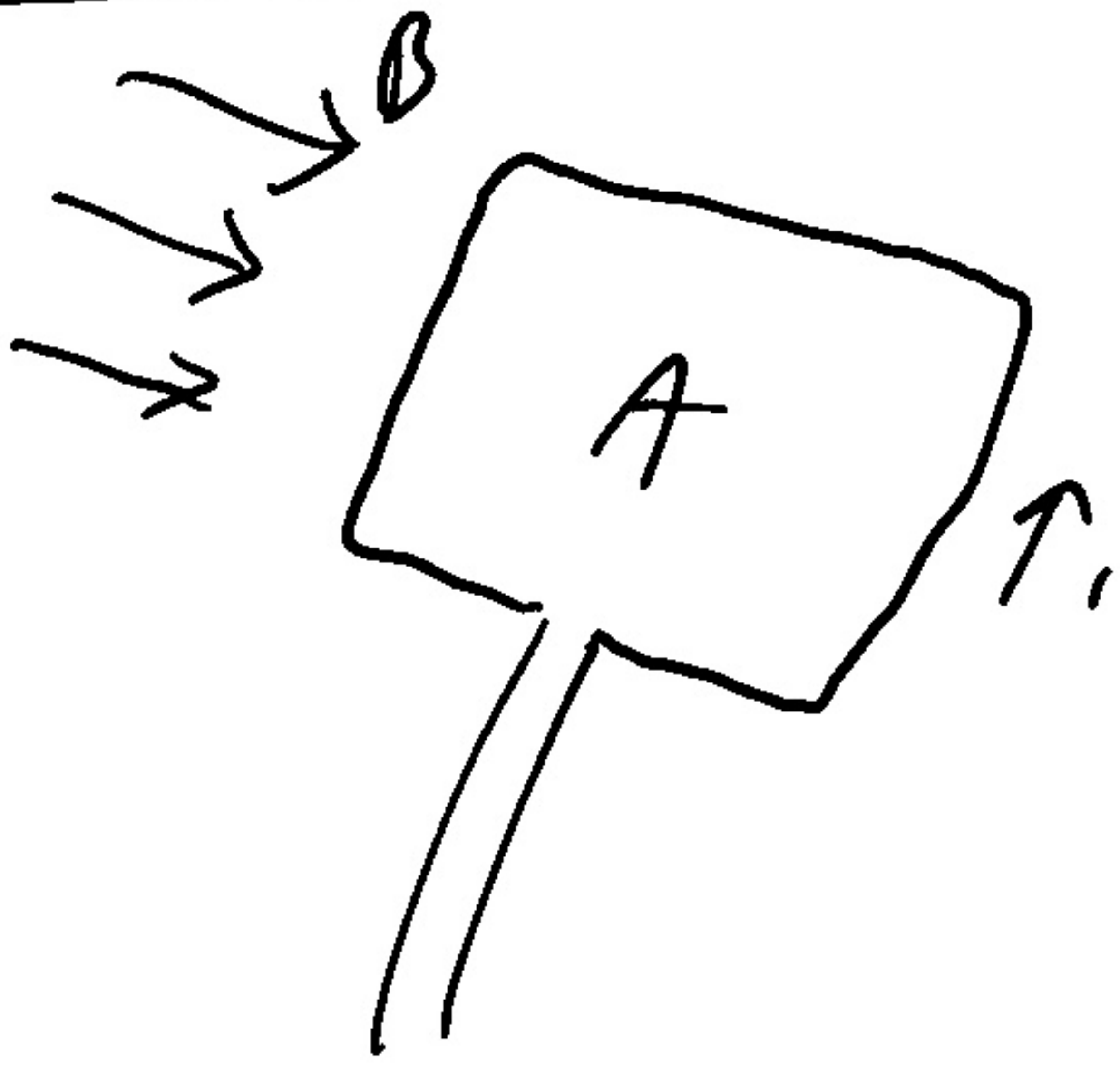
$$= BA\omega\sin(\omega t)$$

- produces oscillating emf
- maximum when $\vec{B}\perp\vec{A}$ so flux changing most rapidly

It's All Energy: AC Motor



Motor



oscillating input current

$i = I \sin \omega t$

$\mu = iA$

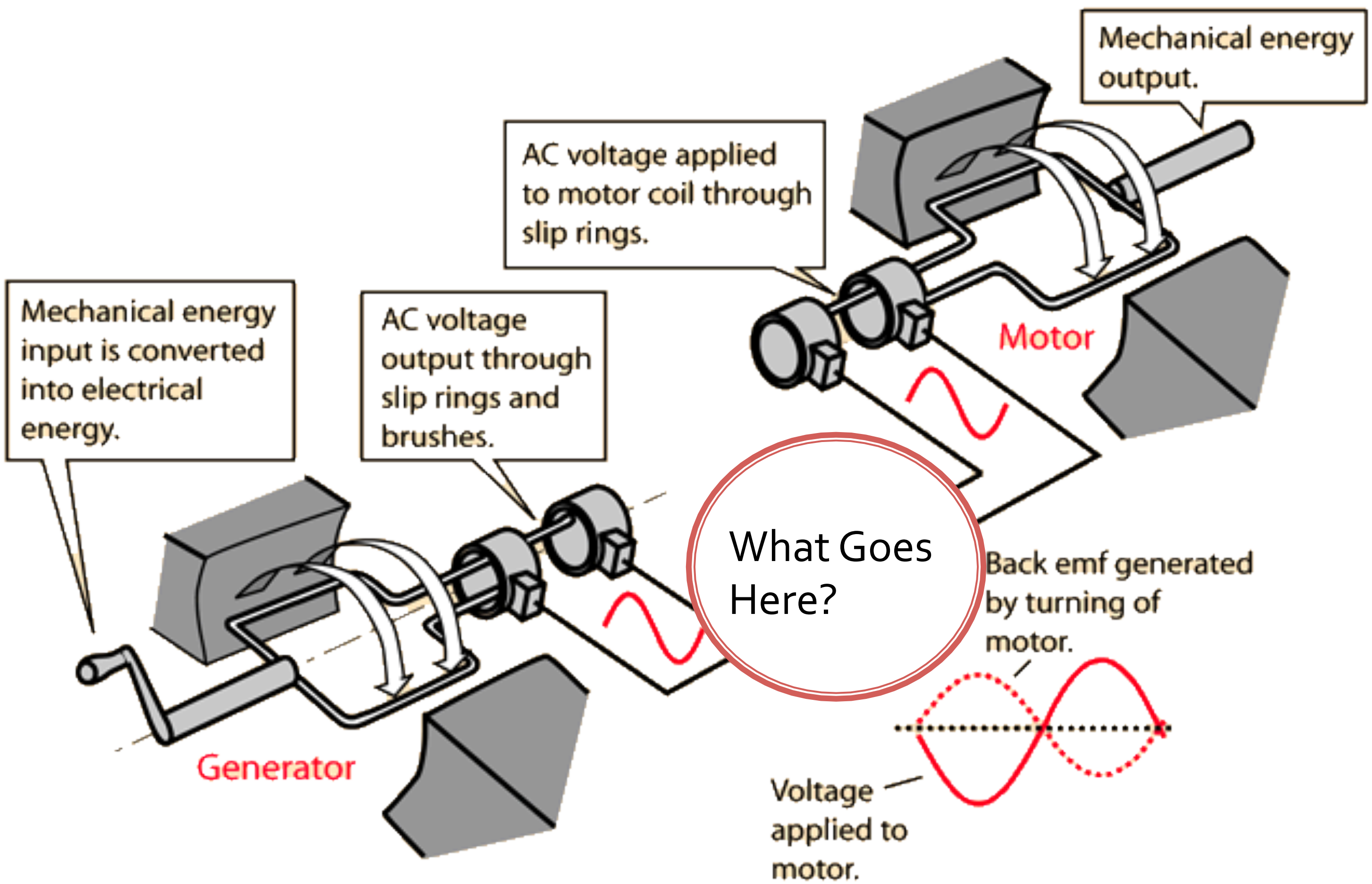
$\tau = \mu \times B$

$= iAB \sin\theta$

$= I \sin \omega t \, AB \sin\theta$

- oscillating torque
- rotates wire loop

Generator and Motor



Concept Check

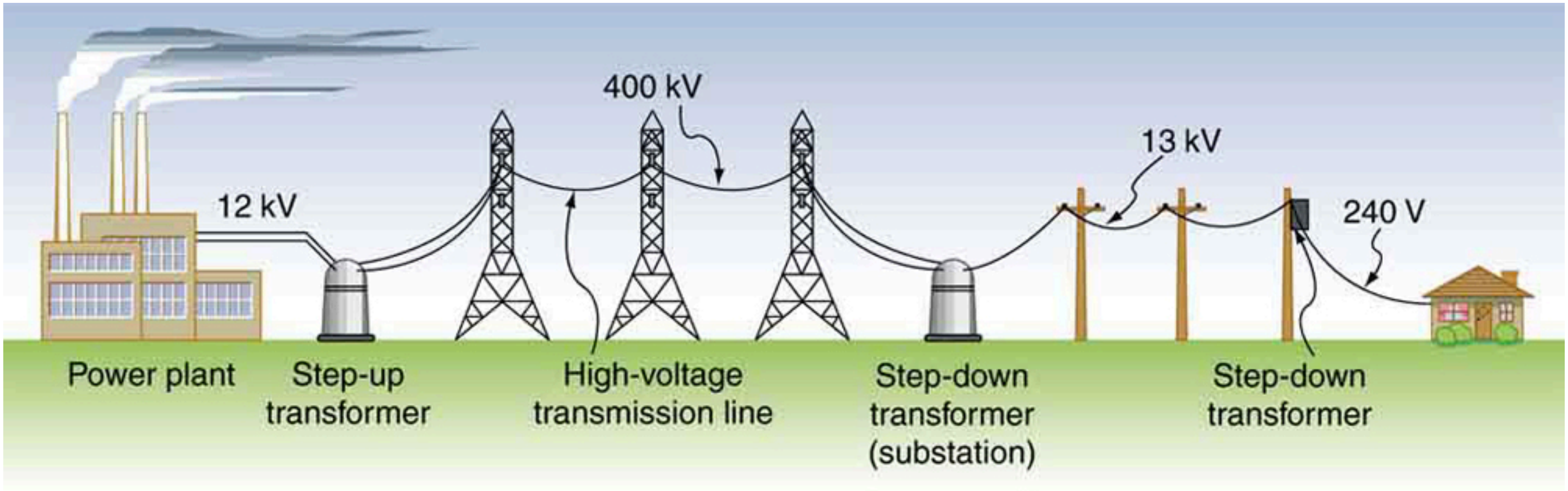
- Imagine you transmit AC electricity with a fixed power P through a long cable. If your wire has some resistance, is it better to transmit your electricity at high or low voltage?

A. High voltage
B. Low voltage
C. Makes no difference

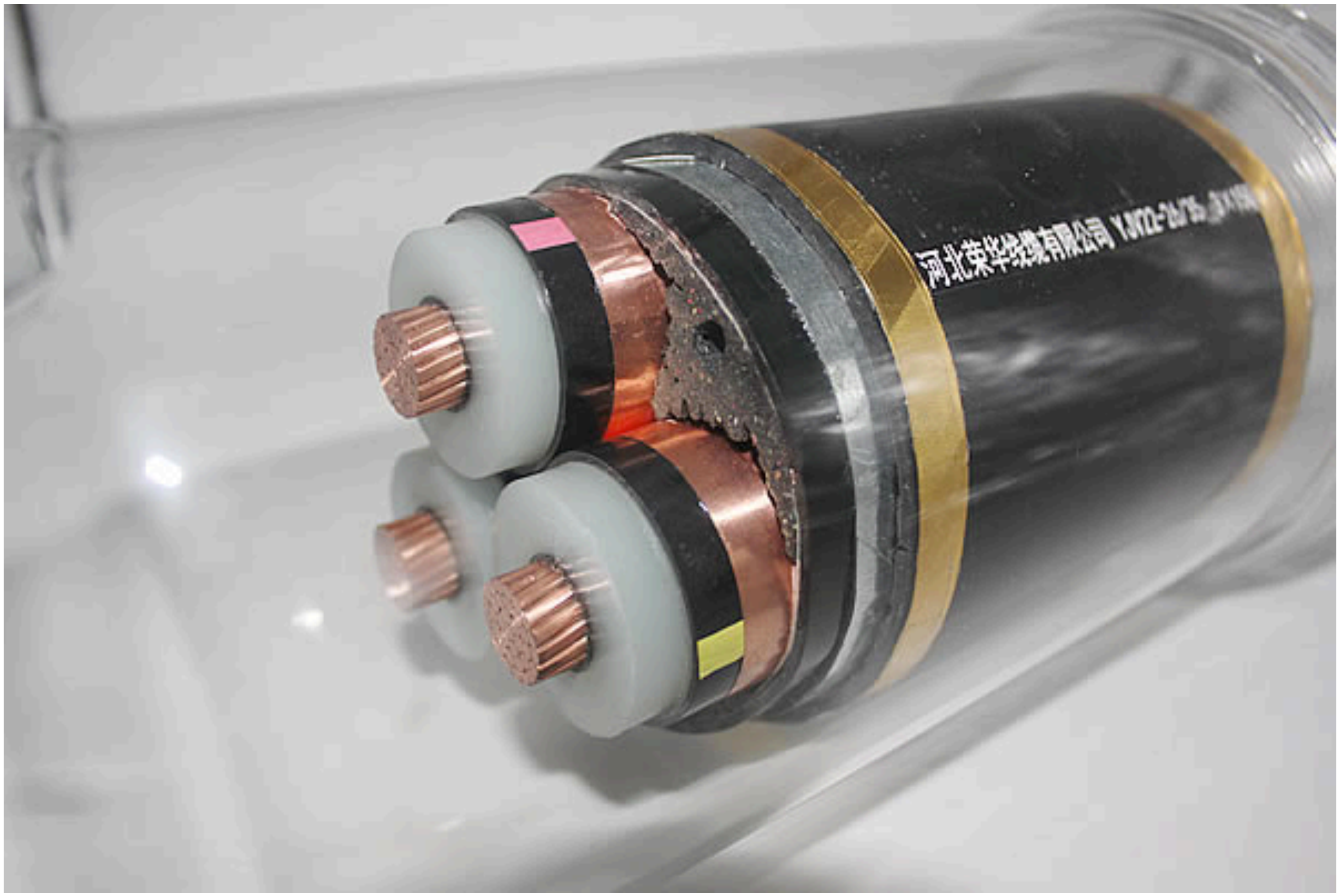
Why High Voltage?

- For fixed power input $P_{in} = V_{in} I_{in}$
  - Power lost to resistive heating in the line
  - $P_{lost} = I_{in}^2 R = (P_{in}/V_{in})^2 R$

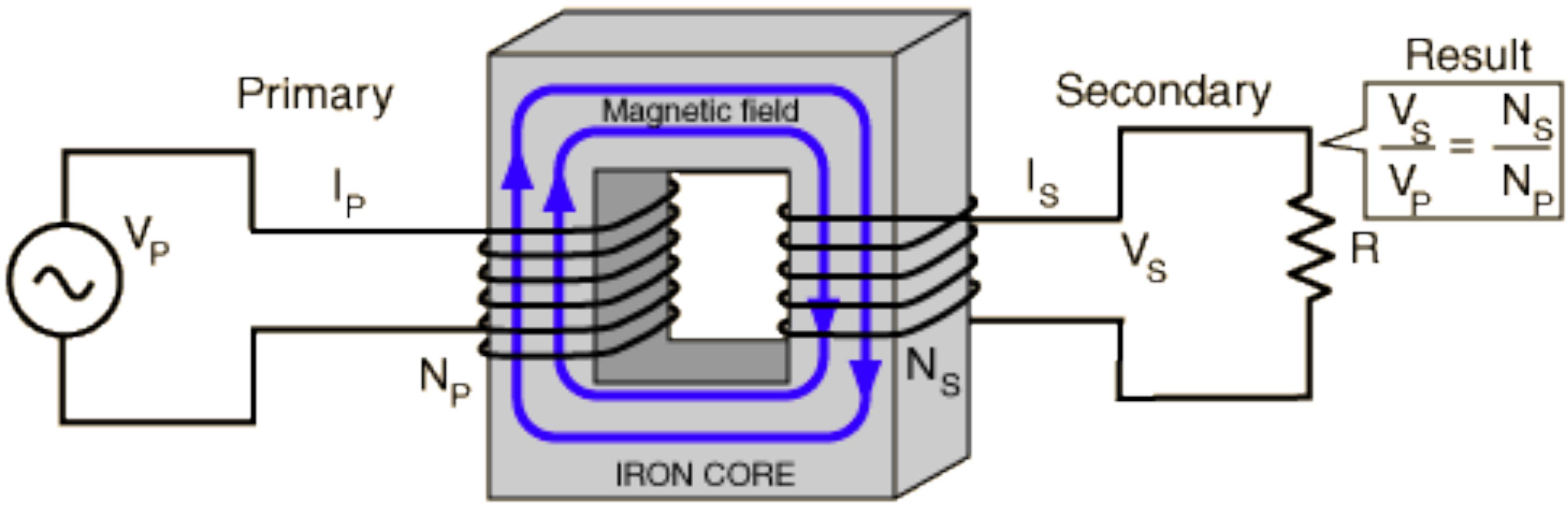
Power Transmission



High Voltage Transmission Cable

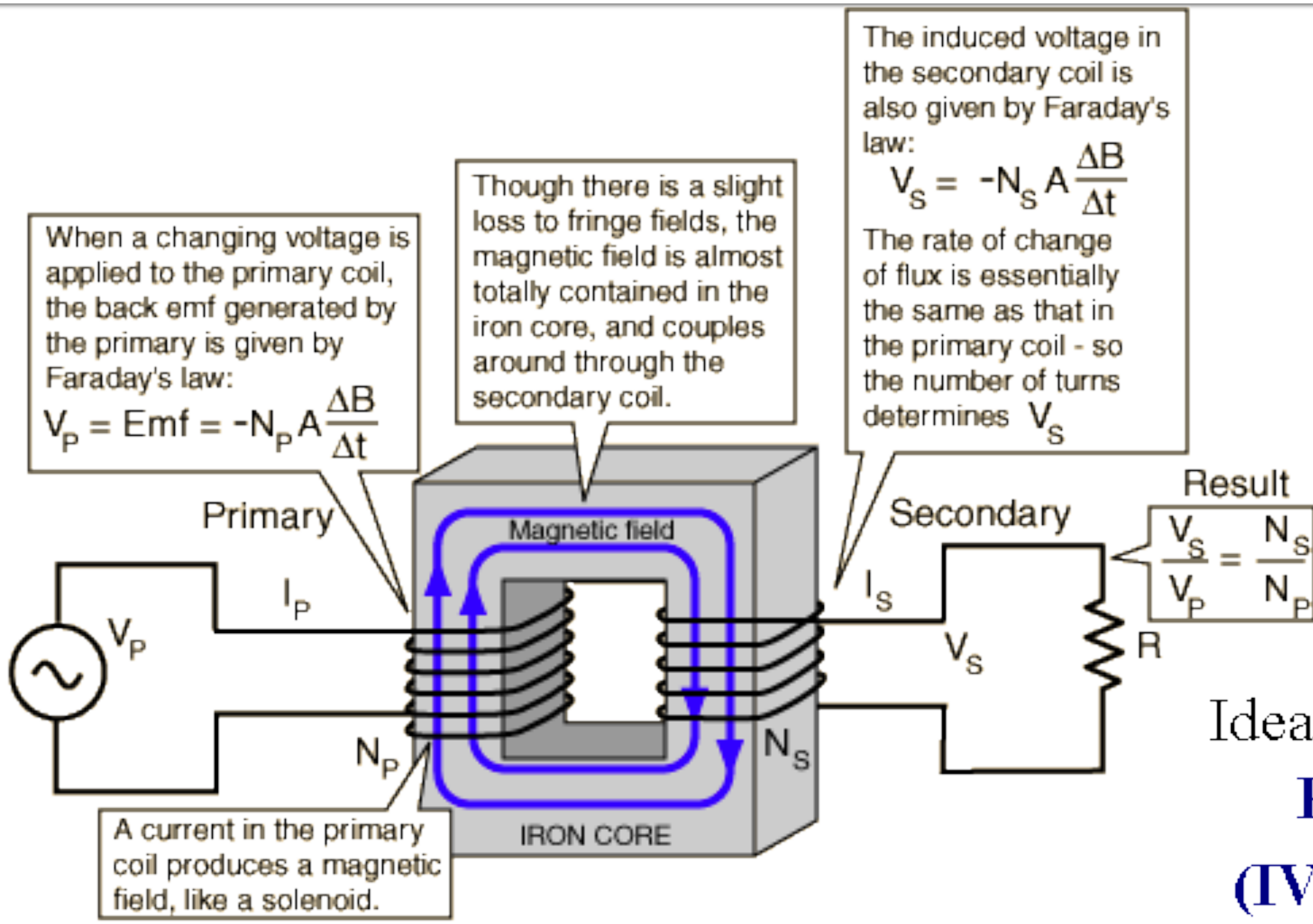


Transformers



More than meets the eye!

Transformers



Ideal Transformer:

$$P_{out} = P_{in}$$

$$(IV)_{out} = (IV)_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{I_{in}}{I_{out}}$$