Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Announcements I

- First homework (math and conceptual sections) due tonight at 11 pm on Wiley Plus

Announcements II

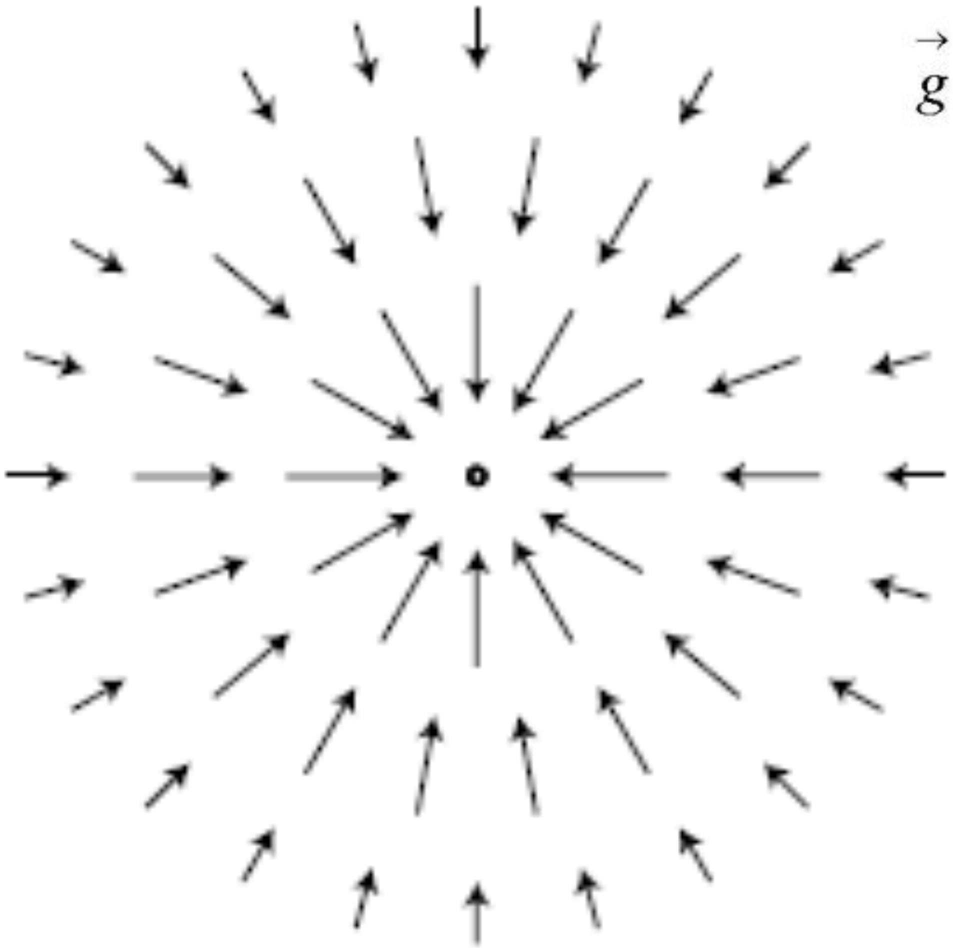
- University Hawk Shop temporarily out of lab manuals
- Iowa book may have some old copies (23 left as of Wednesday afternoon)
- Wherever you check, check under 1512, 1612, and 1702 (same lab manual for all)

- If none of these avenues pan out by Monday, I have a copy of the pre-lab questions and worksheets for E1 that I can send to you

Announcements III

- Please contact me if you are in the late lab section and wish to attend the caucuses on Monday night (which I highly encourage you to do, despite the fact that it will complicate our lives slightly!)

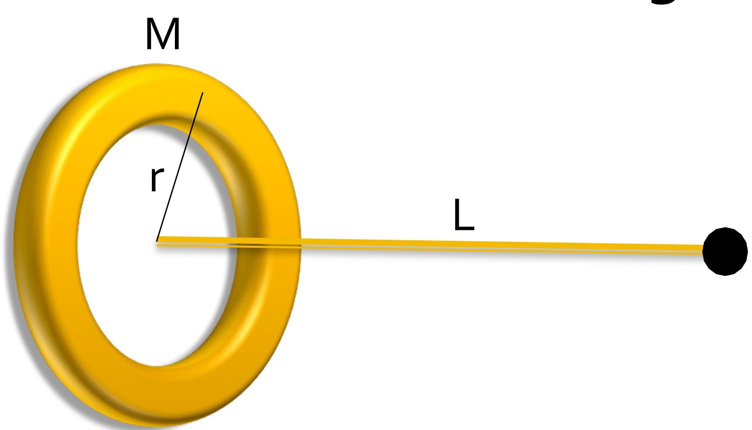
Gravitational Field



$$\vec{g} = \frac{\vec{F}_g}{m} = -G\frac{mM}{mr^2}\hat{r} = -\frac{GM}{r^2}\hat{r}$$

Superposition and Ring of Mass

- What is the gravitational field due to a thin ring of matter with radius r and total mass M, at a point a distance L along the line from the center of the ring?



- $-GML/(r^2 + L^2)^{3/2}$
- $1/(r^2 + L^2)$ from inverse square law
- Extra factor of $L/\sqrt{(r^2 + L^2)}$ because only axial components add (radial components cancel)

The principle of symmetry

- Since the ring is rotationally symmetric, nothing would change if we rotate the ring around its center

- This actually directly implies that the field at a point along the line from the center has to be in the axial direction

- To see this, imagine that it wasn't
  - This would imply that the field would rotate when you rotated the ring, but this clearly can't be since the ring is completely symmetric

Gravity of a spherical shell

- The gravitational field of a spherical shell can be calculated by just adding up the field of many little rings.
  - This turns out to be a rather painful integral (which I will spare you).
- The conclusion is somewhat remarkable
  - The field outside a shell of matter is equal to the field if the entire mass of the shell was at the center of the shell
  - The field inside a shell of matter is zero

Gravity at Earth's Surface

- By the shell theorem, the gravity at the Earth's surface is equivalent to the gravity from a point mass at the center of the Earth

- $M_E = 5.972 \times 10^{24}$ kg
- $r_E = 6378000$ m (at equator)

- $g = GM_E/r_E^2 = 9.792\ m/s^2$

Gravity Below Earth's Surface

- Knowing that the field outside of a shell of matter is proportional to its mass divided by the square of the distance from its center, and that the field inside a shell of matter is zero, can you predict how the field inside the Earth varies with radius r?

A. g proportional to $1/r^2$
B. g proportional to $1/r$
C. g constant
D. g proportional r
E. g proportional to r^2

$\vec{g}$ outside shell of mass M and radius r

$$= -G\, M/r^2\ \hat{r}$$

$\vec{g}$ of a sphere is just that of many shells

so $\vec{g} = -G\, M_{inside}/r^2\ \hat{r}$

where M_{inside} is all the mass inside the radius r

What about inside a sphere?

All that counts is mass inside

$$M_{inside} = \rho V_{inside}$$

$$= \frac{M_{tot}}{V_{tot}} V_{inside}$$

$$= M_{tot} \frac{4/3\,\pi r^3}{4/3\,\pi R^3} = M_{tot}\ r^3/R^3$$

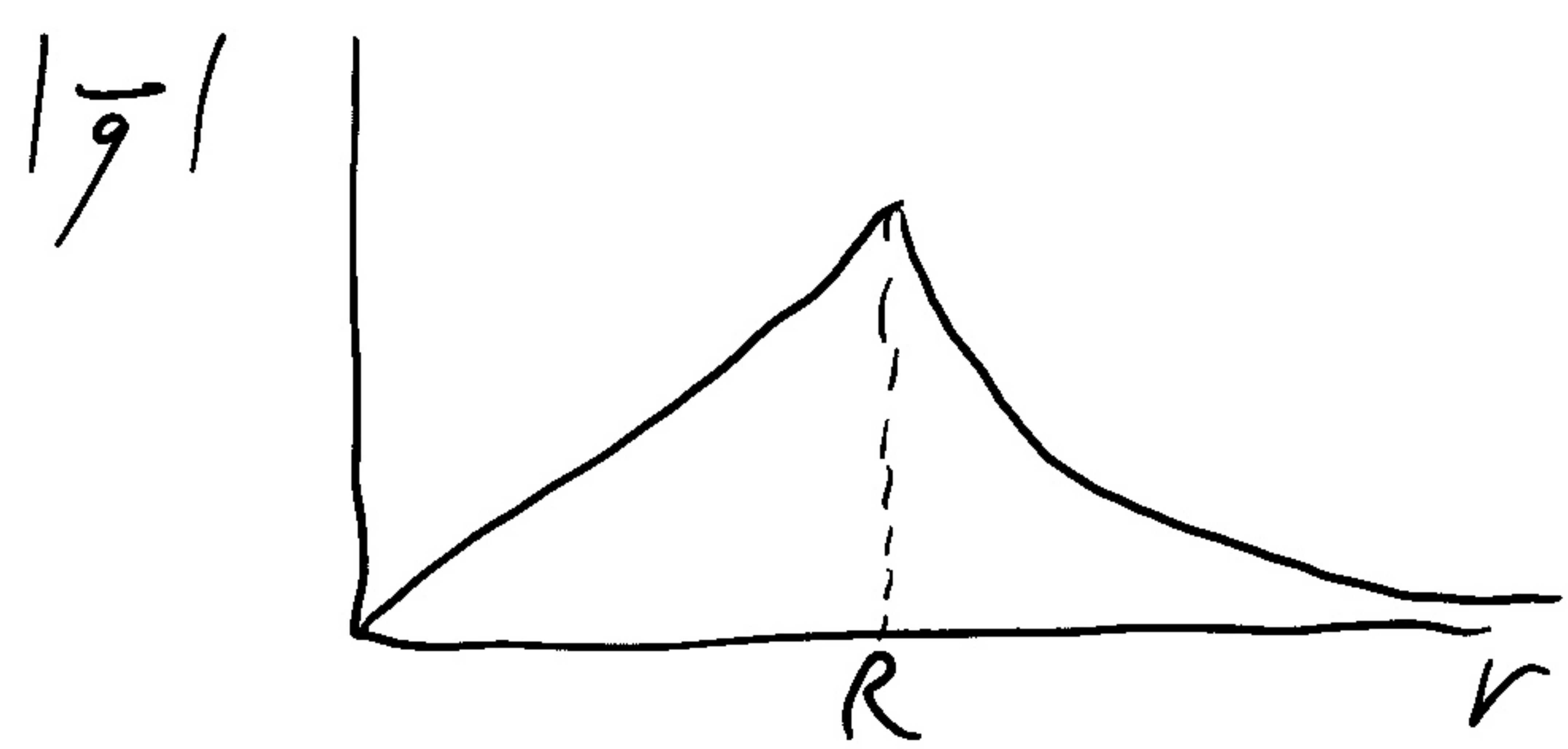
$$\vec{g} = -\frac{G M r^3/R^3}{r^2}\hat{r} = -\frac{G M r}{R^3}\hat{r}$$

Full solution:

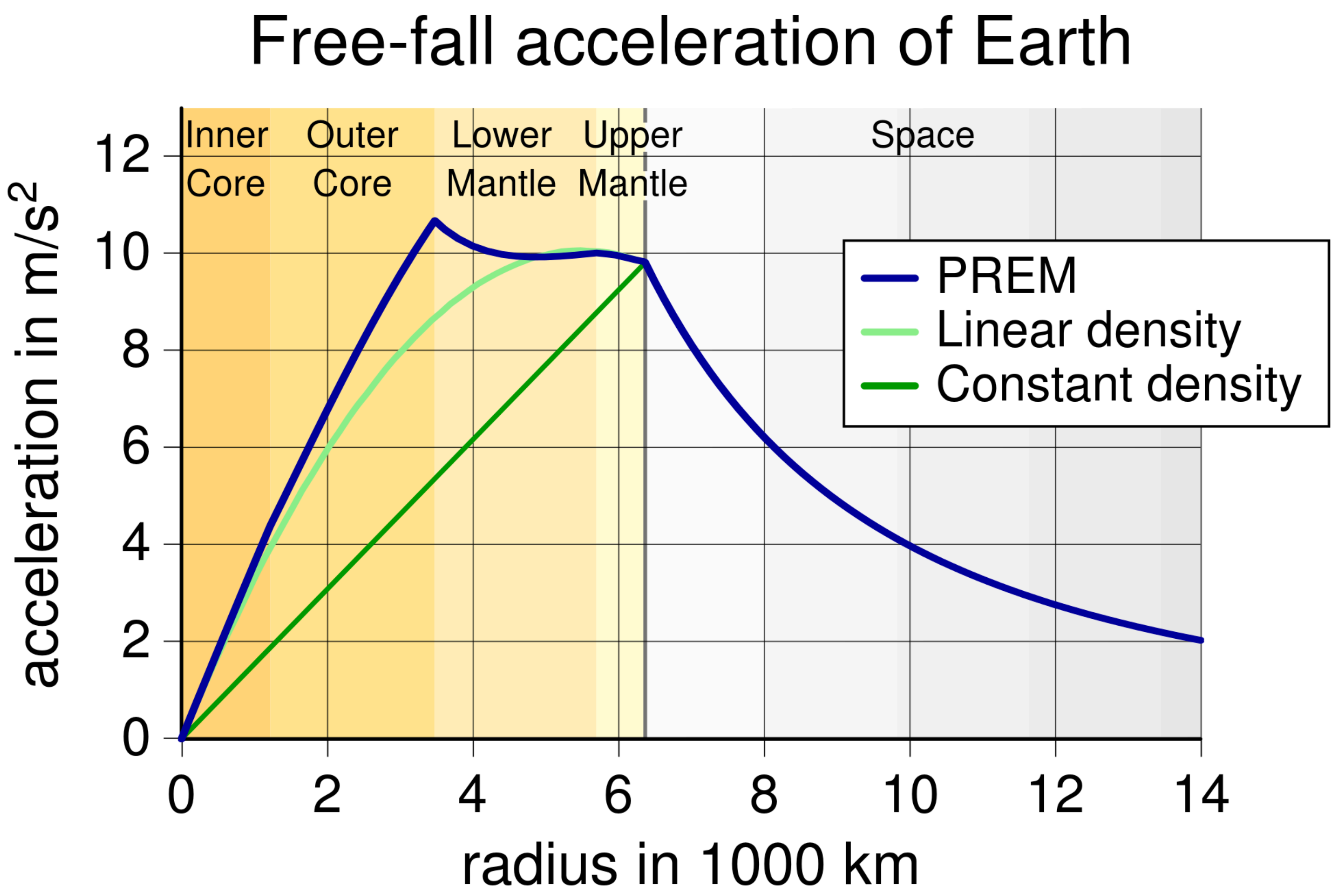
$$\bar{g} = -GMr/R^3 \,\hat{r} \qquad r < R$$

$$= -GM/r^2 \,\hat{r} \qquad r > R$$

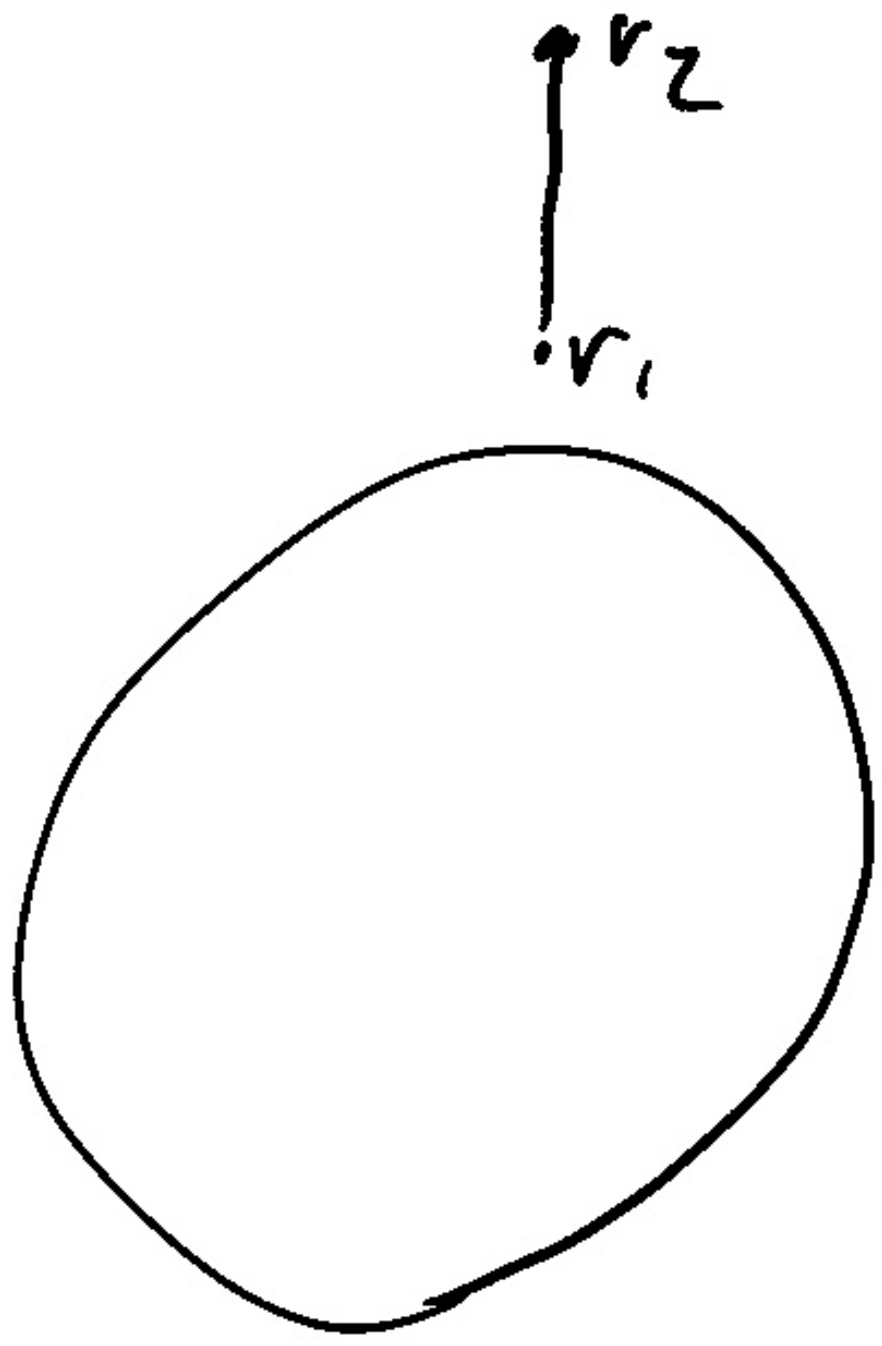
@ $r = R$ these are equal



Actual Gravity Inside Earth



Gravitational Potential Energy:



$$W_{12} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r}$$

- since force radial, only need to worry about r-coordinate

$$\vec{F} = -\frac{GMm}{r^2}\hat{r} \qquad d\vec{r} = dr\,\hat{r}$$

$$\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \int_{r_1}^{r_2} -\frac{GMm}{r^2}\,\hat{r} \cdot dr\,\hat{r}$$

$$= \int_{r_1}^{r_2} -\frac{GMm}{r^2}\,dr$$

$$= \frac{GMm}{r}\Big|_{r_1}^{r_2} = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

$$\Delta U = -W$$

$$= \frac{GMm}{r_1} - \frac{GMm}{r_2}$$

$$= U_2 - U_1$$

convention is to set $U = 0$ @ $r = \infty$

say $r_1 = \infty$
$U_1 = 0$

$$\Rightarrow \boxed{U = -\frac{GMm}{r}}$$

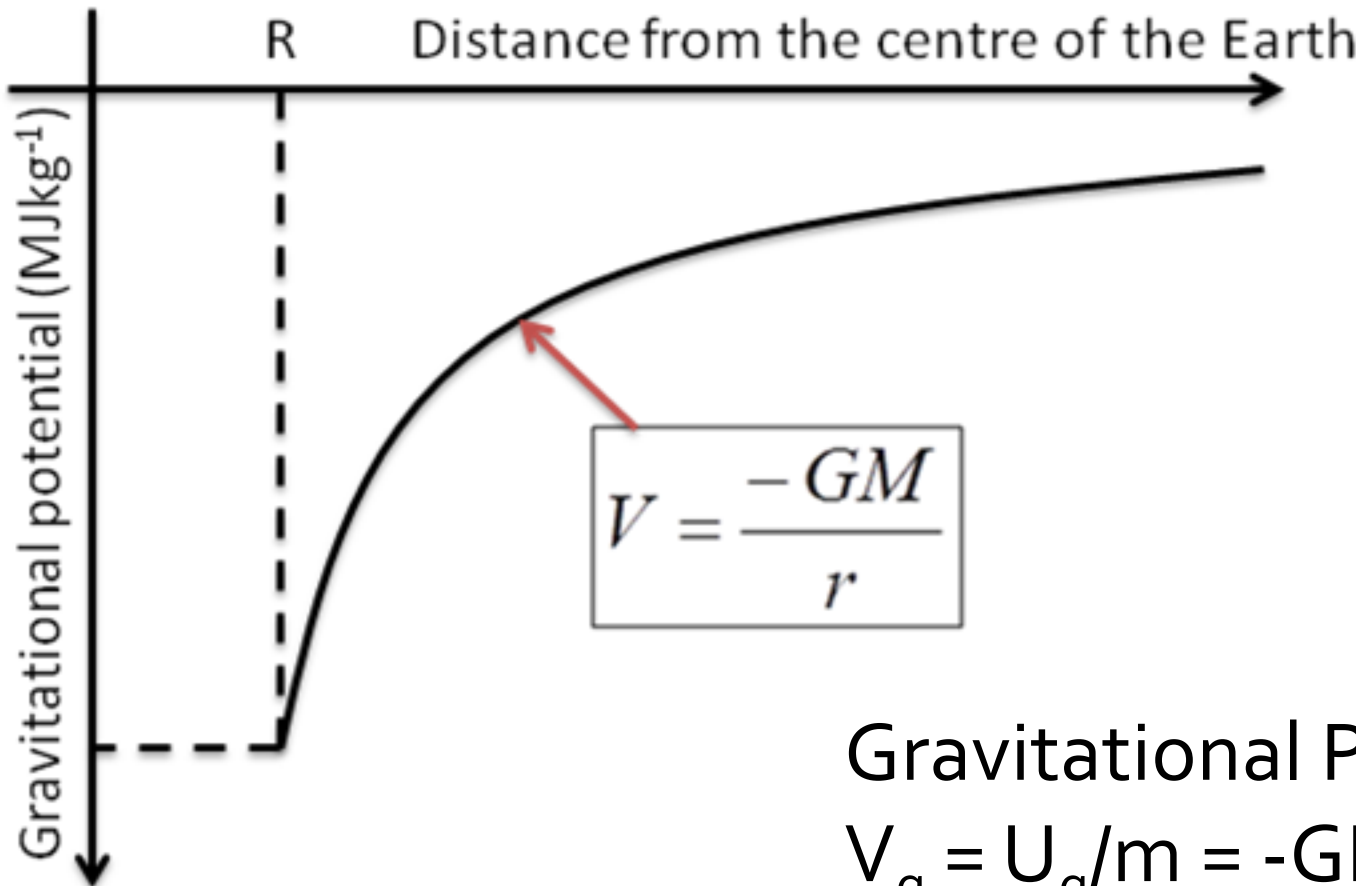
Note $F_r = -\partial U / \partial r$

$$= -GMm / r^2$$

= Newton's Law

Gravitational Potential

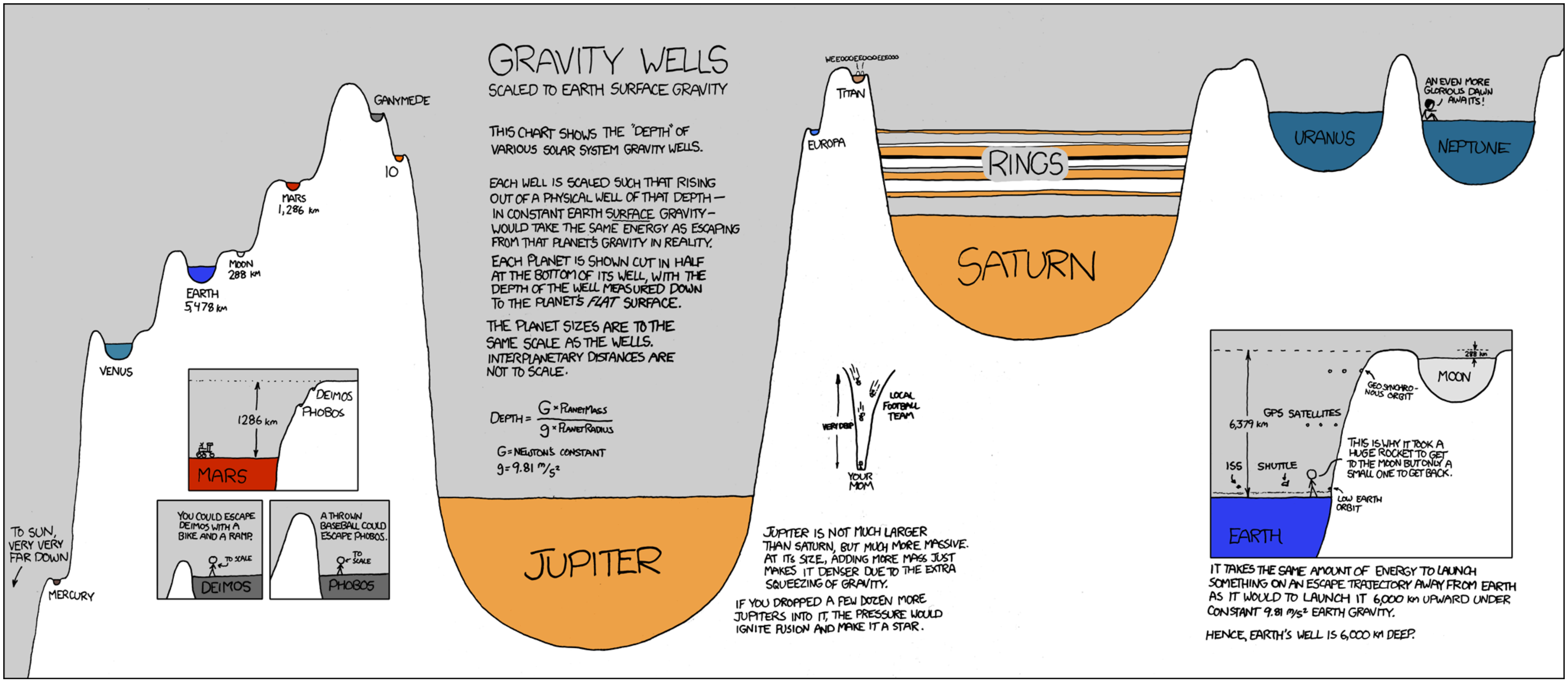
- Depends on both masses involved, just like the force
- Normalize by mass to get a common quantity V_g



Gravitational Potential

$V_g = U_g/m = -GM/r$

Obligatory XKCD



Concept Check

- To escape a body's gravitational pull, an object has to have non-negative kinetic energy at an infinite distance from the object. Knowing this, and the gravitational potential energy –GMm/r, can you predict the escape velocity?

- $\sqrt{(2GMm/r)}$
- $\sqrt{(2GM/r)}$
- $\sqrt{(GM/r)}$
- GM/r
- $2GM/r$