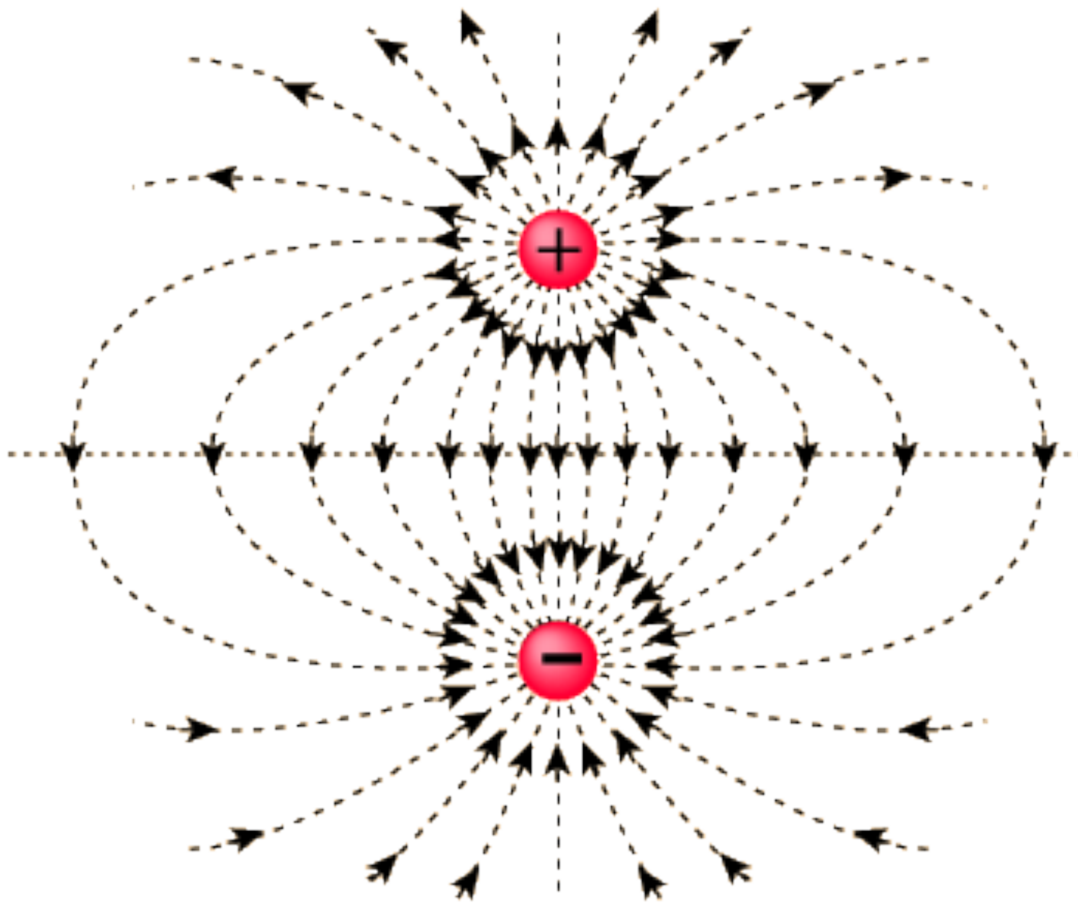
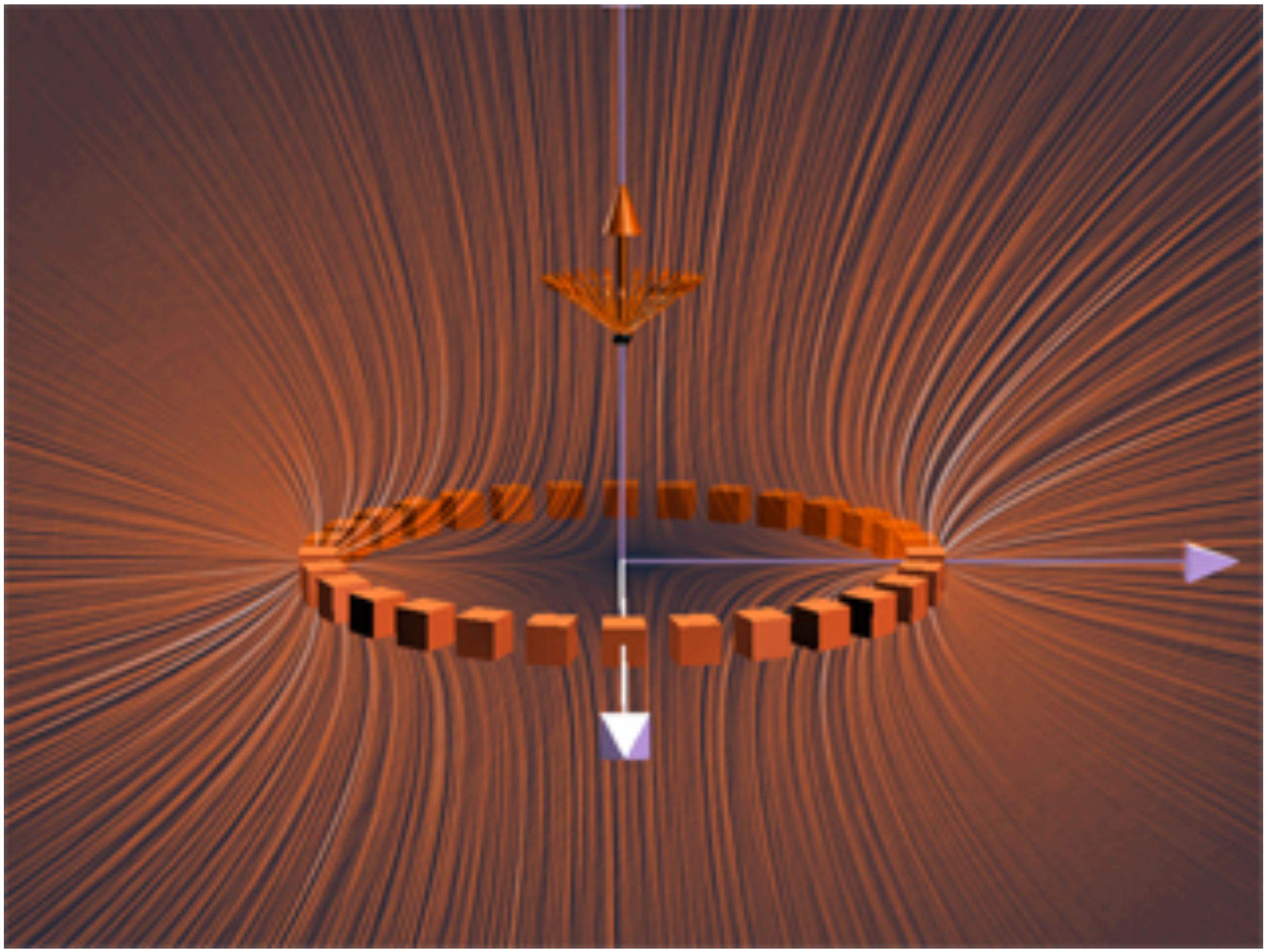
Physics II: 1702
Gravity, Electricity, & Magnetism

Professor Jasper Halekas
Van Allen 70 [Clicker Channel #18]
MWF 11:30-12:30 Lecture, Th 12:30-1:30 Discussion

Dipole Electric Field

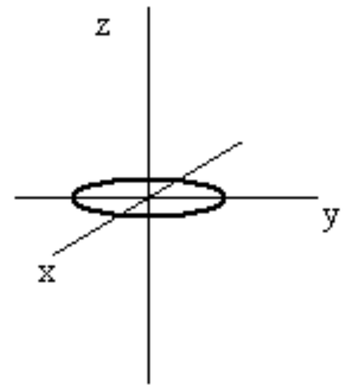


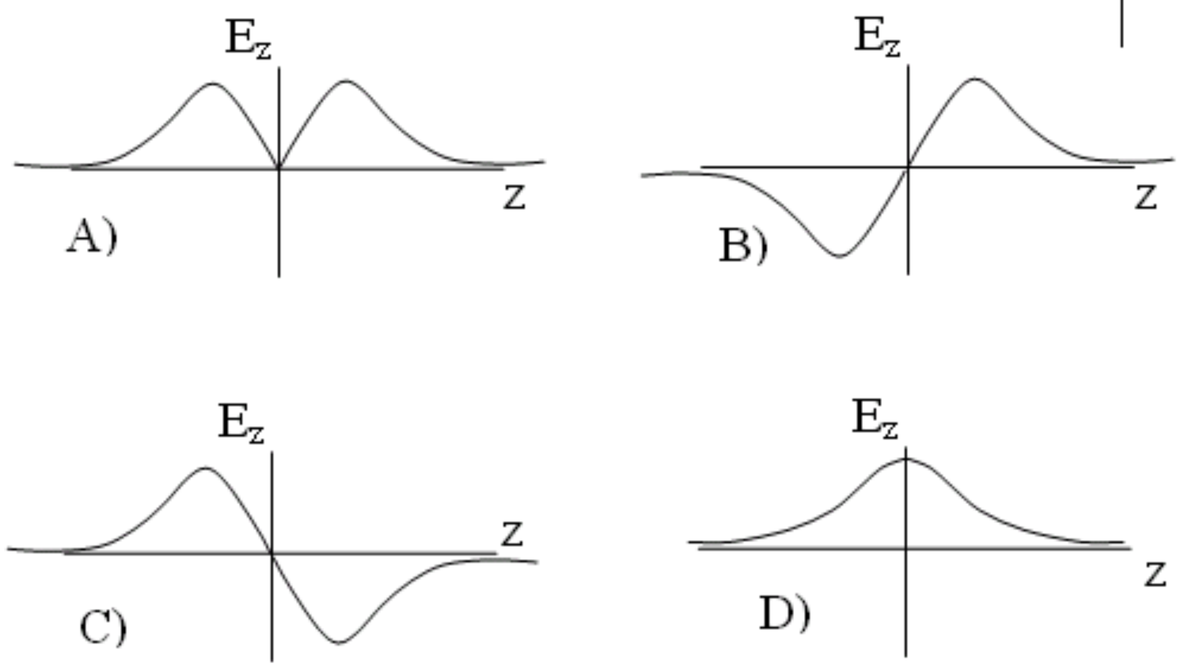
Field of Charged Ring



Concept Check

A circular ring uniformly charged with positive charge Q is in the xy plane centered on the origin as shown. On the z-axis, $\vec{E} = E_z \hat{z}$. Which graph accurately represents the electric field E_z on the z-axis?





E: None of these is an accurate representation of E_z

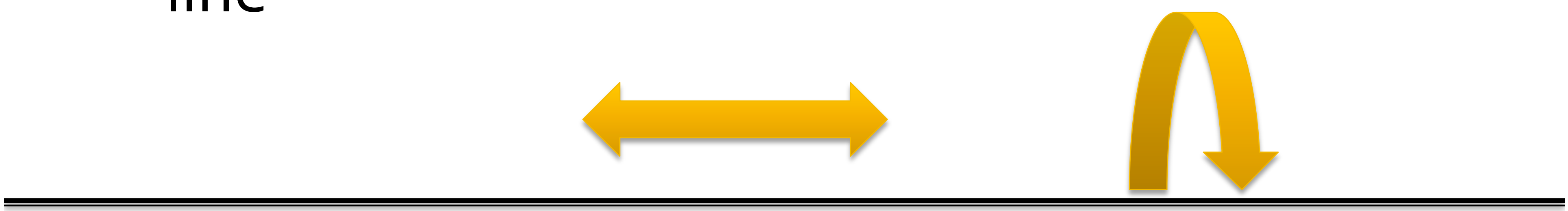
Symmetry

- Symmetry allowed us to conclude that the field at a point along the axis from a ring must be along the axis (since no matter how you rotate the ring, it looks the same from the axis, and so must the field)

- We can use similar arguments for the field of an infinite line of charge, or an infinite plane

Infinite Line Symmetry

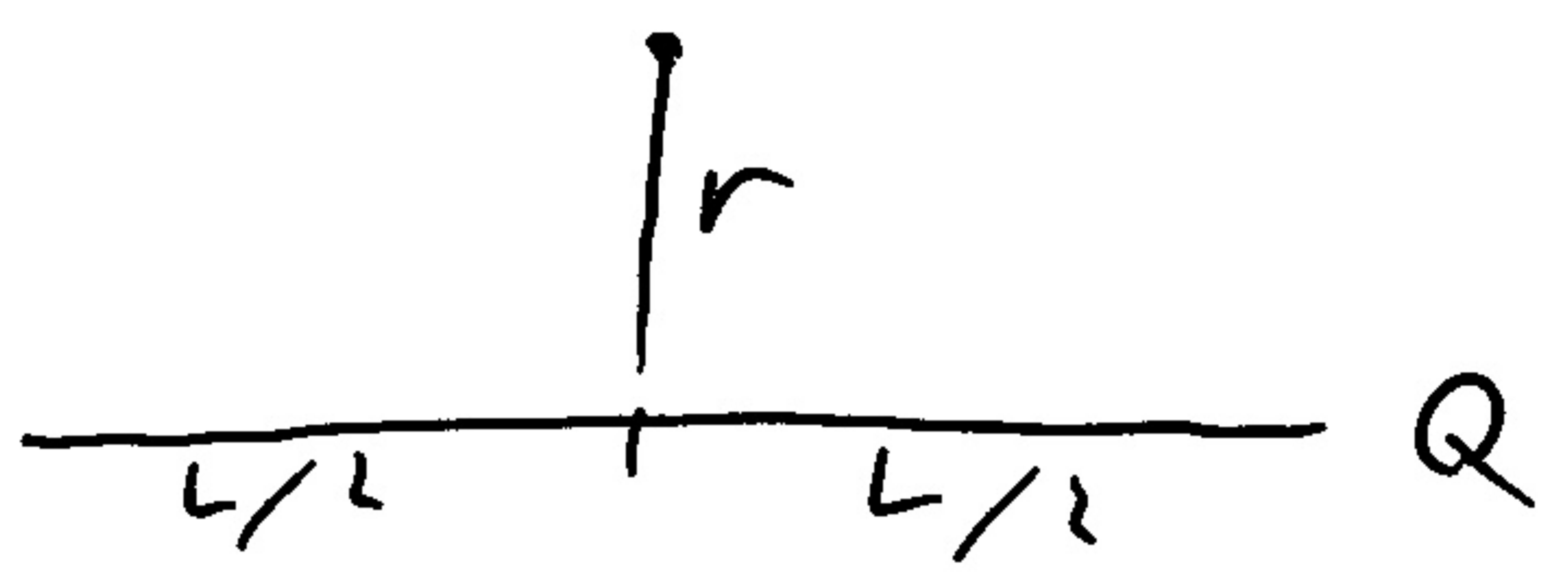
- At a given distance r from an infinite line, it looks the same no matter how far you move along the line, or how you rotate around the line



  - This implies that the field must be radially outward from the line of charge

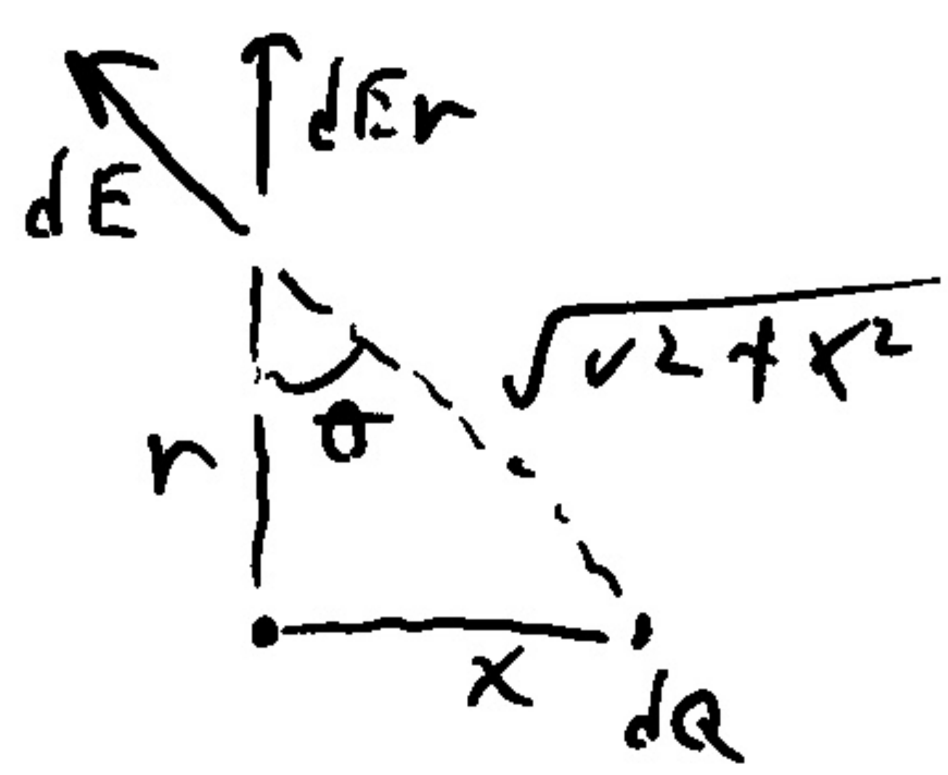
Line

Field above midpoint of line of charge.



$$\lambda = Q/L$$

— By symmetry, field must be out from line



$$dE = \frac{1}{4\pi\varepsilon_0} \frac{dQ}{(r^2+x^2)}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\lambda\, dx}{r^2+x^2}$$

$$dE_r = dE \cos\theta$$

$$= \frac{\lambda\, dx \cdot r}{4\pi\varepsilon_0 (r^2+x^2)^{3/2}}$$

$$E_r = \int_{-L/2}^{L/2} \frac{\lambda \, dx \, r}{4\pi\varepsilon_0 (x^2+r^2)^{3/2}}$$

Look up $\int dx/(x^2+r^2)^{3/2}$

$$= \frac{x}{r^2\sqrt{x^2+r^2}}$$

$$E_r = \frac{\lambda r}{4\pi\varepsilon_0} \frac{x}{r^2\sqrt{x^2+r^2}} \Bigg|_{-L/2}^{L/2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\left[\frac{L/2}{\sqrt{r^2+(L/2)^2}} - \frac{-L/2}{\sqrt{r^2+(L/2)^2}}\right]$$

$$= \frac{\lambda L}{4\pi\varepsilon_0 r \sqrt{r^2+(L/2)^2}}$$

$$= \boxed{\frac{Q}{4\pi\varepsilon_0 r\sqrt{r^2+L^2/4}}}$$

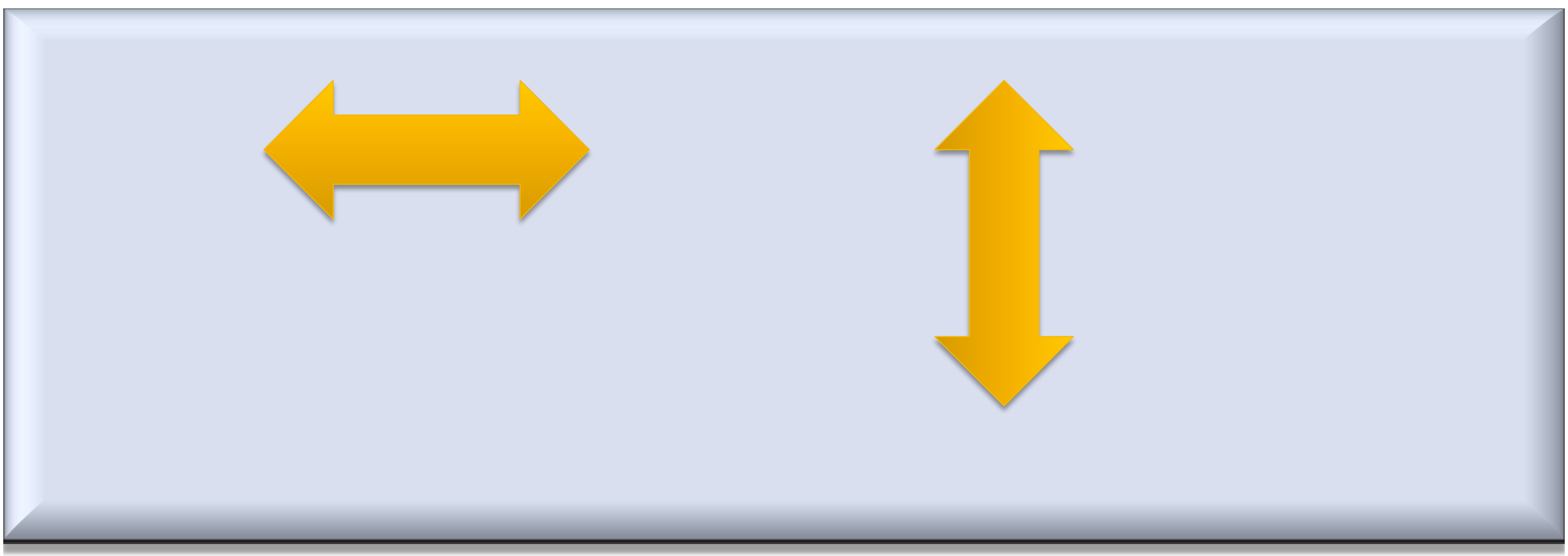
Infinite Line (or $L \gg r$)

$$L/\sqrt{r^2+L^2/4} \rightarrow L/\sqrt{L^2/4} = L/L/2 = 2$$

$$E_r \rightarrow \frac{\lambda \cdot 2}{4\pi\varepsilon_0 r} = \boxed{\frac{\lambda}{2\pi\varepsilon_0 r}}$$

Infinite Plane Symmetry

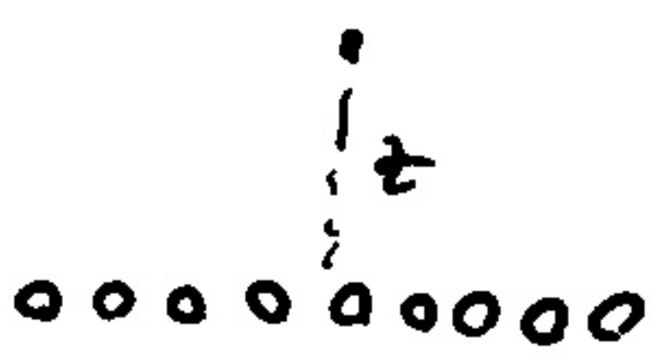
- At a given distance from an infinite plane of charge, no matter how far you move laterally in any direction, it looks the same
  - This implies that the field is directly out of the plane everywhere
  - That in turn implies that the field must be constant!

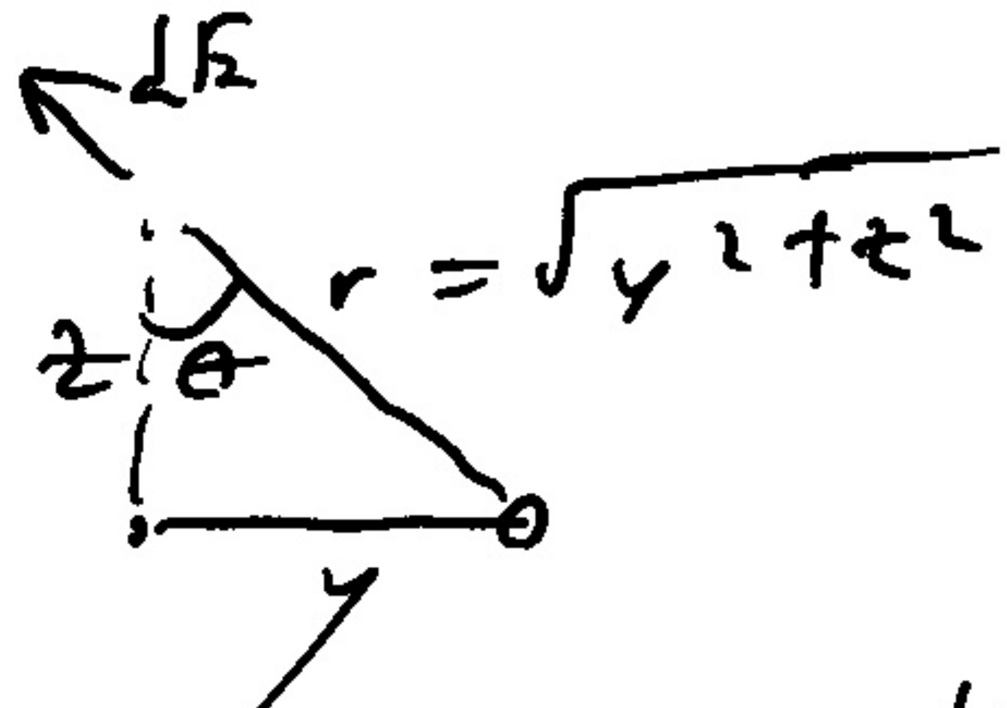


Infinite plane

Build up from lines

- λ is charge density of line
- we need σ = charge density of plane





- we can put

$$dE = \lambda / 2\pi\varepsilon_0 r \rightarrow \frac{\sigma\, dy}{2\pi\varepsilon_0 r}$$

$$dE_z = dE \cos\theta$$
$$= dE \cdot z / \sqrt{y^2+z^2}$$

$$E_z = \int dE_z = \int \frac{\sigma\, dy\, z}{2\pi\varepsilon_0 (y^2+z^2)}$$

$$= \frac{\sigma z}{2\pi\varepsilon_0} \int dy/(y^2+z^2)$$ Look up integral!

$$= \frac{\sigma z}{2\pi\varepsilon_0} \cdot 1/z \tan^{-1}(y/z) \Big|_{-\infty}^{\infty}$$

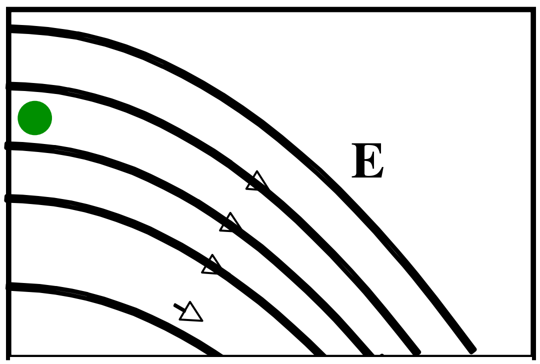
$$= \frac{\sigma}{2\pi\varepsilon_0} \cdot (\pi/2 - (-\pi/2))$$

$$= \boxed{\sigma/2\varepsilon_0}$$ Constant!

Electric Fields: So What?

- Particles follow F = ma, with F given by qE
  - => a = dv/dt = qE/m
- Particles with opposite charges go in opposite directions
  - This means an electric field can cause a net current (flow of charge)

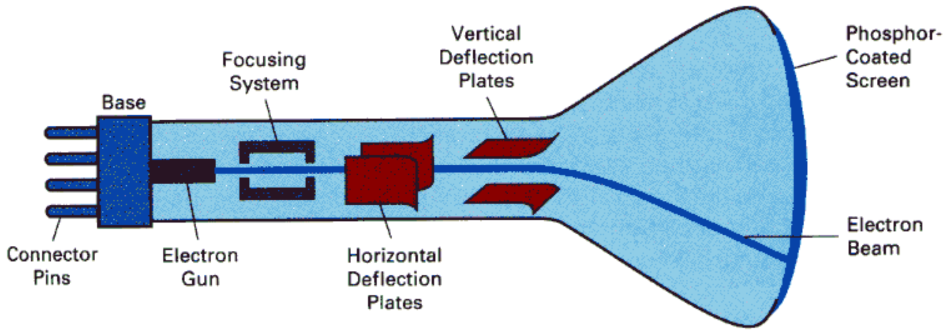
Concept Check



A proton starts at rest at the green spot.
How will it move?

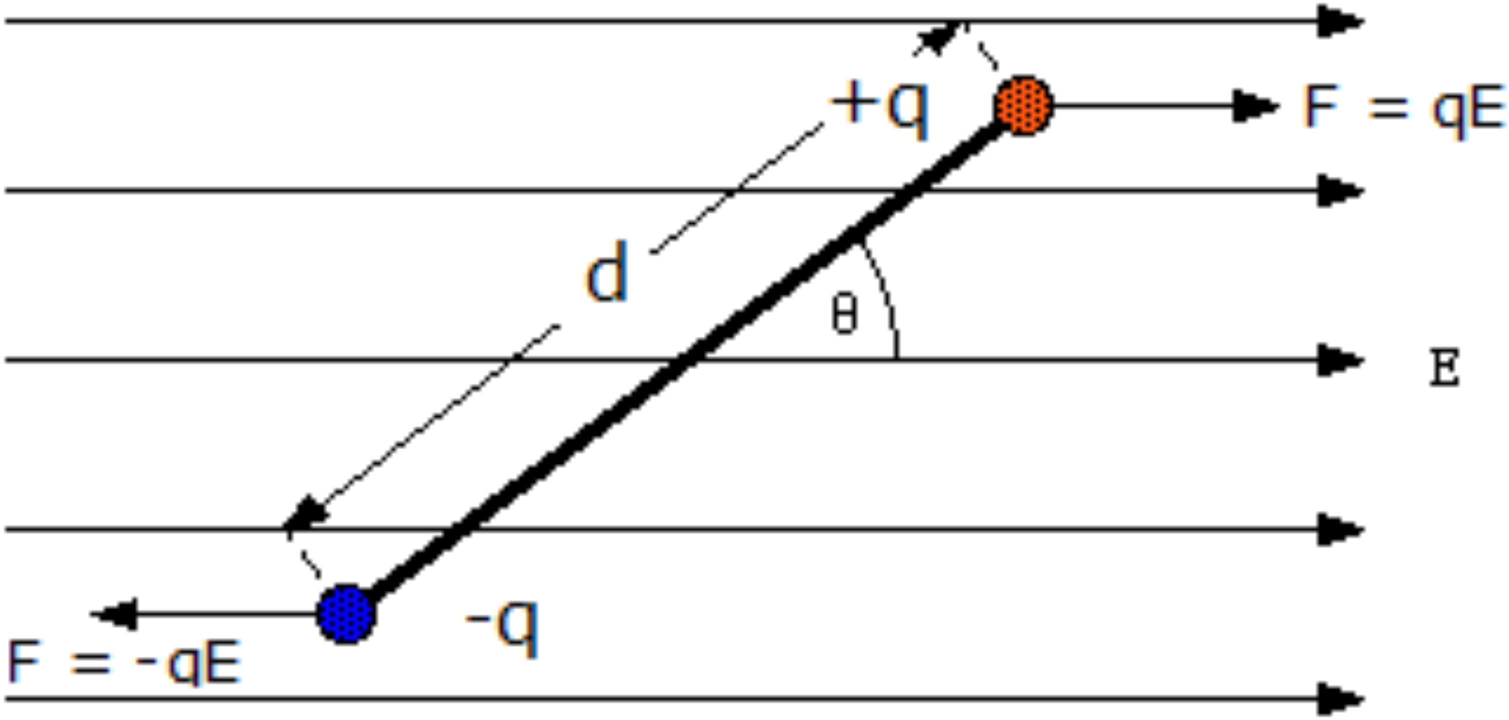
A) Stays at rest
B) Follows a path between the field lines
C) It moves to the right, curving downward but ultimately crossing a field line

Applications of Bending Electrons



Cathode Ray Tube = Old-School Television

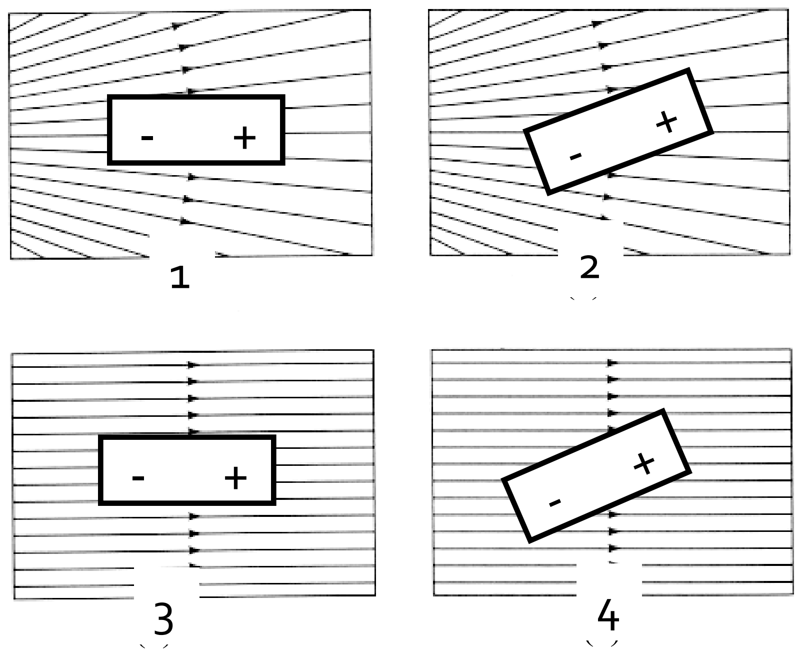
Force and Torque on a Dipole



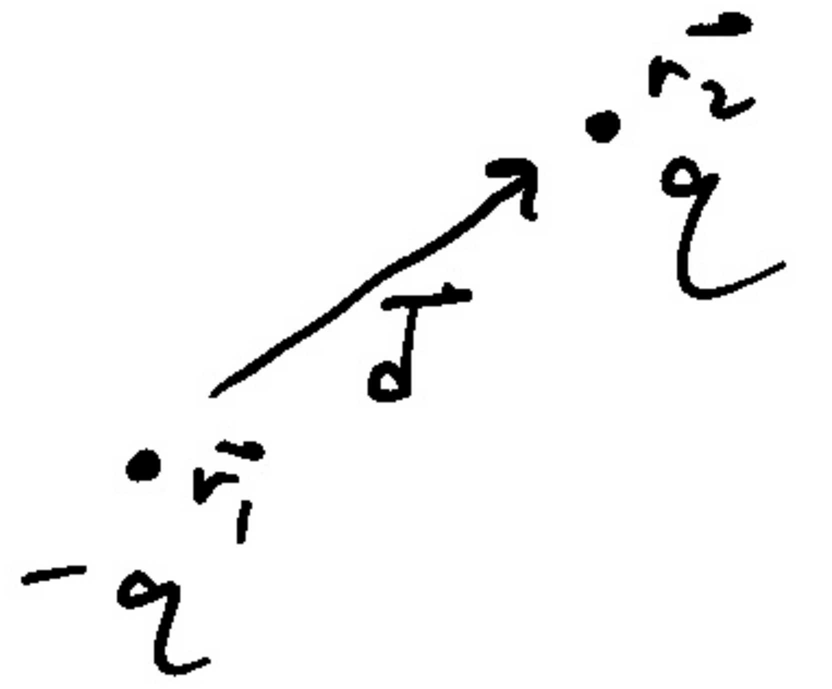
Dipole Concepts I

A dipole is placed in an external field as shown. In which situation(s) is there a net force on the dipole?

A) 1 only
B) 2 only
C) 1 and 2
D) 3 and 4
E) 2 and 4



Force on Dipole



$\vec{p} = q\vec{d} = q(\vec{r_2} - \vec{r_1})$

- Assume dipole held together
- Then $\vec{F}_{dip} = \vec{F}_{-q} + \vec{F}_q$

$$= -q\vec{E}(\vec{r_1}) + q\vec{E}(\vec{r_2})$$

- Do local expansion around point $\vec{r_1}$

In 1-d $\quad f(x+dx) = f(x) + f'(x)dx \ldots$

In 3-d $\quad f(\vec{r} + \delta\vec{r}) = f(\vec{r}) + \nabla f \cdot \Delta\vec{r}$

so $\quad \vec{E}(\vec{r_2}) \sim \vec{E}(\vec{r_1}) + (\vec{r_2} - \vec{r_1}) \cdot \nabla \vec{E}$

$$\Rightarrow \vec{F}_{dip} \sim -q\vec{E}(r_1) + q\left(\vec{E}(r_1) + (\vec{r_2} - \vec{r_1}) \cdot \nabla \vec{E}\right)$$

$$= q(\vec{r_2} - \vec{r_1}) \cdot \nabla \vec{E}$$

$$= \boxed{\vec{p} \cdot \nabla \vec{E}}$$

Explicit Expression:

$$\vec{F}_{dip} =$$

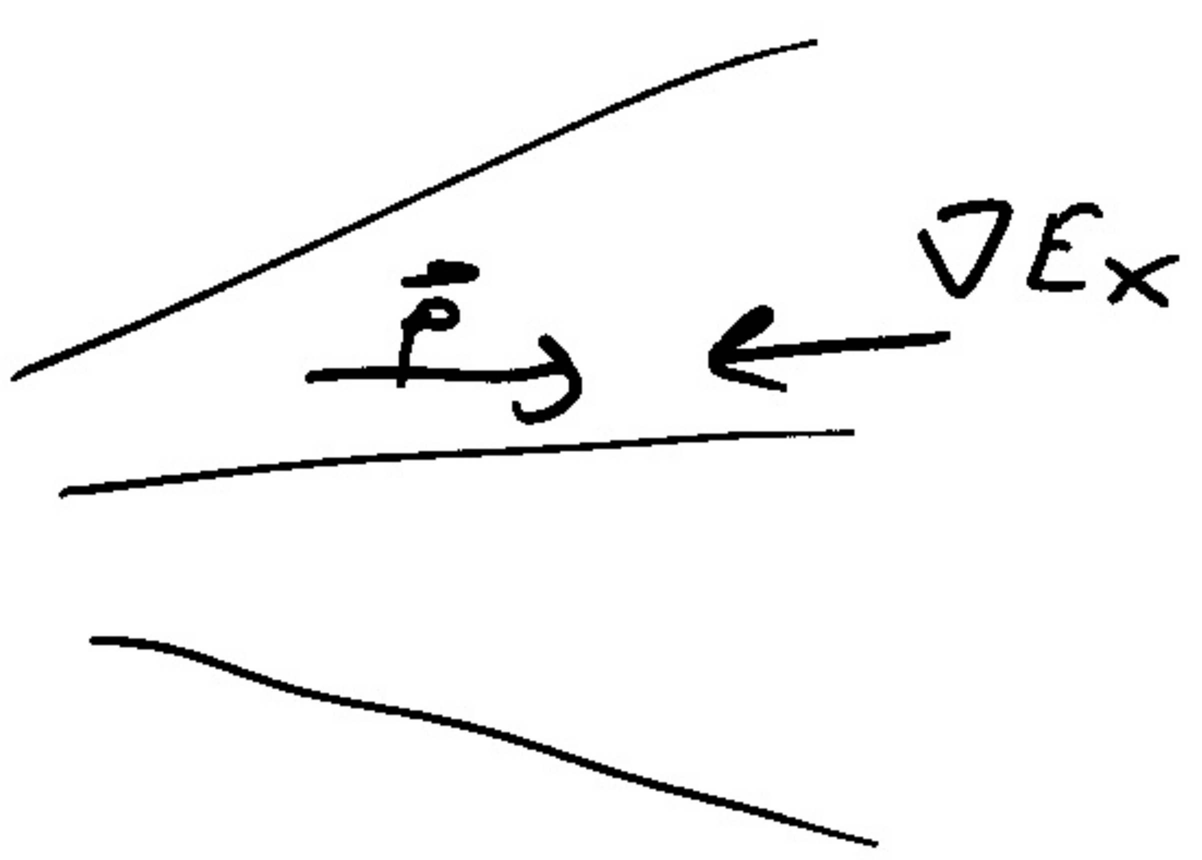
$$[\vec{p}\cdot\nabla E_x, \vec{p}\cdot\nabla E_y, \vec{p}\cdot\nabla E_z]$$

For simplicity, say

$$\vec{p} = p\,\hat{x}$$

$$\vec{F} = [p\,\partial E_x/\partial x, p\,\partial E_y/\partial x, p\,\partial E_z/\partial x]$$

$$F_x = p\,\partial E_x/\partial x$$



$F_x = p\,\partial E_x/\partial x$

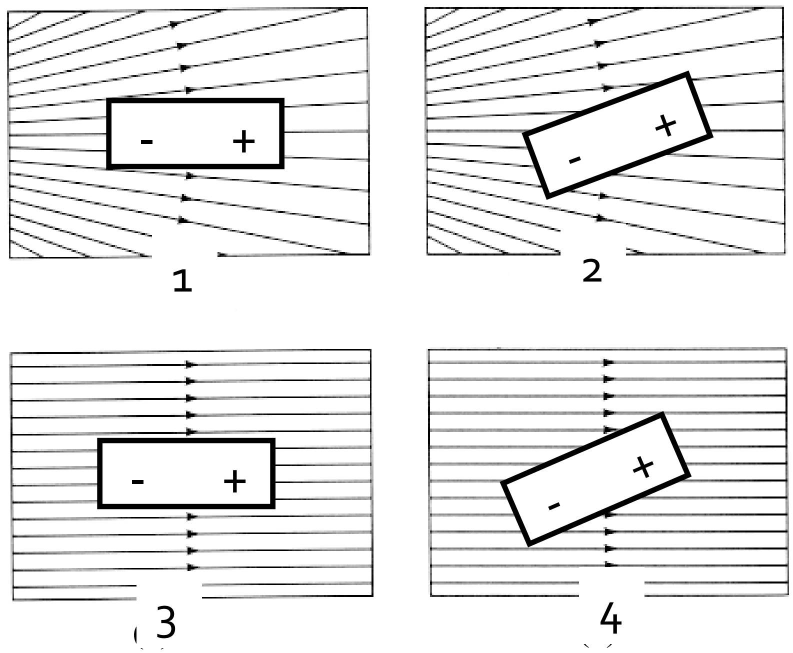
< 1

so to left

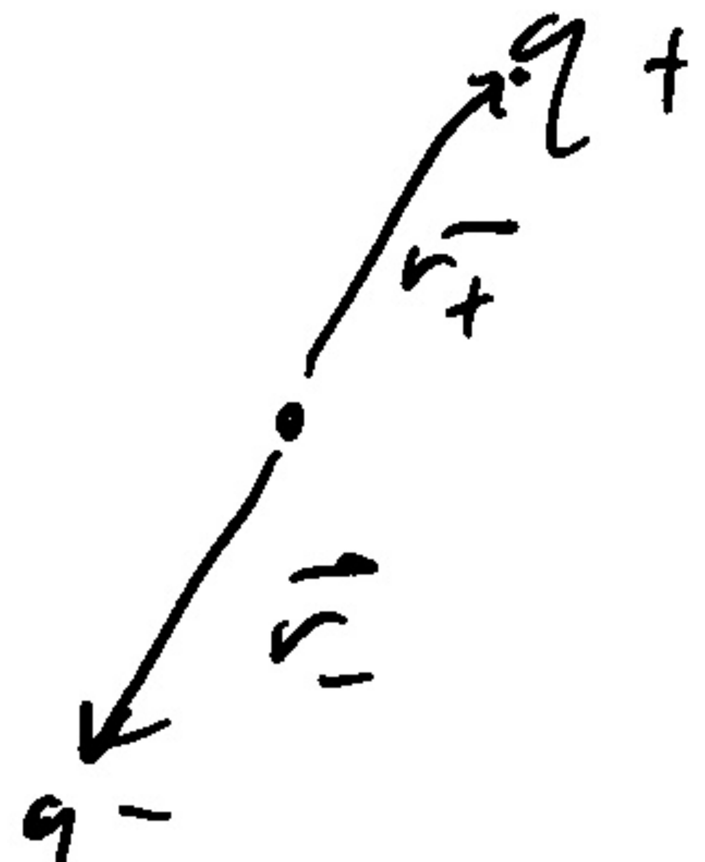
Dipole Concepts II

A dipole is placed in an external field as shown. In which situation(s) is there a net *torque* on the dipole?

A) 1 only
B) 2 only
C) 1 and 3
D) 2 and 4
E) 1 and 2



Torque on Dipole



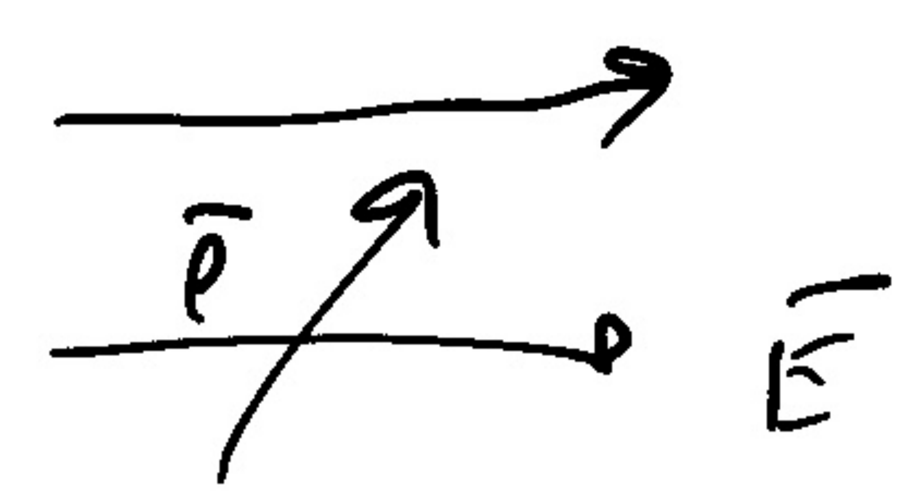
$$\vec{d} = \vec{r}_+ - \vec{r}_-$$
$$\vec{p} = q\vec{d}$$

- put origin @ dipole center for convenience

$$\vec{\tau}_{dip} = \vec{\tau}_- + \vec{\tau}_+$$
$$= \vec{r}_- \times \vec{F}_- + \vec{r}_+ \times \vec{F}_+$$
$$= \vec{r}_- \times -q\vec{E}(\vec{r}_-) + \vec{r}_+ \times q\vec{E}(\vec{r}_+)$$
$$= q(\vec{r}_+ - \vec{r}_-) \times \vec{E}(\vec{r})$$

assuming $\vec{E}(\vec{r}_-) \sim \vec{E}(\vec{r}_+)$

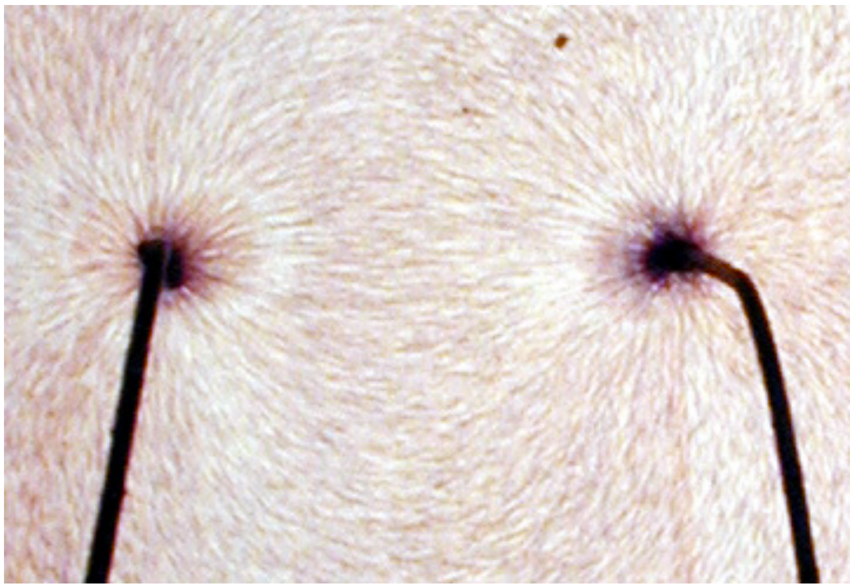
$$= q\vec{d} \times \vec{E}$$
$$= \vec{p} \times \vec{E}$$



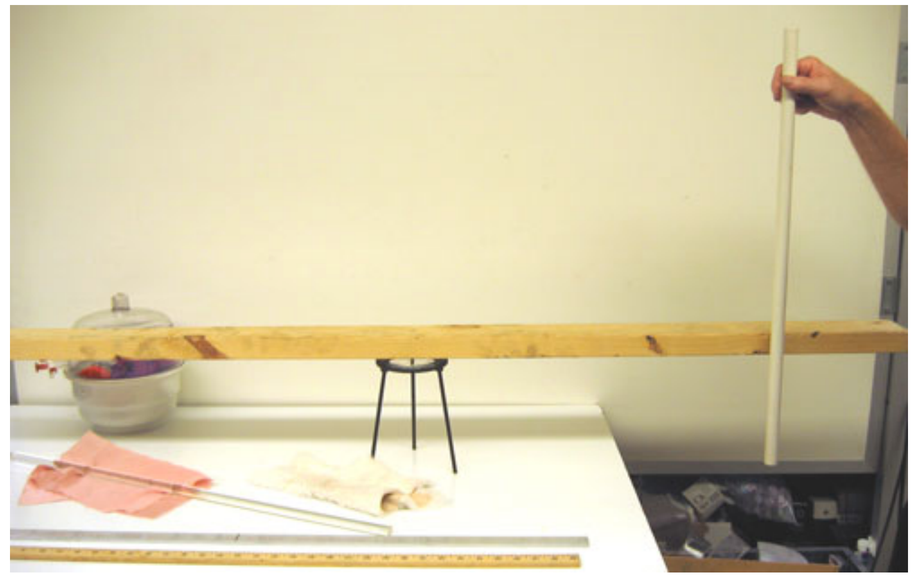
$$\vec{\tau} = \vec{p} \times \vec{E}$$

- into board so clockwise
- rotates $\vec{p}$ to align w/ $\vec{E}$

Dipole Force/Torque Implications



Torque on little dipoles (felt fibers) aligns them with the electric field



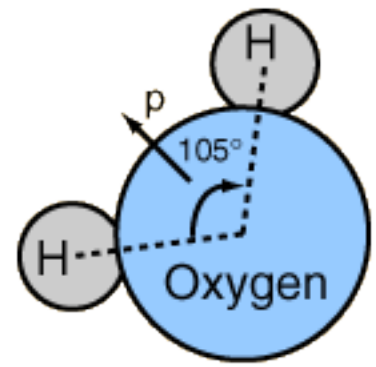
Force on little dipoles (molecules in wood) from diverging electric field of charged object pulls them toward the charge

How Does a Microwave Work?

- Dipoles want to align with the external electric field

- What if you switch the electric field back and forth rapidly?

- The dipoles try to flip too, which generates heat as they rub against other molecules nearby



- Water molecules are especially good dipoles, and they absorb a lot of heat from this process