

Physics II: 1702

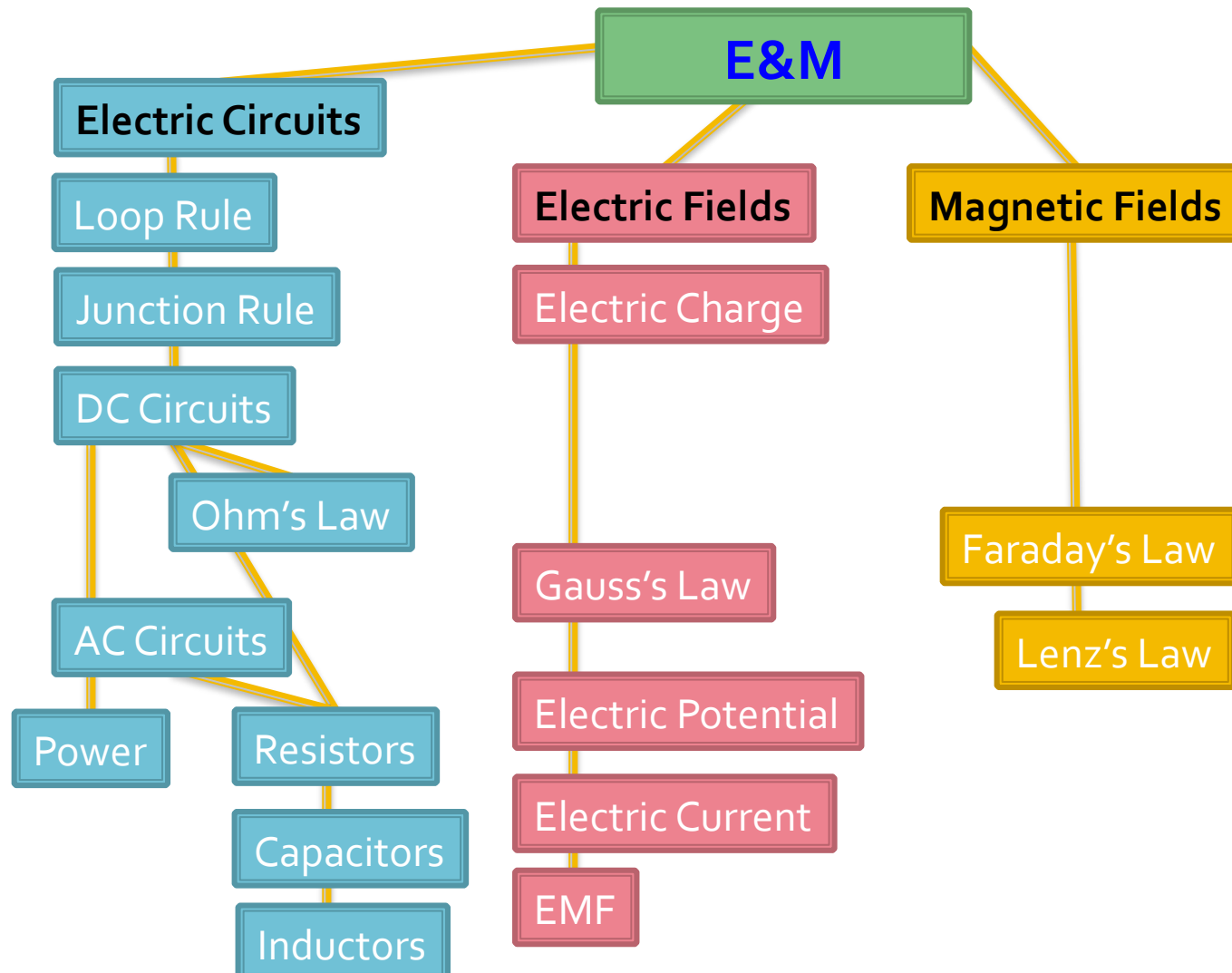
Gravity, Electricity, & Magnetism

Professor Jasper Halekas

Van Allen 70 [Clicker Channel #18]

MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Circuits



Junction Rule

- The sum of currents entering and departing from any junction is **always** zero
- If this were not true, charge would **very** quickly build up that would oppose the non-equilibrium current

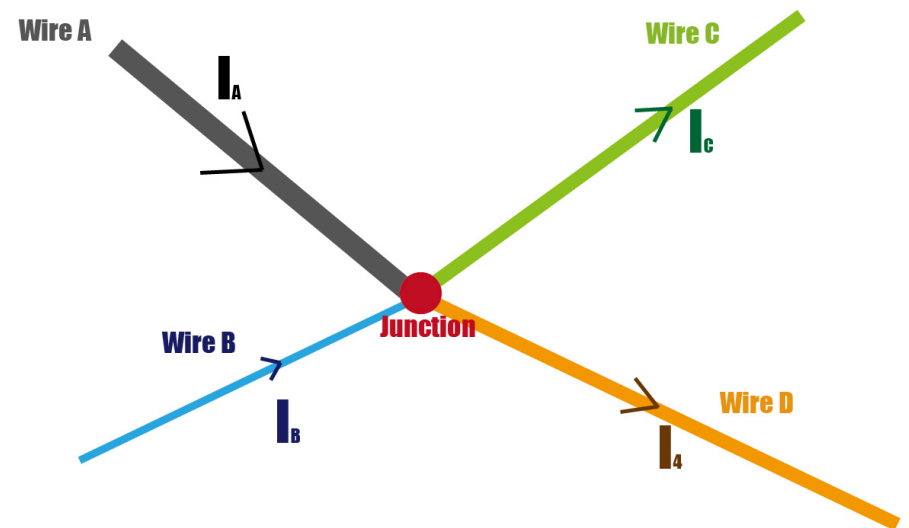
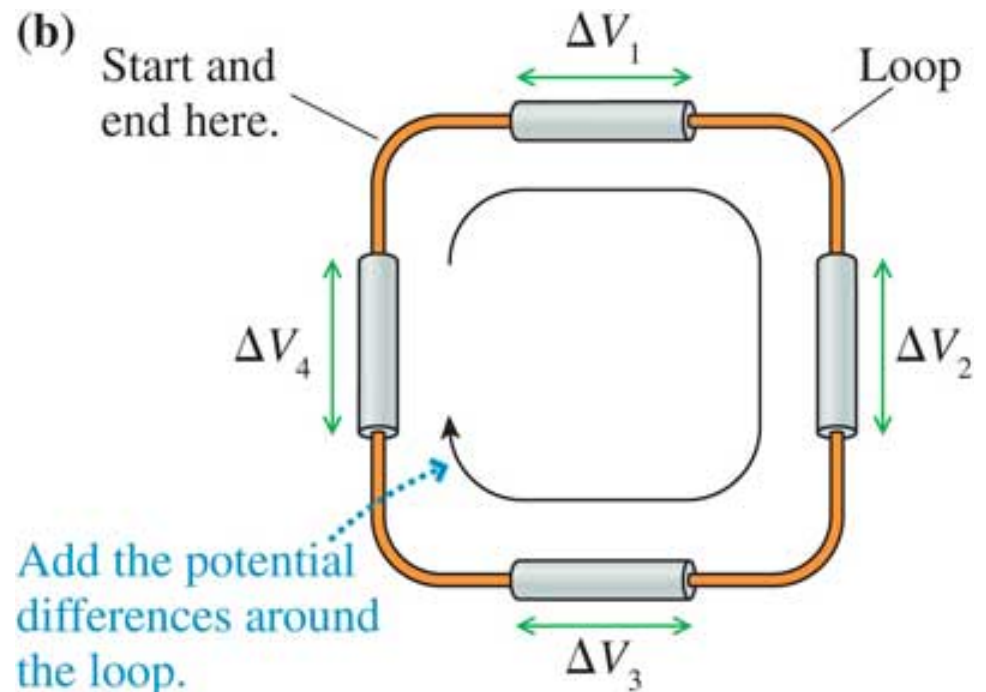


Figure 2

$$I_A + I_B = I_C + I_D$$

Loop Rule

- The sum of the voltage drops around a loop (excluding any induced EMF from externally applied magnetic field variations) is equal to zero
- Since $\Delta V = \Delta U/q$, this statement is equivalent to conservation of energy



Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

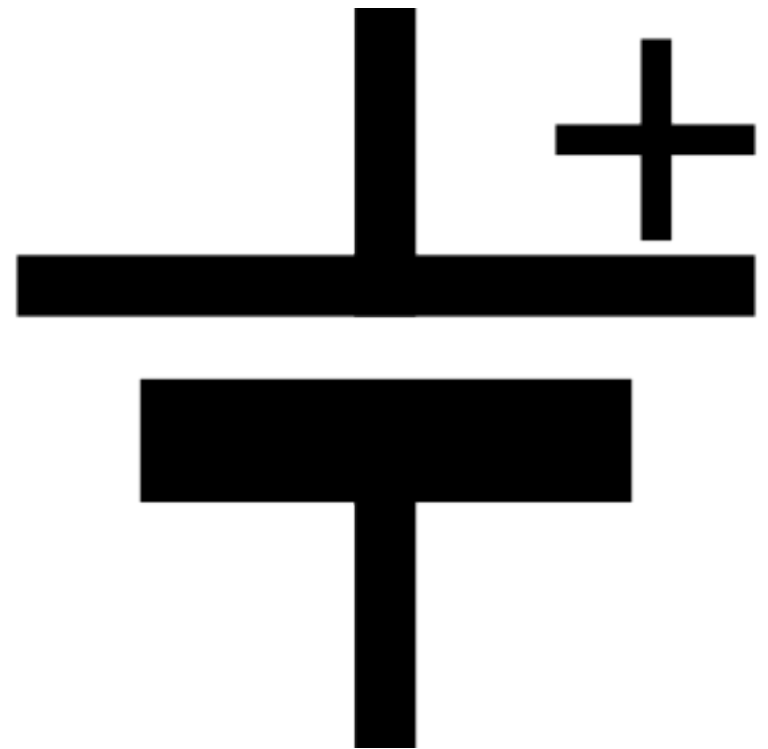
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Loop Rule Special Case

- A changing magnetic field can produce an external EMF which may not be apparent from the circuit itself
 - The energy for this comes from the magnetic field and it does not contradict conservation of energy
- As long as this extra EMF is added properly to any battery EMF the loop rule can still be applied

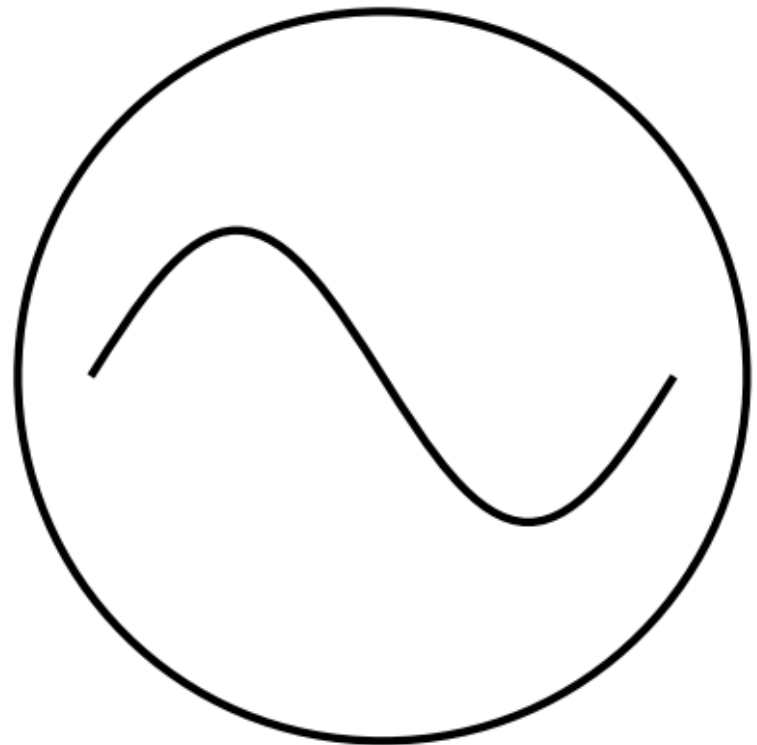
Batteries

- A battery (neglecting any losses) provides a fixed EMF to a circuit
- Traversing a battery from minus to plus terminals gives a positive ΔV for the loop rule



AC Voltage Source

- An alternating voltage source provides a voltage of the form $\epsilon_m \sin(\omega t)$ to a circuit

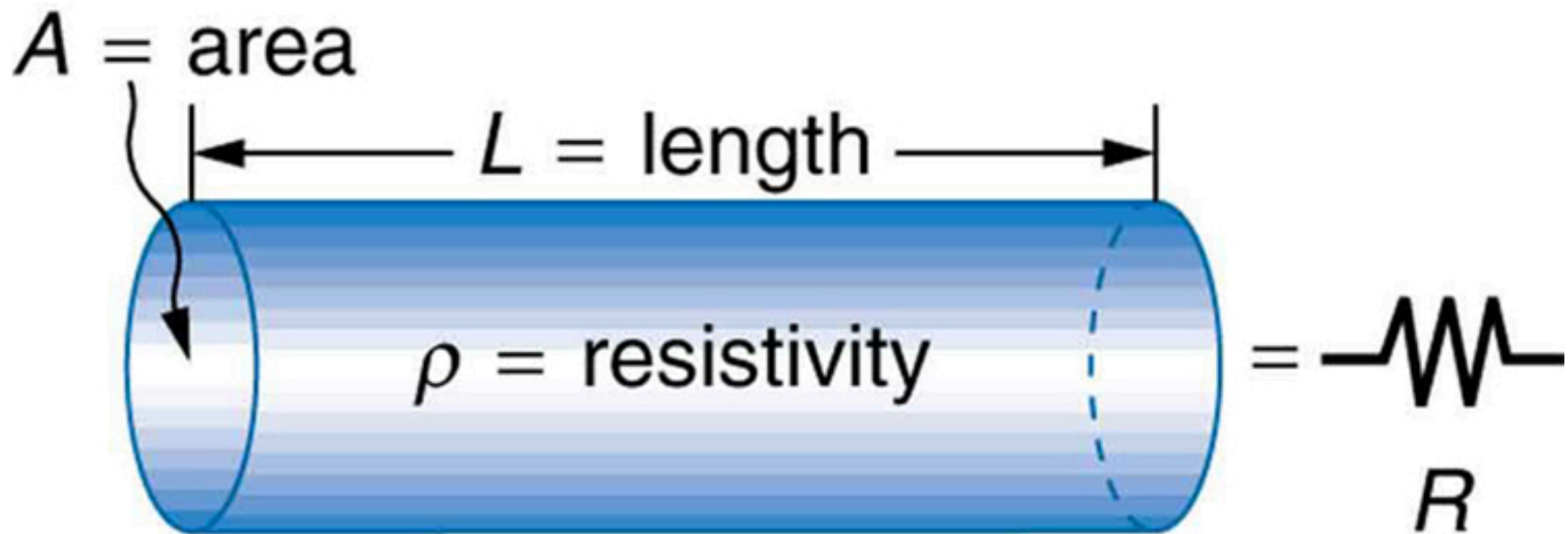


Resistor

- A resistor dissipates energy in the form of heat due to collisions between current-carrying electrons and molecules in the resistor
 - Ohm's Law states $\Delta V = IR$
- Traversing a resistor in the direction of current flow gives a negative ΔV for the loop rule



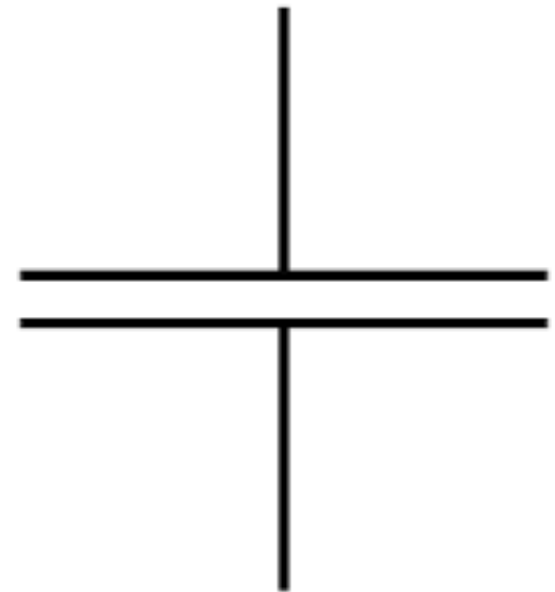
Resistor Properties



$$R = \rho \frac{L}{A}$$

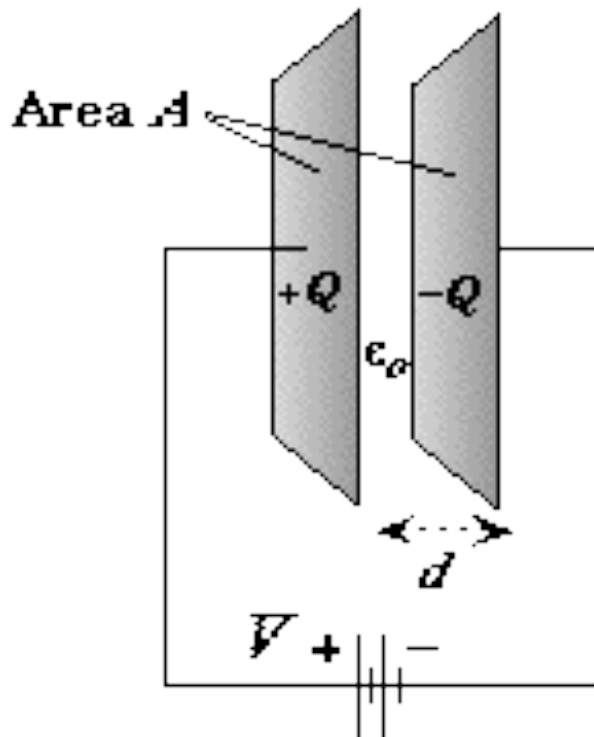
Capacitor

- A capacitor stores charge built up by the flow of current
 - By the definition of capacitance $\Delta V = Q/C$
- Traversing a capacitor in the direction of current flow gives a negative ΔV for the loop rule

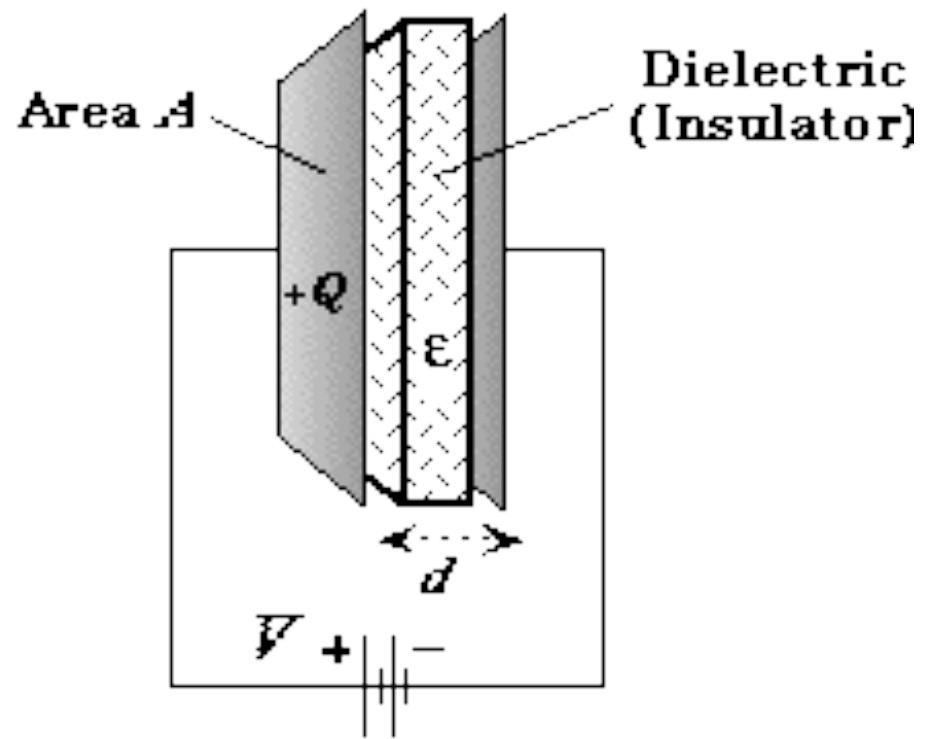


Capacitor Properties

$$C = \epsilon_0 \frac{A}{d}$$



$$C = \epsilon \frac{A}{d} \quad \epsilon = K\epsilon_0$$

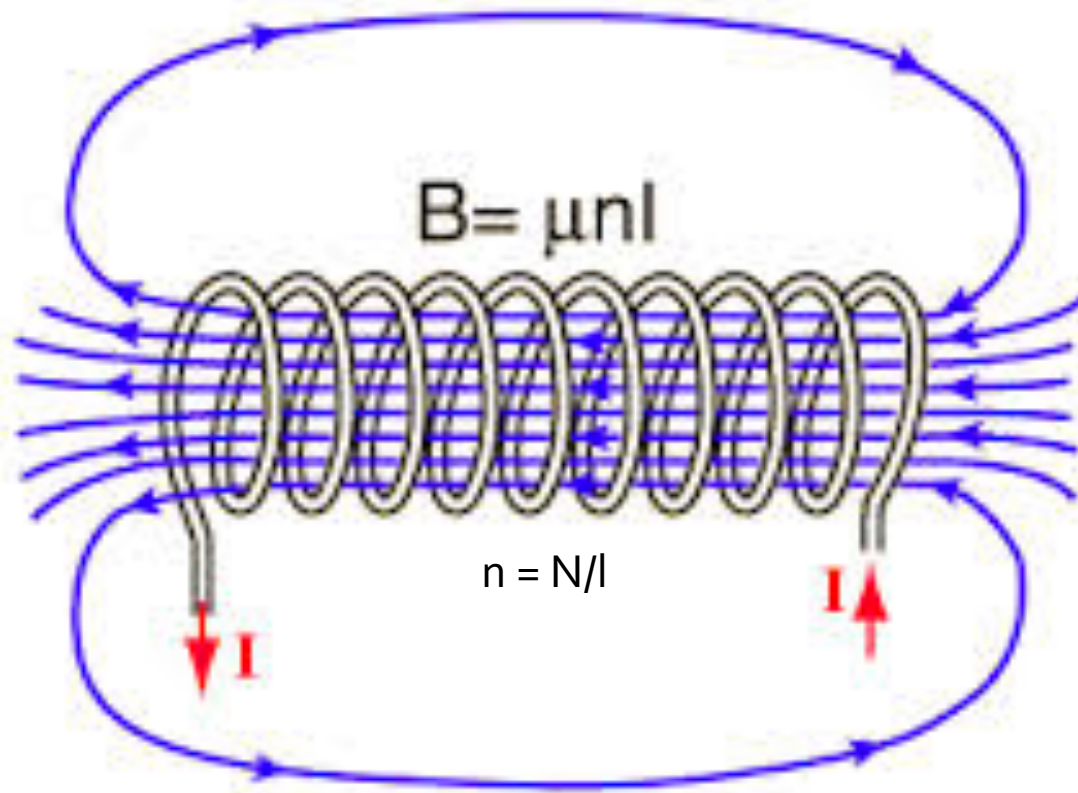


Inductor

- An inductor produces a voltage drop that opposes the change in current
 - By the definition of inductance $\Delta V = -Ldi/dt$
- Traversing an inductor in the direction of the change in the current gives a negative ΔV for the loop rule



Inductor Properties



$$\phi = BA = \frac{\mu_0 N^2 AI}{l}$$

but, $\phi = LI$

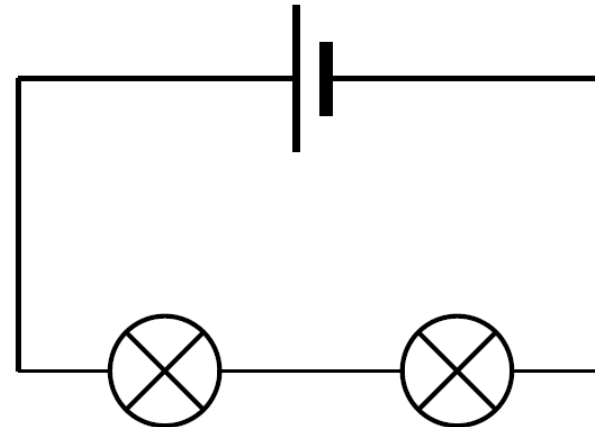
$$\text{So, } LI = \frac{\mu_0 N^2 AI}{l}$$

or, coefficient of self induction

$$L = \frac{\mu_0 N^2 A}{l}$$

Series Combinations

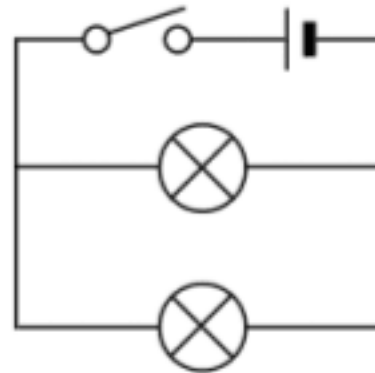
- Components in series share the same current
- The voltage drops add



- Thus resistances and inductances in series add, and inverse capacitances add

Parallel Combinations

- Components in parallel share the same voltage drop



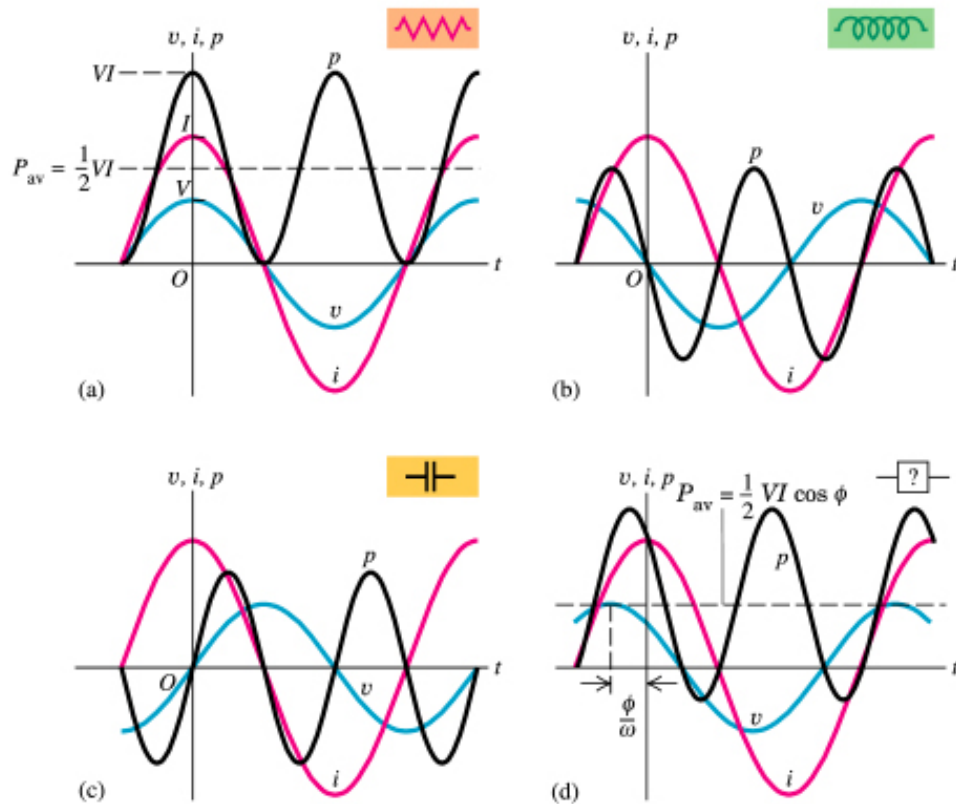
- The currents add

- Thus capacitances in parallel add, and inverse resistances and inverse inductances in parallel add

Power Dissipation

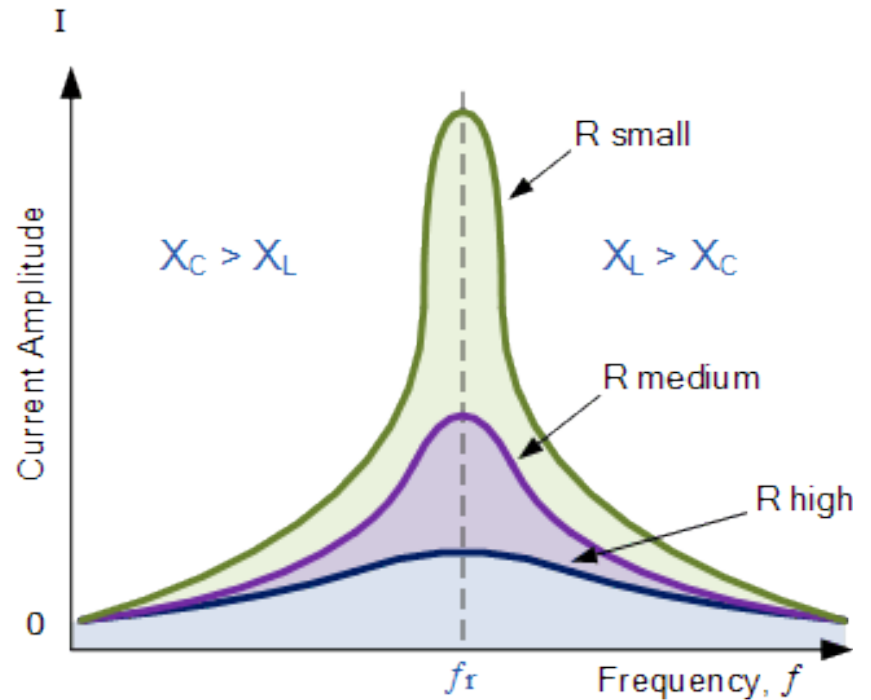
- $P = dU/dt = Vdq/dt = VI$ for circuit element
- For Ohmic component (resistor)
 - $P = I^2R = V^2/R$
- For inductor or capacitor, power dissipation can be positive or negative (energy can be stored!)

AC Power Dissipation & Lead/Lag

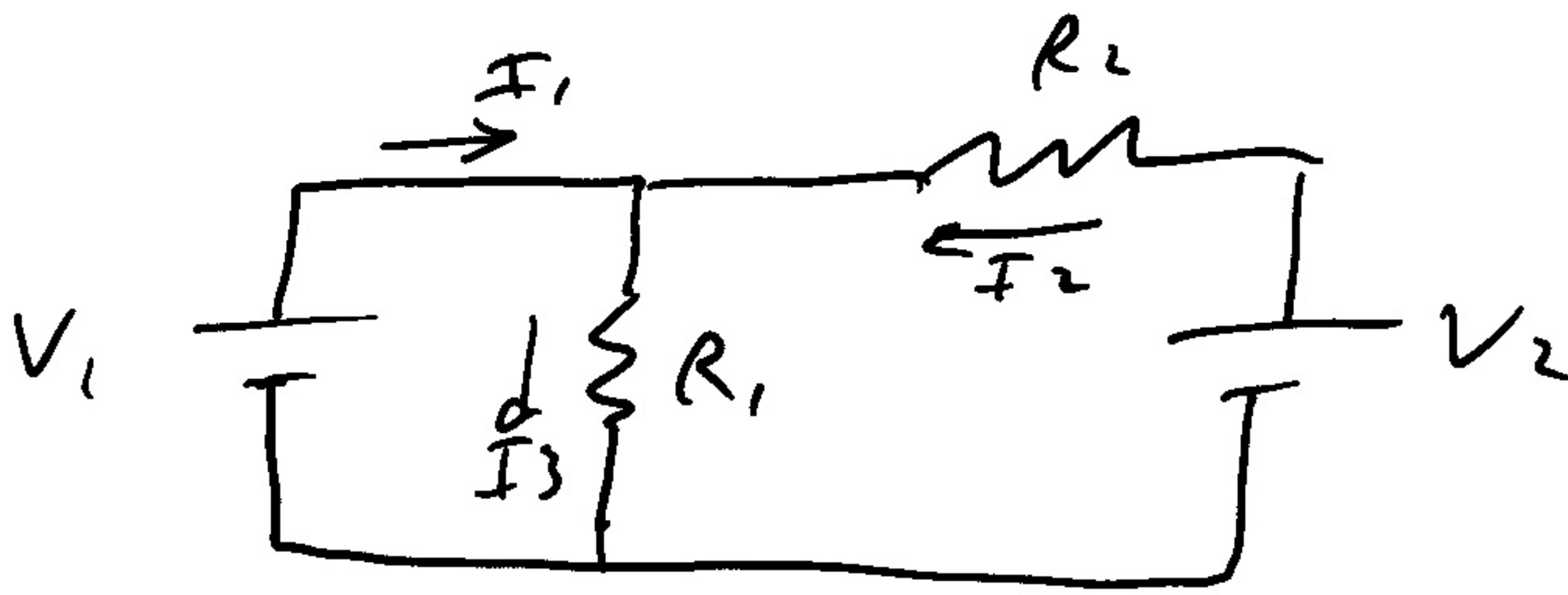


KEY: i — v — p —

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R-circuits



$$I_3 = I_2 + I_1$$

$$V_1 - I_3 R_1 = 0$$

$$V_2 - I_2 R_2 - I_3 R_1 = 0$$

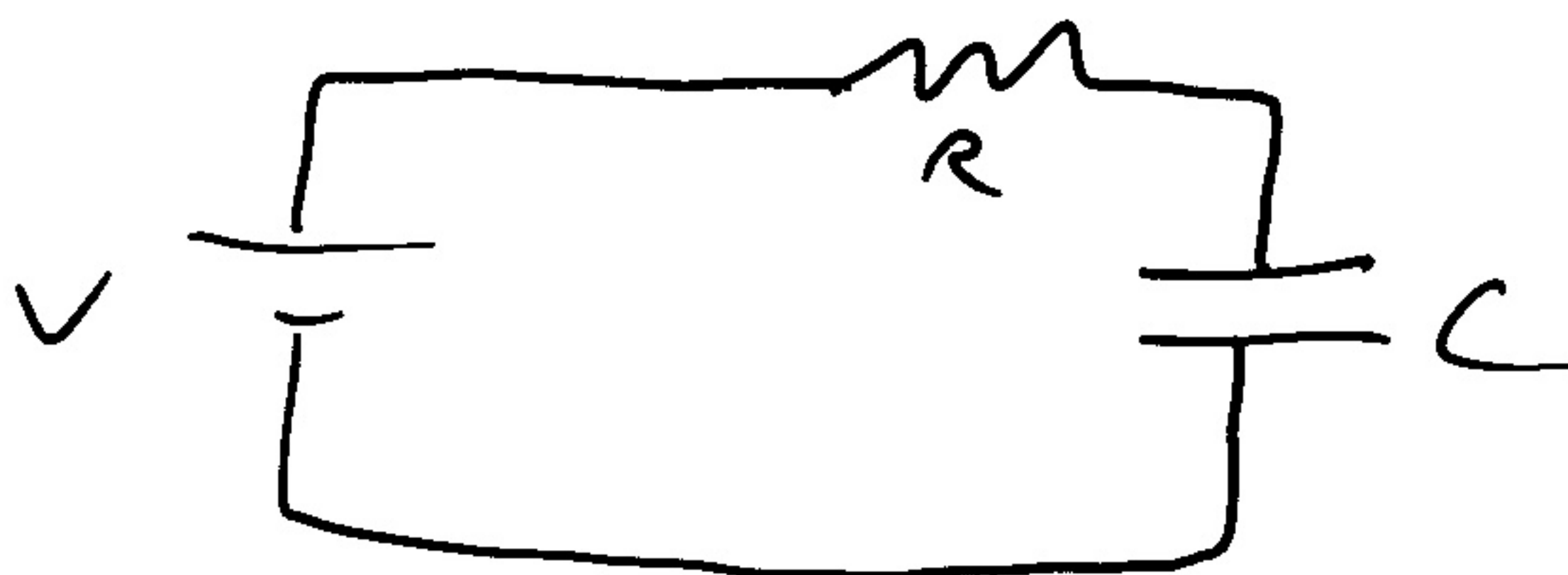
$$\Rightarrow V_2 - V_1 - I_2 R_2 = 0$$

$$\Rightarrow I_2 = \frac{V_2 - V_1}{R_2}$$

$$I_3 = V_1 / R_1$$

$$I_1 = I_3 - I_2 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

RC



$$V - IR - Q/C = 0$$

$$V - R \frac{dQ}{dt} - \frac{1}{C} Q = 0$$

exponential w/ time constant
RC

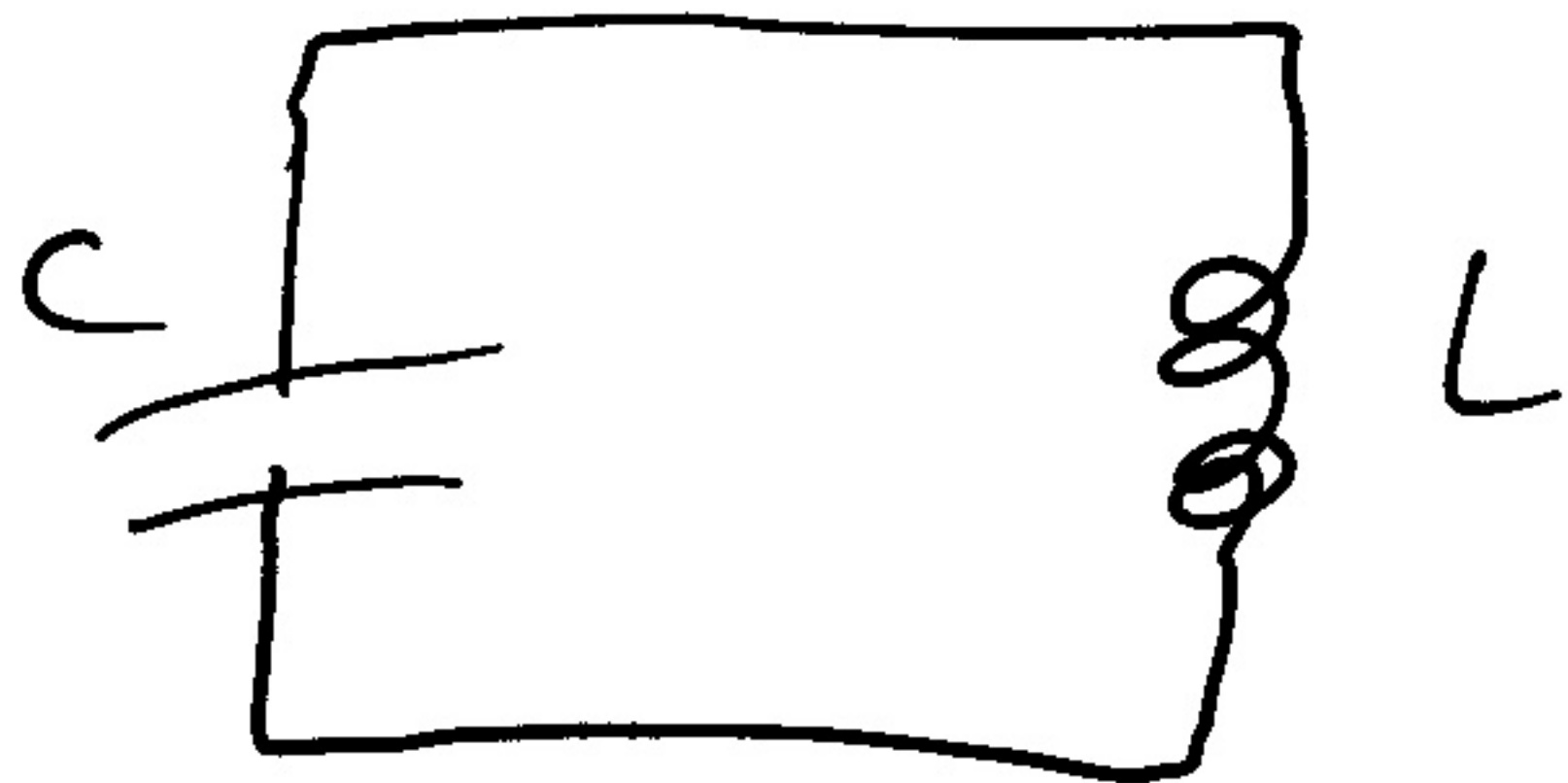
RL



$$V - IR - L \frac{dI}{dt} = 0$$

exponential w/ time
constant L/R

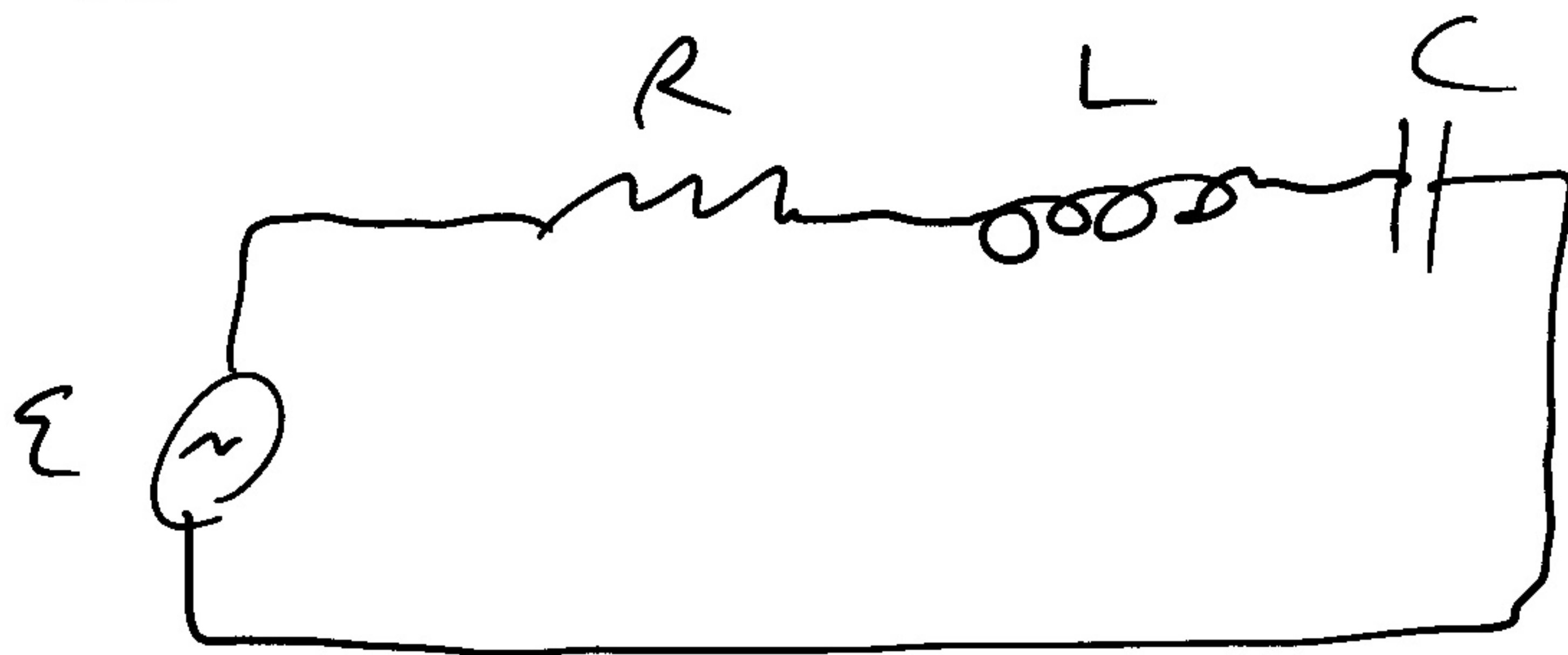
LC



$$Q/C + L \frac{d^2 Q}{dt^2} = 0$$

$$\sin(\omega t - \phi) \quad \omega = \frac{1}{\sqrt{LC}}$$

RLC



$$\varepsilon = \varepsilon_m \sin(\omega t)$$

$$\varepsilon - iR - L \frac{di}{dt} - q/C = 0$$

magnitudes:

$$\varepsilon_m - IZ = 0$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$i = I \sin(\omega t - \varphi)$$

$$X_L = \omega L$$
$$X_C = 1/\omega C$$

$$\varphi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$\langle P \rangle = \langle \varepsilon i \rangle = \frac{\varepsilon_m^2}{2Z} \cos \varphi$$

$$\cos \varphi = R/Z = \text{"power factor"}$$