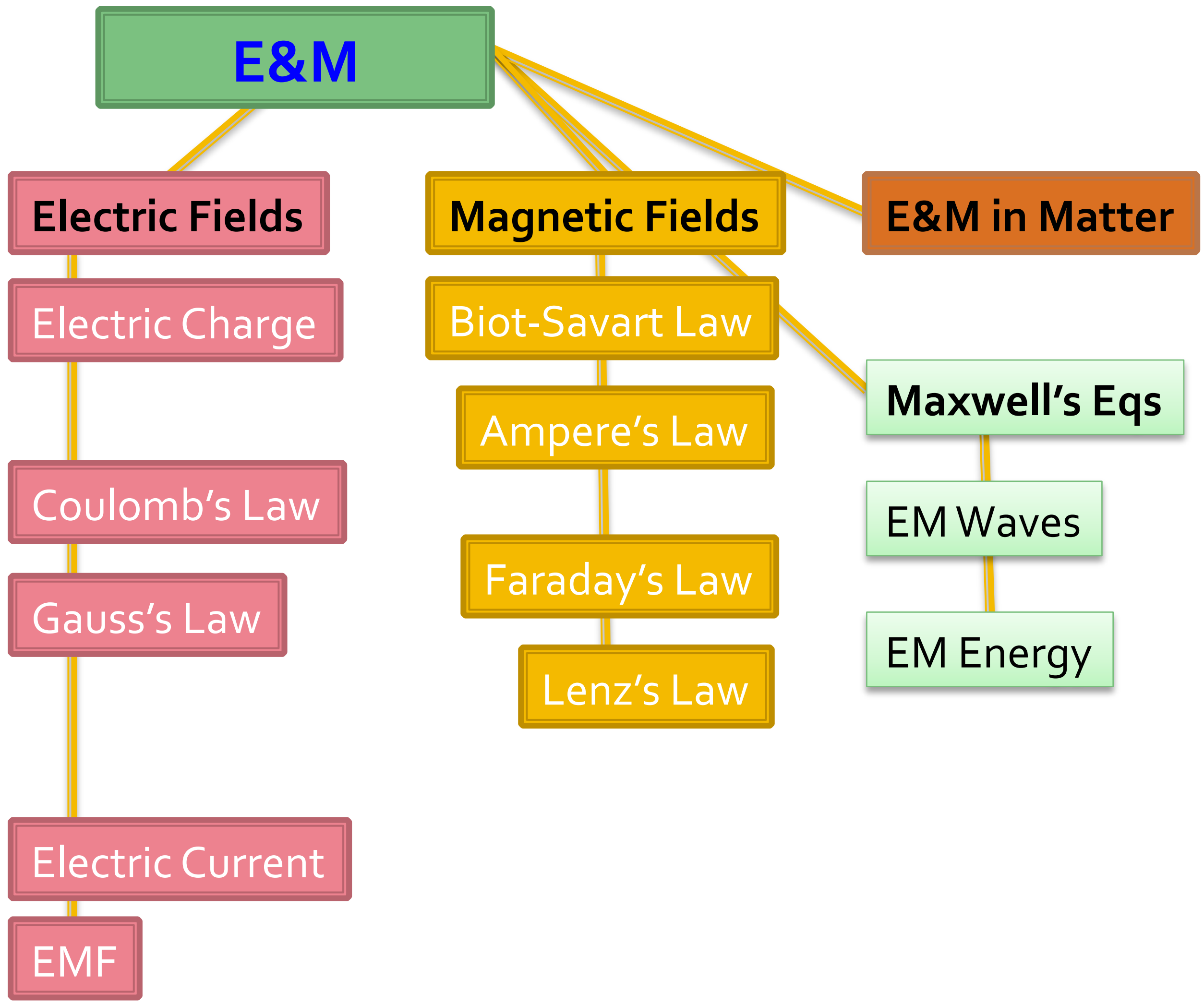
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Maxwell's Equations



Maxwell's Equations

$$\oint \mathbf{E} \cdot d\mathbf{A} = q / \varepsilon_0$$

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathbf{B}} / dt$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \varepsilon_0 d\Phi_{\mathbf{E}} / dt$$

Gauss's Law: What it Means

- Top Level: Electric field strength is proportional to net charge, and inversely proportional to distance from the charge

- Next Level: Electric flux through a surface is proportional to charge enclosed

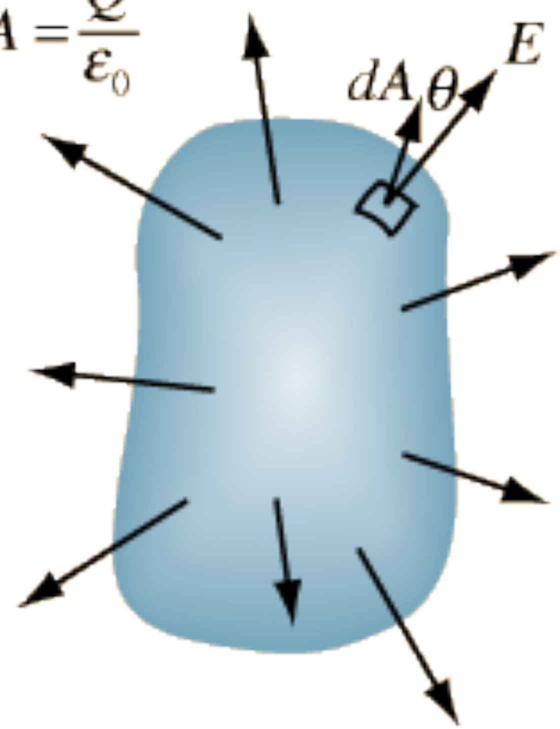
- Implications: Electric field lines start and end on charges

Gauss's Law: How to Use It

- Conceptually: Add up field lines poking out of closed surface

- Mathematically: Integrate the dot product of the electric field and the infinitesimal area vector over a closed surface

$$\oint \vec{E} \cdot \vec{dA} = \frac{Q}{\varepsilon_0}$$

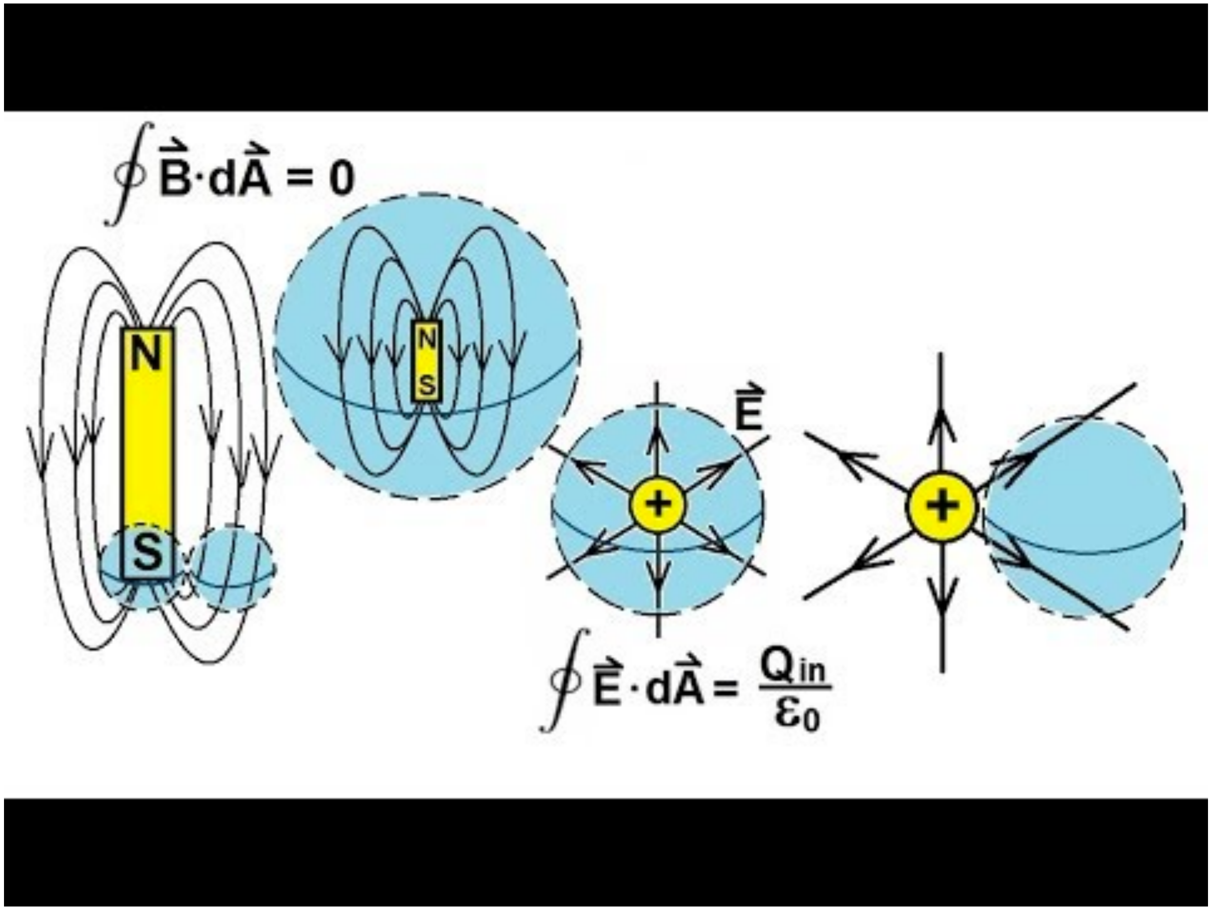


Magnetic Gauss's Law: What it Means

- Top Level: Magnetic field lines always make closed loops
- Next Level: Net magnetic flux through any closed surface is zero
- Implications: There are no magnetic monopoles

Magnetic Gauss's Law: How to Use It

- Conceptually: Any magnetic field line goes all the way through a closed volume (or stays completely within it)

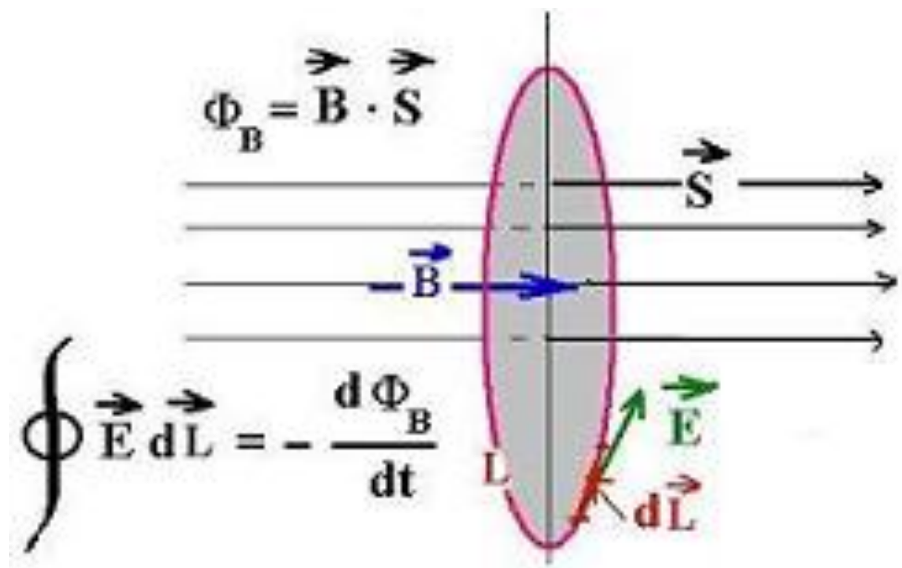


Faraday's Law: What it Means

- Top Level: A changing magnetic field produces an electric field
- Next Level: The EMF around a loop (or the circulation of the electric field) is proportional to the rate of change of the magnetic field flux through the area enclosed
- Implications: Changing magnetic fields drive currents that oppose the change in magnetic field

Faraday's Law: How to Use It

- Conceptually: Measure the change in the number of magnetic field lines passing through a loop
- Mathematically: Integrate the dot product of the magnetic field and the infinitesimal area vector over a surface, take the derivative, and relate to the integral of the dot product of the electric field and the infinitesimal vector tangent to the loop over a closed loop around the edge of the surface

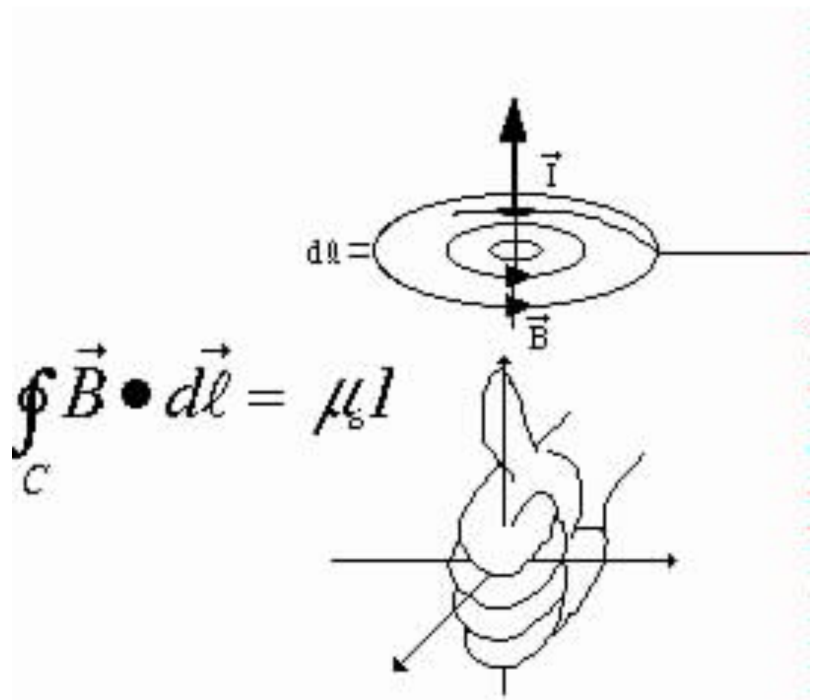


Ampere's Law: What it Means

- Top Level: Magnetic field strength is proportional to current, and inversely proportional to distance from the current
- Next Level: The circulation of the magnetic field is proportional to the current enclosed
- Implications: Moving charges make magnetic fields

Ampere's Law: How to Use It

- Conceptually: Add up the current flowing through a surface
- Mathematically: Integrate the dot product of the magnetic field and the infinitesimal vector tangent to the loop along a closed loop and relate to the current that passes through the loop



Ampere's 2nd term: What it Means

- Top Level: A changing electric field produces a magnetic field
- Next Level: The circulation of the magnetic field around a loop is proportional to the rate of change in the electric flux through the area enclosed
- Implications: Changing electric fields make changing magnetic fields make changing electric fields make changing magnetic fields…

Ampere's 2nd term: How to Use It

- Conceptually: Measure the change in the number of electric field lines passing through a loop

- Mathematically: Integrate the dot product of the electric field and the infinitesimal area vector over a surface, take the derivative, and relate to the integral of the dot product of the magnetic field and the infinitesimal vector tangent to the loop over a closed loop around the edge of the surface

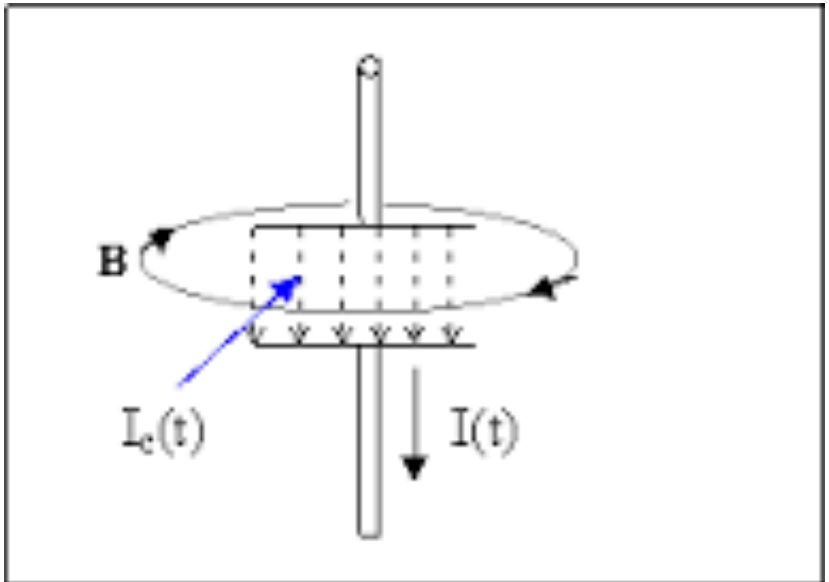
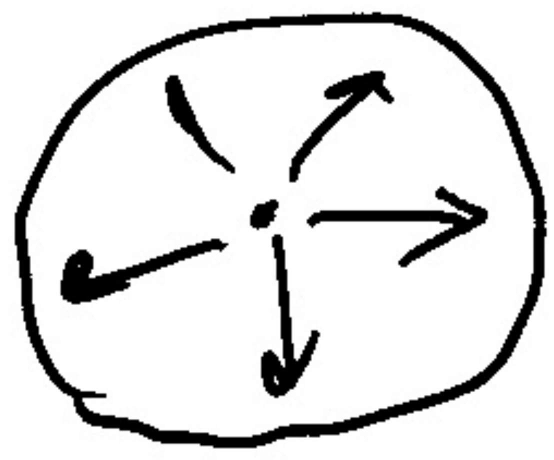


Figure (3.12.5) Magnetic action of $I_c(t)$

Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$$

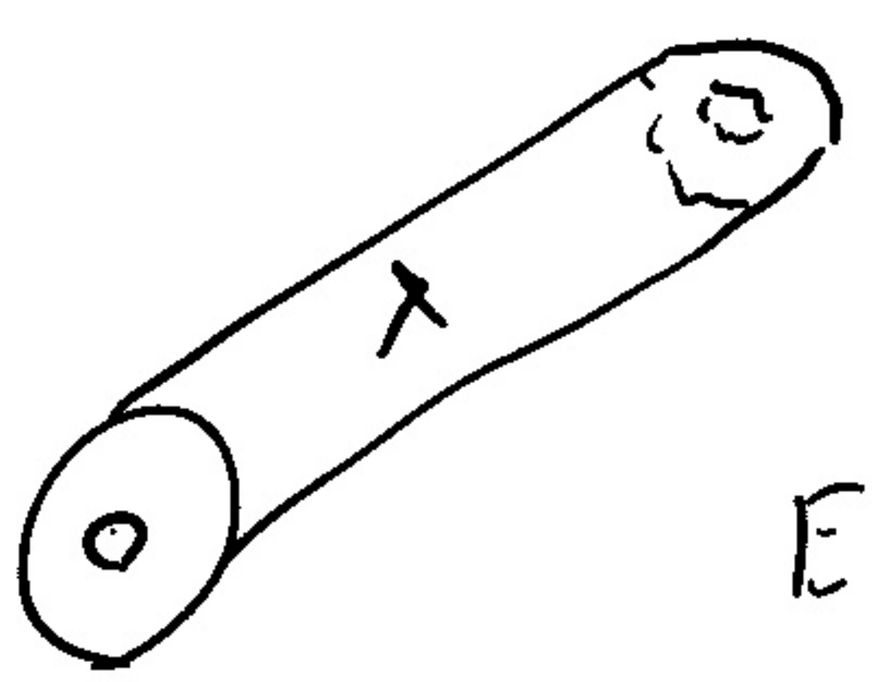


can evaluate if sphere centered and no other field

$$\oint \vec{E} \cdot d\vec{A} = EA = E \cdot 4\pi r^2$$

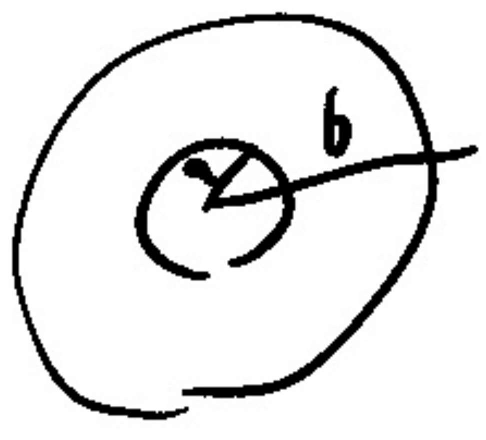
$$\Rightarrow |\vec{E}| = q/4\pi\varepsilon_0 r^2$$

back to



$$E = \lambda/2\pi\varepsilon_0 r$$

for $r > b$



but what about inside?

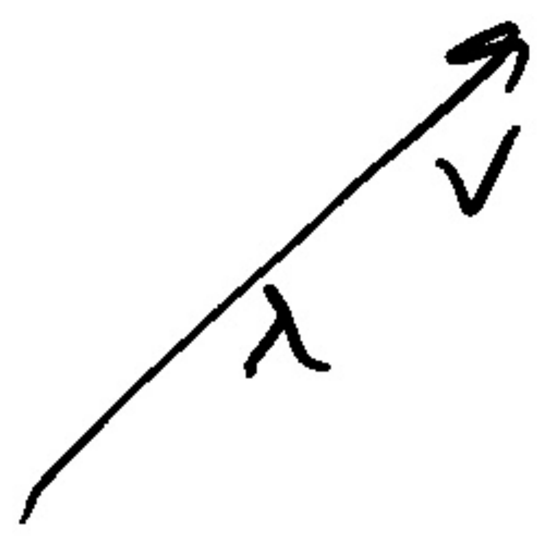
$r < a$, $q_{enc} = 0$ so $E = 0$

$a < r < b$, $q_{enc} = \rho L \cdot (\pi r^2 - \pi a^2)$

$$\rho = \lambda L / (\pi b^2 - \pi a^2)$$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}\left[\frac{r^2 - a^2}{b^2 - a^2}\right]$$

Moving line of charge



$$i = \lambda v$$
$$= [Q/L][L/t] = [Q/t]$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

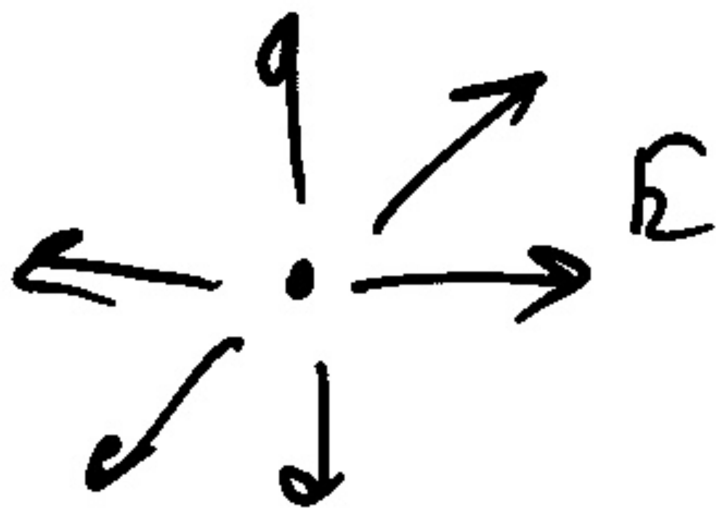
take loop centered on wire

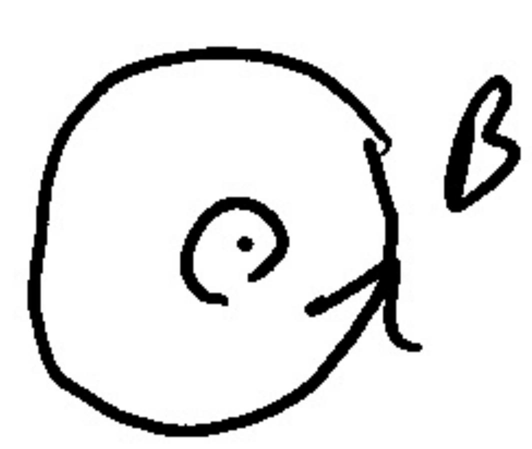
$$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r = \mu_0 i$$

$$\Rightarrow |\vec{B}| = \mu_0 i / 2\pi r$$

very similar to
$$E = \lambda / 2\pi \varepsilon_0 r$$

but





Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -d\Phi_B/dt$$

$$B = at^2 + bt + c$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$= [at^2 + bt + c]\,\pi r^2$$

$$d\Phi_B/dt = [2at + b]\,\pi r^2$$

$$= -\oint \vec{E} \cdot d\vec{l}$$

$$= -E \cdot 2\pi r$$

$$\Rightarrow E = -(2at + b) \cdot r/2$$