1\. a. $U_{total} = U_1 + U_2$

$$= -\frac{GMm}{R/2} - \frac{G \cdot 2M \cdot m}{R/2}$$

$$= \boxed{-\frac{6GMm}{R}}$$

b. $F_{total} = F_1 + F_2$

$$= -\frac{GMm}{(R/2)^2} + \frac{G \cdot 2Mm}{(R/2)^2}$$

$$= \boxed{\frac{4GMm}{R^2}} \text{ to the right}$$

c. Need $\dfrac{G \cdot 2M \cdot m}{(R-x)^2} = \dfrac{G \cdot M \cdot m}{x^2}$

or $2x^2 = (R-x)^2 = R^2 - 2Rx + x^2$

or $\boxed{R^2 - 2Rx - x^2 = 0}$

full solution (not required)

is $\left(2R \pm \sqrt{4R^2 + 4R^2}\right)/-2$

$= -R \mp \sqrt{2}R$ want + root

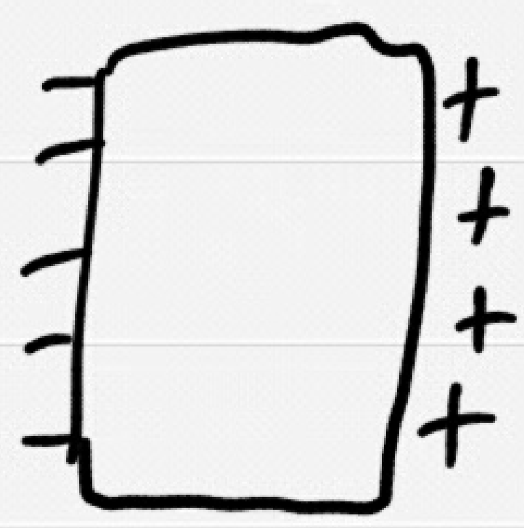
$(\sqrt{2} - 1)R \sim 0.414\ R$

d. No other point w/ $F = 0$

since any other point w/ $F_x = 0$

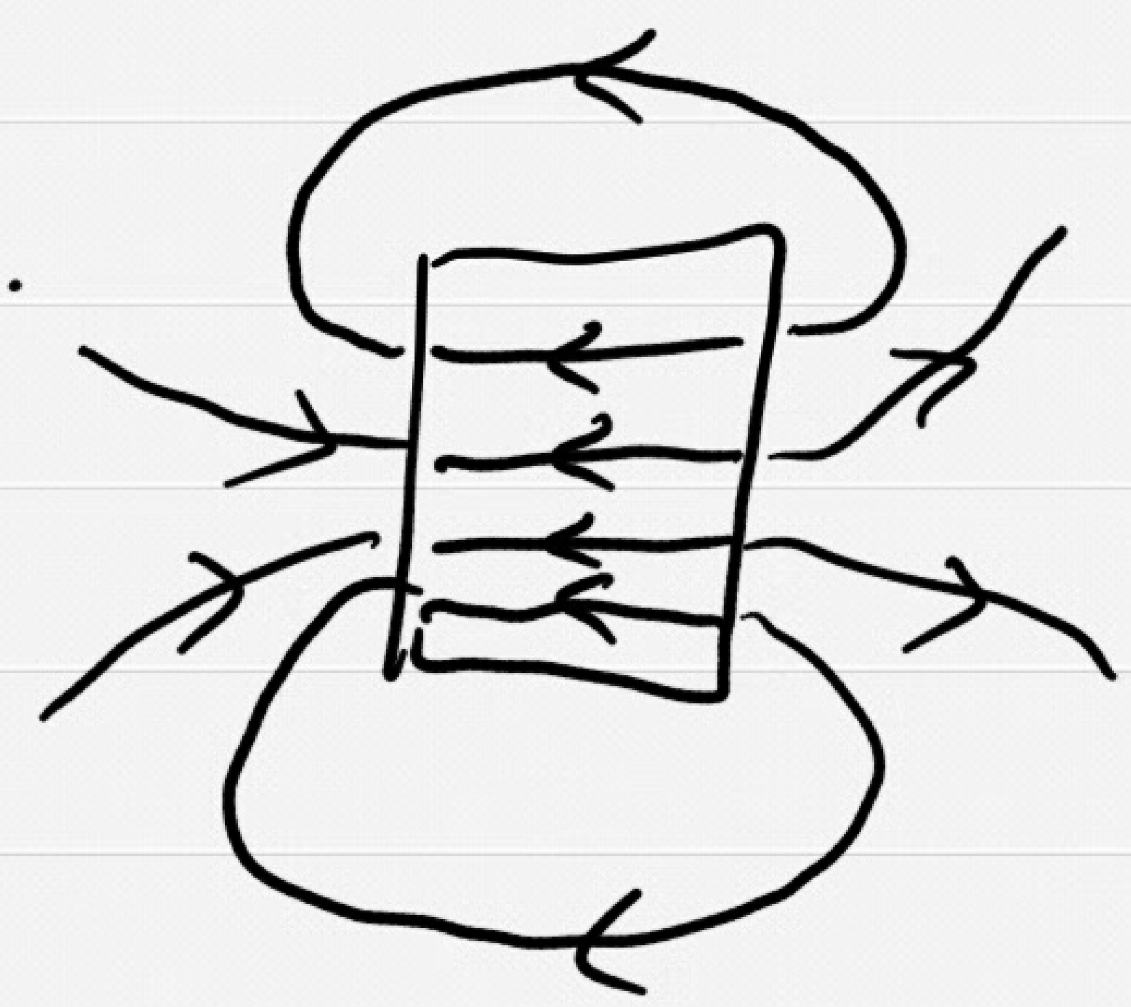
has $F_y \neq 0$

2. A.

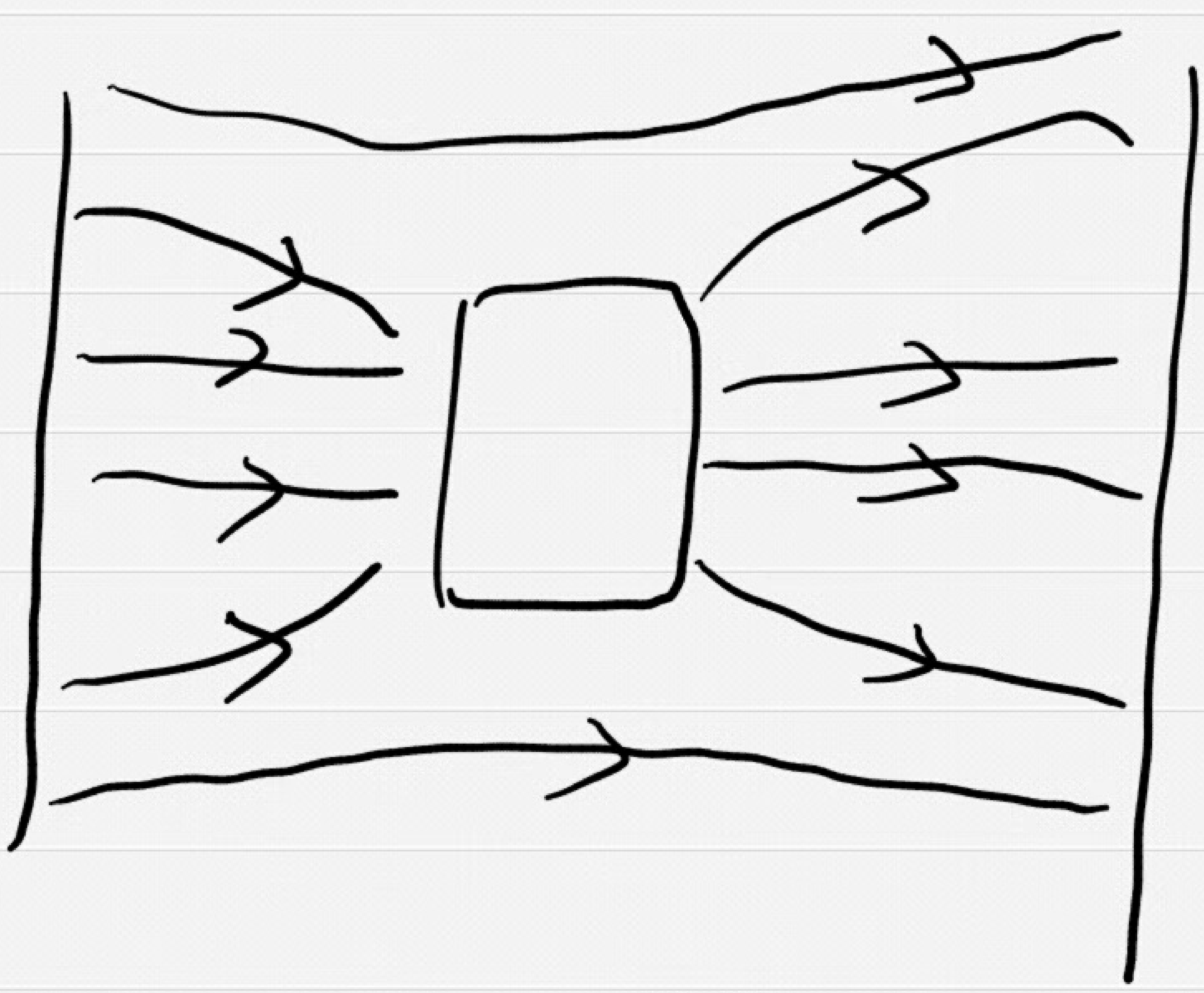


— Charge cancels out applied field

B.

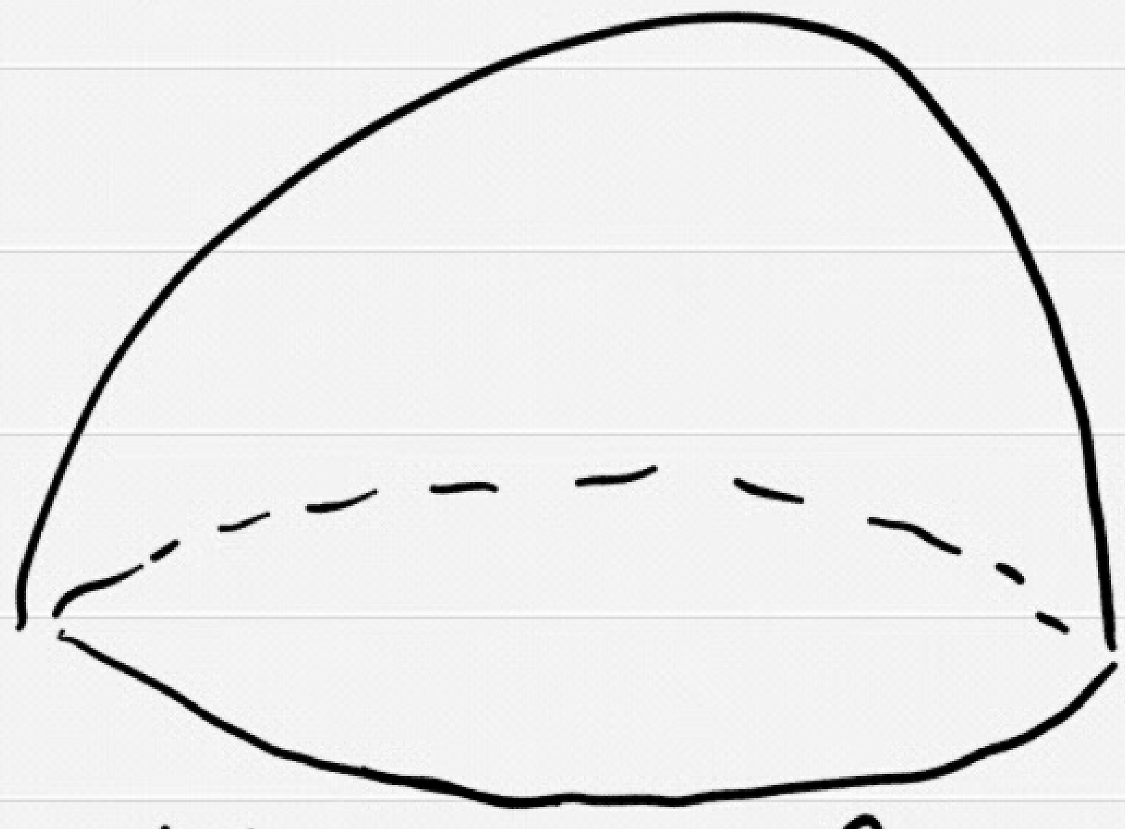


C.



D. In insulator, field wouldn't be entirely canceled, just partially reduced

3.



- Uniform E so same # of field lines go through bottom of bowl as top

$$\int_{bowl} \vec{E} \cdot d\vec{A} = -\int_{lid} \vec{E} \cdot d\vec{A} = \boxed{E \cdot \pi R^2}$$

- since $\vec{E} \parallel d\vec{A}$ for the "lid" that forms the bottom surface the bowl rests on

4. a. $V_{tot} = V_1 + V_2$

$V(\text{ring}) = Q / 4\pi\epsilon_0\sqrt{R^2+(x-x_0)^2}$

(by adding contribution from every point on ring, all at distance $\sqrt{R^2+(x-x_0)^2}$)

so $\boxed{V_A = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+(\ell/2)^2}} + \frac{Q}{4\pi\epsilon_0\sqrt{R^2+(\ell/2)^2}}}$

$$= \frac{2Q}{4\pi\epsilon_0\sqrt{R^2+(5R)^2}} = \frac{Q}{2\pi\epsilon_0 R\cdot\sqrt{26}}$$

$$\sim \frac{Q}{10\pi\epsilon_0 R}$$

b. $\boxed{V_B = \frac{Q}{4\pi\epsilon_0 R} + \frac{Q}{4\pi\epsilon_0\sqrt{R^2+\ell^2}}}$

$$= \frac{Q}{4\pi\epsilon_0 R}\left[1 + \frac{1}{\sqrt{101}}\right]$$

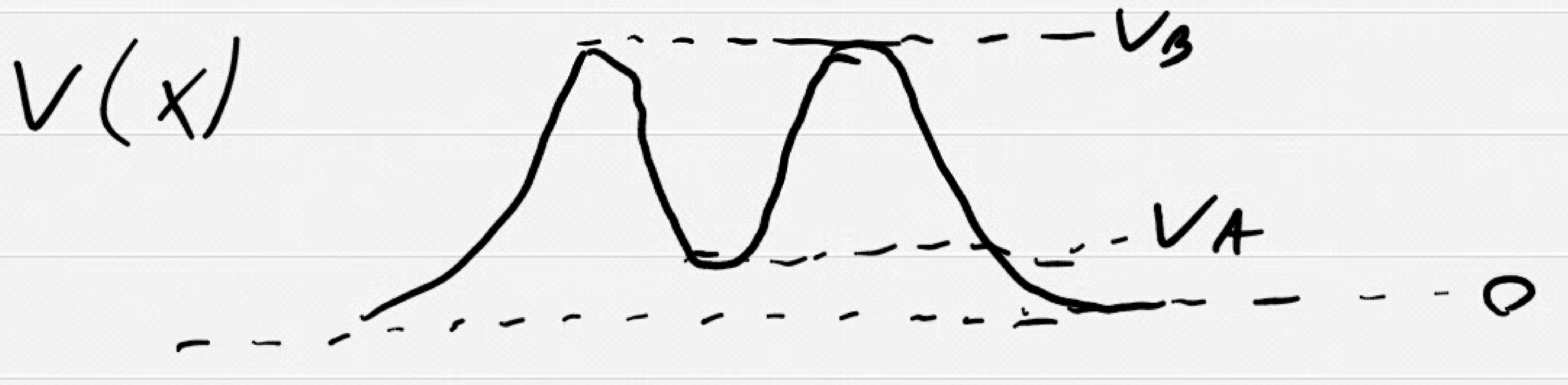
$$\sim Q/4\pi\epsilon_0 R$$

c. need $KE = q(V_B - V_A) = \frac{1}{2}mv^2$

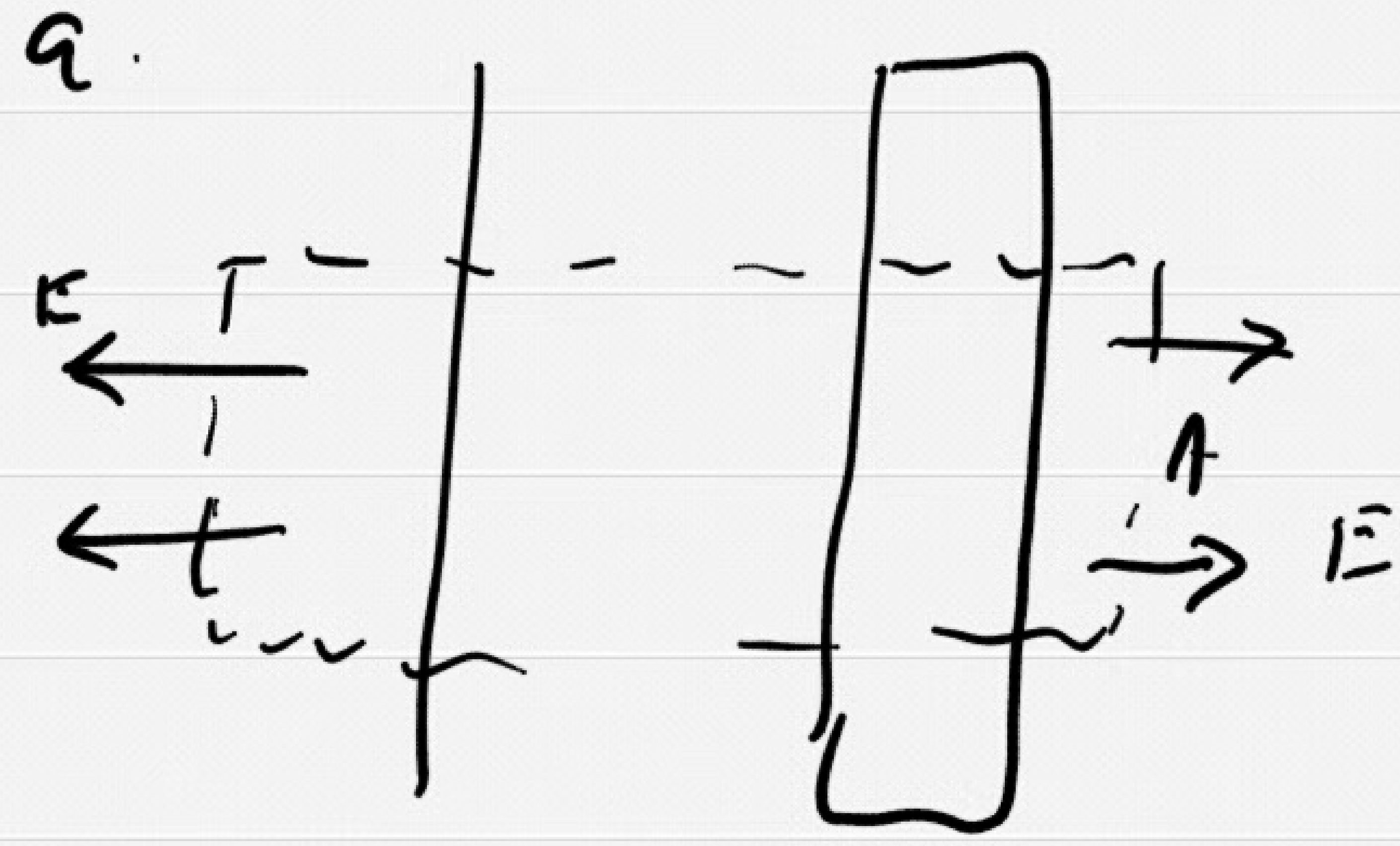
or $\boxed{v_x = \sqrt{2q/m\,(V_B - V_A)}}$

d. need $KE = qV_A$

or $\boxed{v_x = \sqrt{\frac{2q}{m}V_A}}$



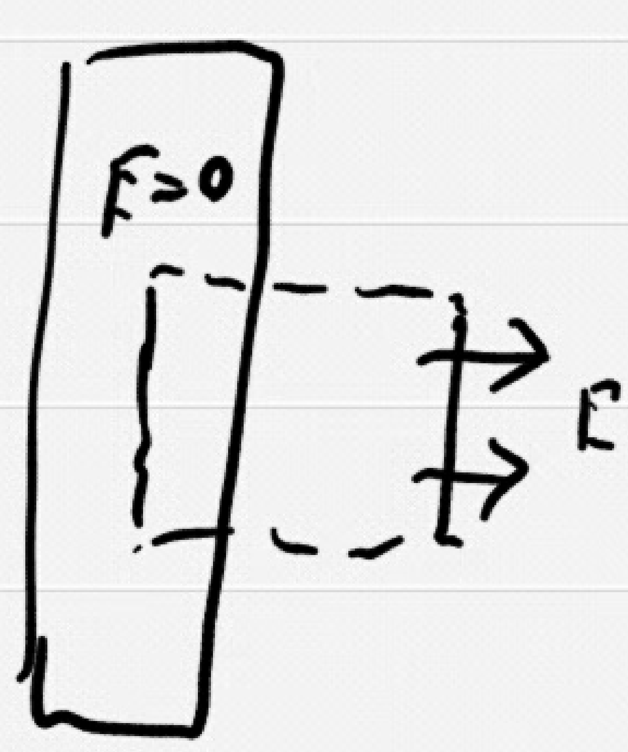
5. a.



$$\oint \vec{E}\cdot d\vec{A} = 2EA = (\sigma + \Sigma)A/\varepsilon_0$$

$$\Rightarrow E = (\sigma + \Sigma)/2\varepsilon_0$$

b. Look @ new Gaussian surface



$$\oint \vec{E}\cdot d\vec{A} = EA$$
$$= Q_{enc}/\varepsilon_0$$

$$E = (\sigma + \Sigma)/2\varepsilon_0$$

so $Q_{enc} = \left(\frac{\sigma + \Sigma}{2}\right) A$

- or charge density on right $= (\Sigma + \sigma)/2$
- this leaves $(\Sigma - \sigma)/2$ on left

C. $E = \sigma/2\varepsilon_0 - \frac{\Sigma + \sigma}{4\varepsilon_0} - \frac{\Sigma - \sigma}{4\varepsilon_0}$

$= \sigma/2\varepsilon_0 - \Sigma/2\varepsilon_0$ (to left)

D. $W = \int \vec{F}\cdot d\vec{x} = qE\Delta x$

$= q\left[(\sigma/2\varepsilon_0 - \Sigma/2\varepsilon_0)\cdot T + 0T + \frac{\sigma + \Sigma}{2\varepsilon_0} T\right]$

$= \boxed{q\sigma/\varepsilon_0\, T}$

6. $V = 3x + 2y^3$

$\vec{E} = -\nabla V$

$= [-3, -6y^2, 0]$

$\vec{E}$ at $[1, 0, 1]$

$= [-3, 0, 0]$

so $\boxed{|\vec{E}| = 3}$

Bonus: - No E_z so no flux through top/bottom

- E_x constant so flux through front/back balances

$\int_{y=0} \vec{E} \cdot d\vec{A} = 0$ since $E_y = 0$ @ $y = 0$

$\oint \vec{E} \cdot d\vec{A} = \int_{y=1} \vec{E} \cdot d\vec{A} = \int_{y=1} -6y^2 \hat{j} \cdot \hat{j} \, dA$

$= \int -6 \, dA$

$= \boxed{-6}$

enclosed charge is negative!