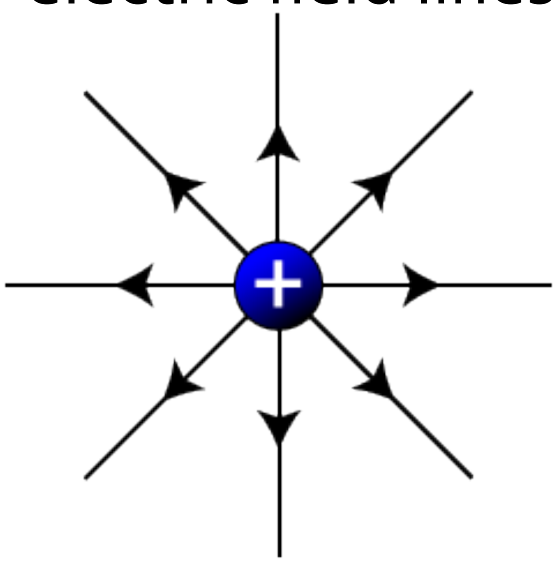
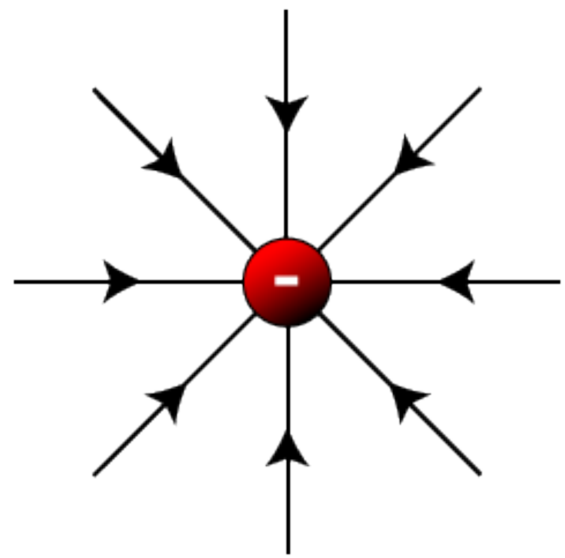
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Electric Field

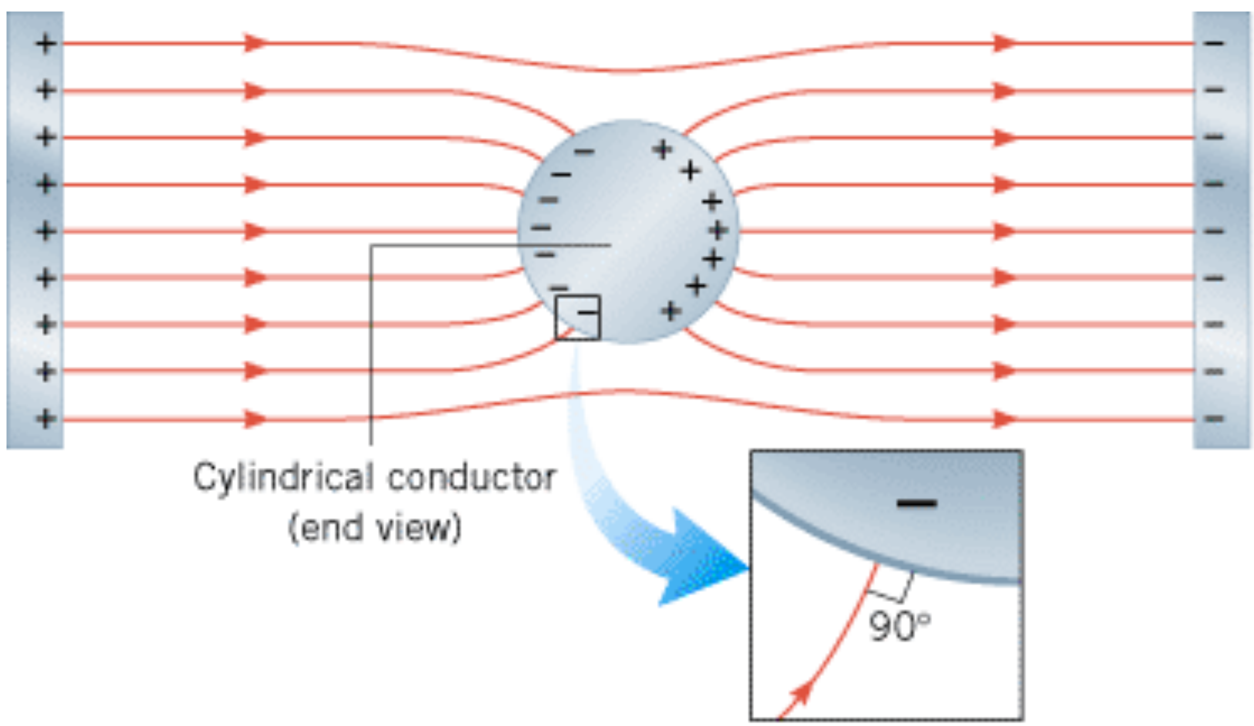
- The electric field is $E = F_e/q$
  - In other words, the electrostatic force per charge on a point charge ("test charge")
  - The electric field can be visualized by drawing "electric field lines"





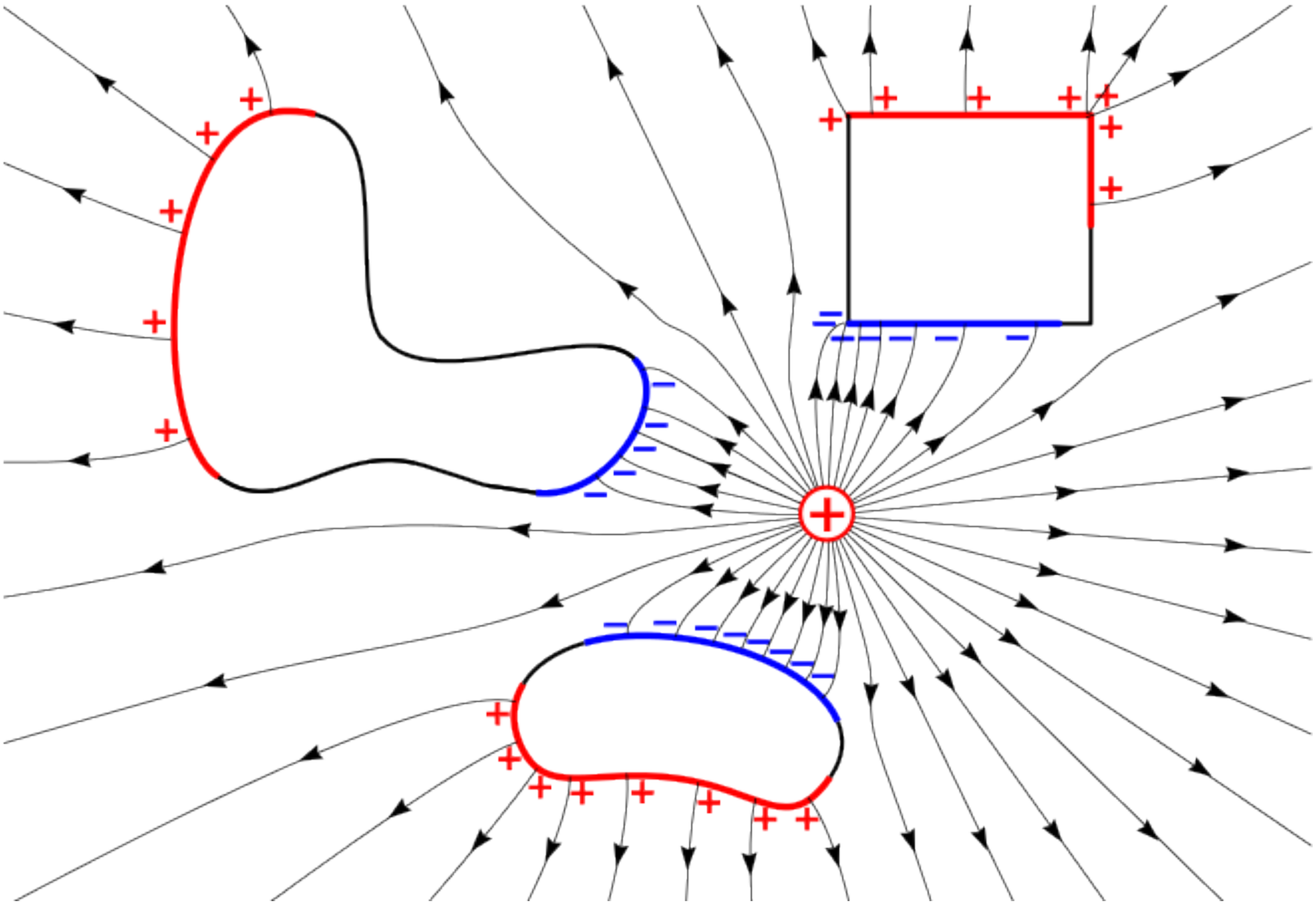
Electric Fields in a Conductor

- None!
  - Any electric field in a conductor causes charge to move in such a way that it shorts out that field
  - Also, electric fields must be perpendicular to the surface (or else they would move charge along the surface)



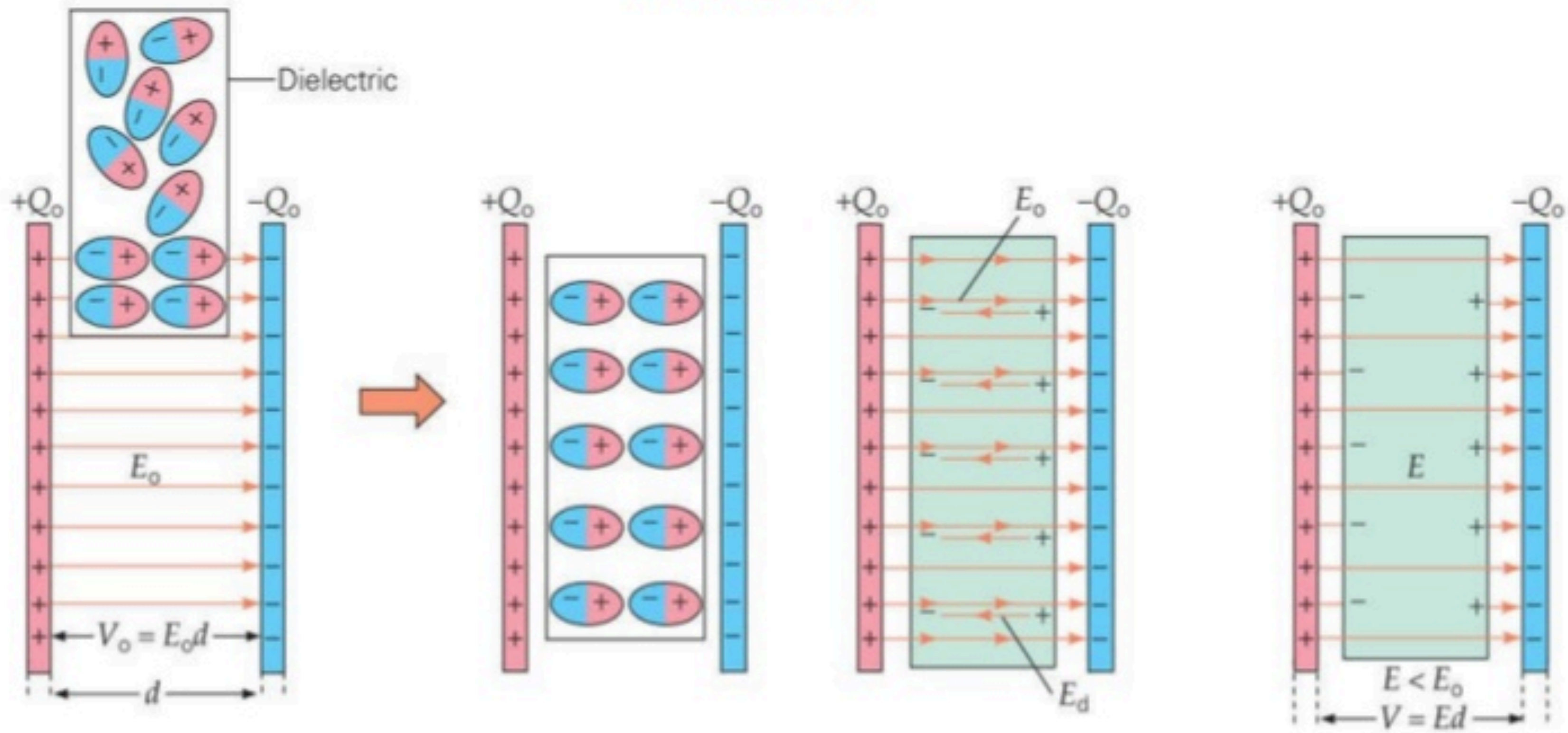
(a)

Electric Fields in a Conductor



Electric Fields in an Insulator

A dielectric in an electric field becomes polarized; this allows it to reduce the electric field in the gap for the same potential difference.



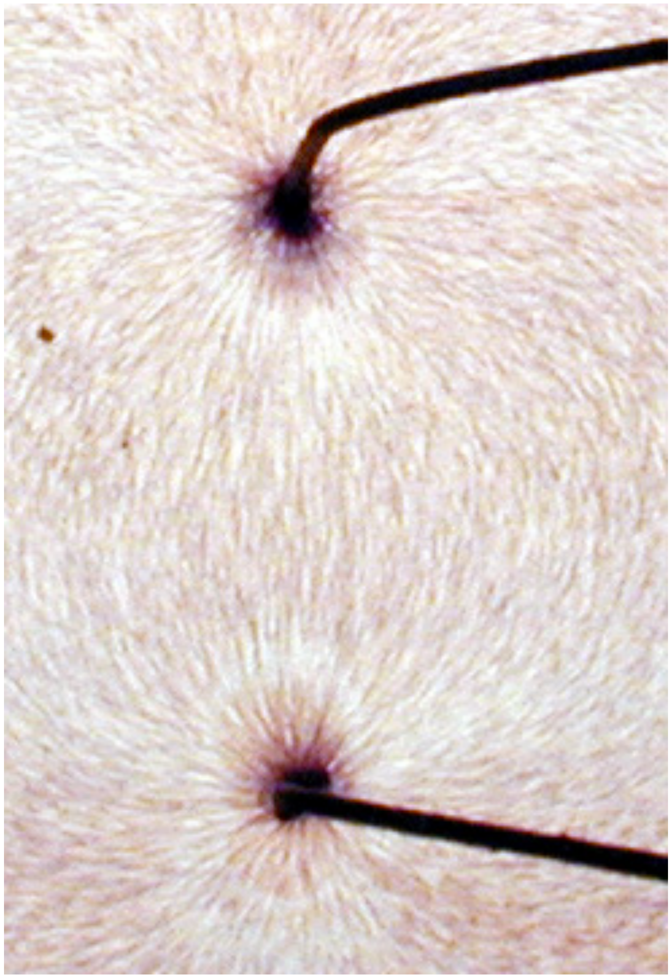
Charged capacitor

Charged capacitor with dielectric inserted

Electric field diagram of same

Effect on electric field and voltage

Electric Field Demo



Computing Electric Fields

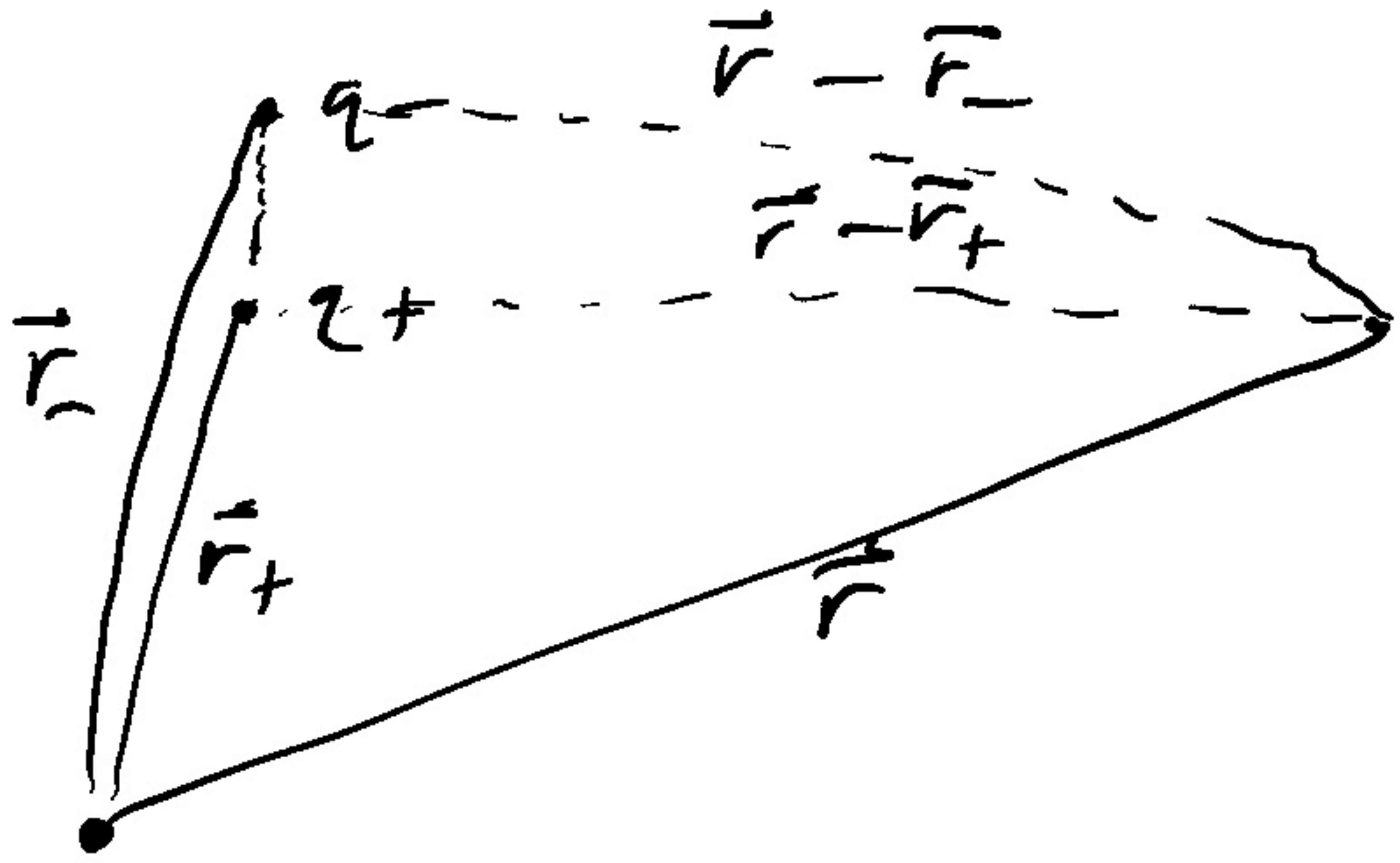
- Basic Idea:
  - Add electric fields from all individual charges
    - Each individual electric field is given by Coulomb's law
- For continuous charge distributions
  - Must add electric fields from infinitesimal bits of charge
  - This means integration

Electric Dipoles

- We have seen a dielectric (insulator) modeled as a bunch of +/- charge pairs
- It turns out that this configuration comes up over and over again in physics

- This should not be surprising:
  - Consider a hydrogen atom = one proton + one electron (most common atom in the universe)
  - This is just a little dipole!

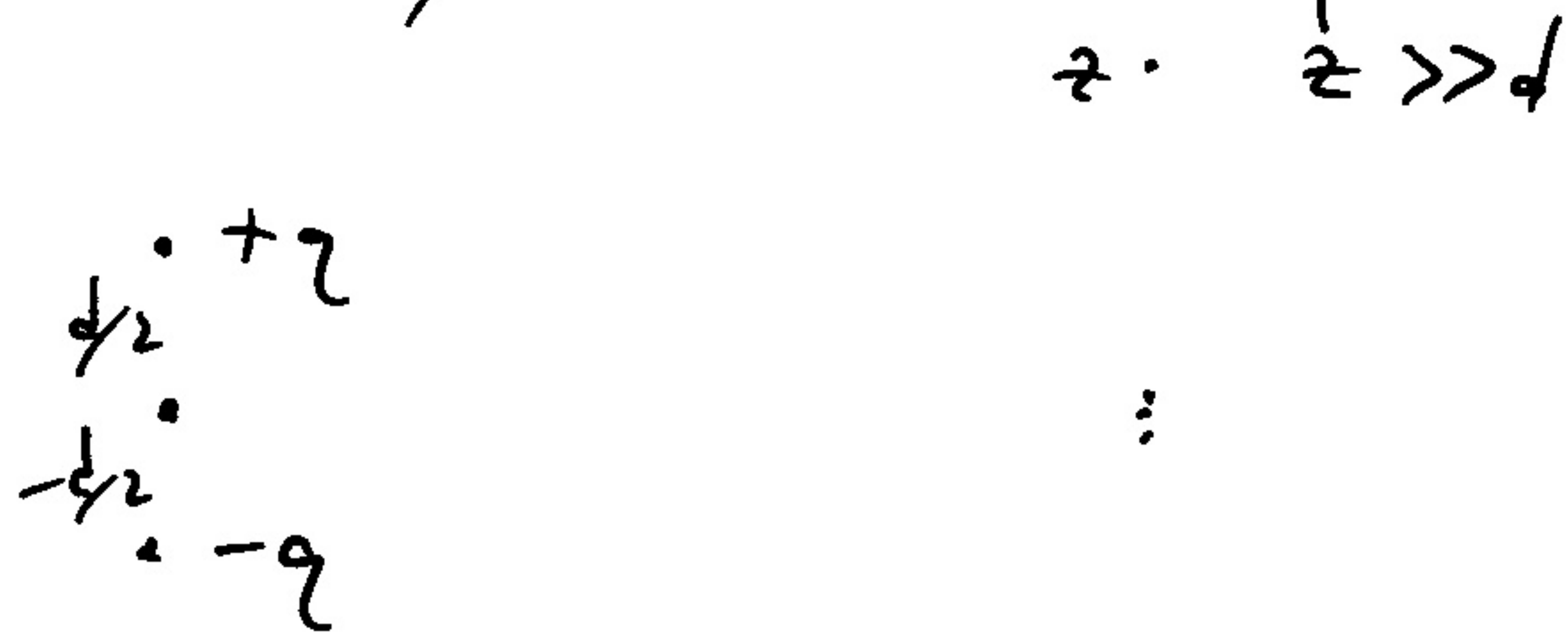
Dipole



$$\vec{E}(\vec{r}) = \frac{q}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}_+}{|\vec{r} - \vec{r}_+|^3} - \frac{q}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}_-}{|\vec{r} - \vec{r}_-|^3}$$

- Can solve this generally but let's look @ special cases.

- Put origin @ dipole center



- Put $\vec{r}$ on z-axis w/ $z \gg d$

$$E_z(z) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(z - d/2)^2} - \frac{1}{(z + d/2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[\frac{1}{(1 - d/2z)^2} - \frac{1}{(1 + d/2z)^2}\right]$$

Taylor Expand

$$\frac{1}{(1 + d/2z)^2} \sim 1 - 2\,(d/2z) \cdots$$

$$\frac{1}{(1 - d/2z)^2} \sim 1 + 2\,(d/2z) \cdots$$

so $E_z\,(z \gg d)$

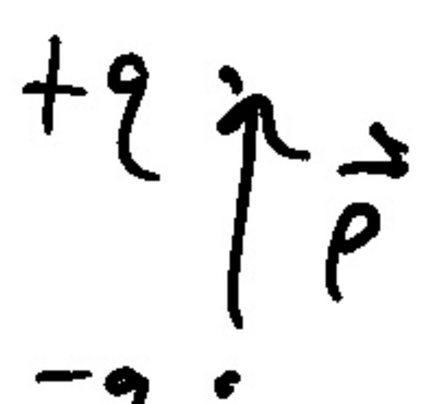
$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[1 + d/z - (1 - d/z)\right]$$

$$= q/4\pi\varepsilon_0 z^2 \cdot 2d/z$$

$$= qd/(2\pi\varepsilon_0 z^3)$$

$qd = p$ = dipole moment

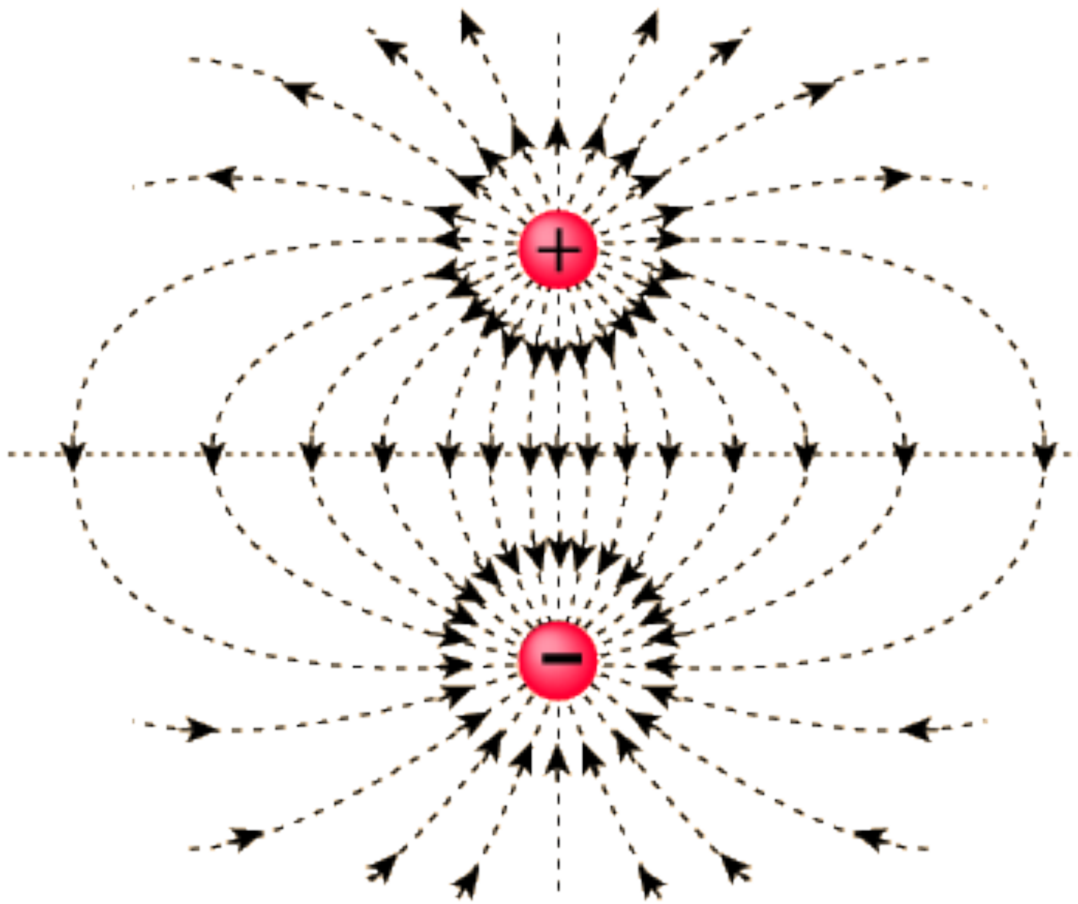
$\vec{p}$ = vector w/ length p from $-q$ to $+q$



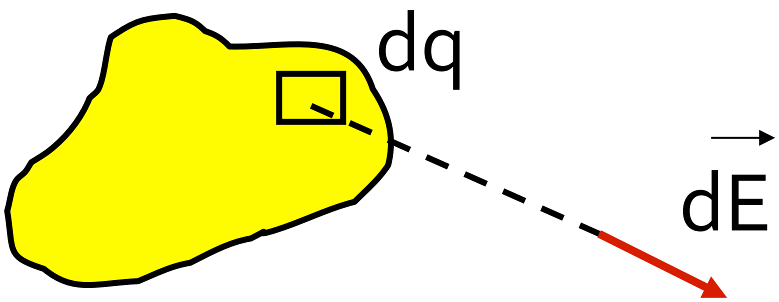
$$\boxed{E = \vec{p}/(2\pi\varepsilon_0 z^3) \text{ along axis}}$$

- off axis different, but still falls off as $1/r^3$

Dipole Electric Field



Continuous Distribution of Charge



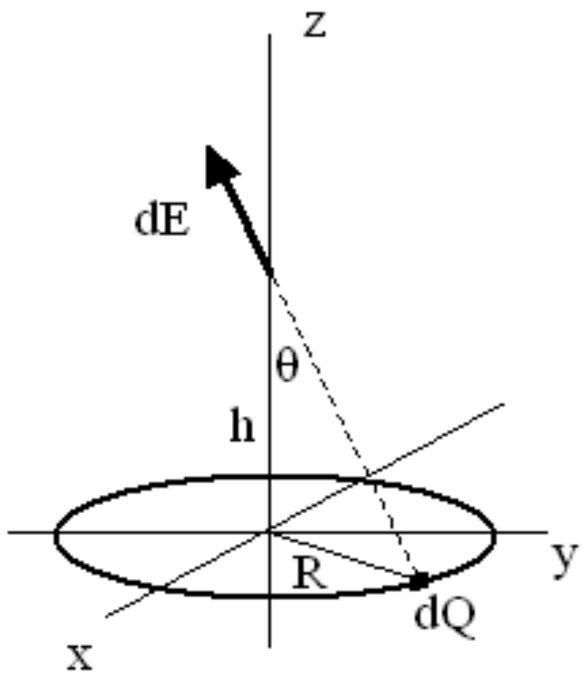
$$\vec{E}_{net} = \sum_i \vec{E}_i$$

$$\vec{E}_{net} = \int d\vec{E} = \int \left(\frac{k\,dq}{r^2} \right) \hat{r}$$

Concept Check

A circular ring of radius R, uniformly charged with total charge +Q, is in the xy plane centered on the origin.

The electric field $d\vec{E}$ at position z = h on the z-axis, due to a small piece of the ring with charge dQ, is shown. What is the magnitude of the field dE?



A: $\dfrac{kQ}{h^2}$ B: $\dfrac{k\,dQ}{h^2}$

C: $\dfrac{k\,dQ}{R^2+h^2}$ D: $\dfrac{k\,dQ}{\sqrt{R^2+h^2}}$

E: None of these.

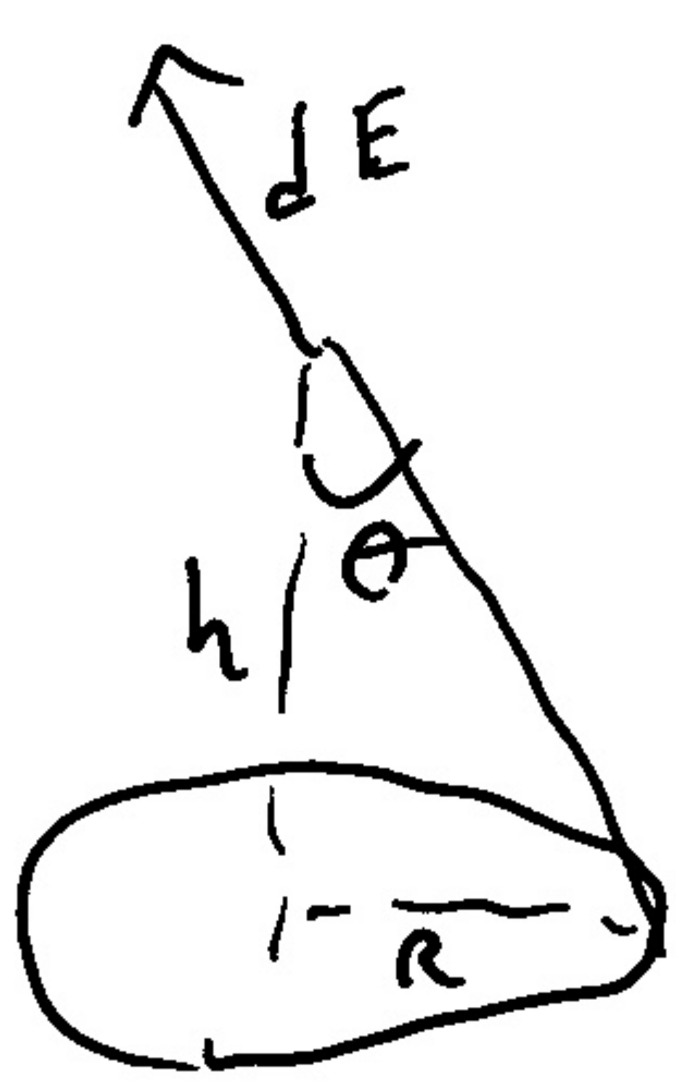
Ring

$$dE = |d\vec{E}| = k\,dQ/(R^2+h^2)$$

$$= 1/(4\pi\varepsilon_0)\; dQ/(R^2+h^2)$$

dE_x and dE_y cancel

$$dE_z = dE\cos\theta = dE\; h/\sqrt{R^2+h^2}$$



$$= \frac{1}{4\pi\varepsilon_0}\frac{dQ\,h}{(R^2+h^2)^{3/2}}$$

could jump to answer by seeing that each dE is the same

Or, put $dQ = \lambda\, dl$

w/ λ = charge density $= Q/L = Q/2\pi R$

Then $E_z = \int dE_z = \int \frac{1}{4\pi\varepsilon_0}\frac{h\lambda\, dl}{(R^2+h^2)^{3/2}}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda h}{(R^2+h^2)^{3/2}}\int dl$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda h L}{(R^2+h^2)^{3/2}}$$ w/ $L = 2\pi R$

But $\lambda L = Q$

so $$\boxed{\vec{E} = E_z\hat{z} = \frac{Qh}{4\pi\varepsilon_0(R^2+h^2)^{3/2}}}$$