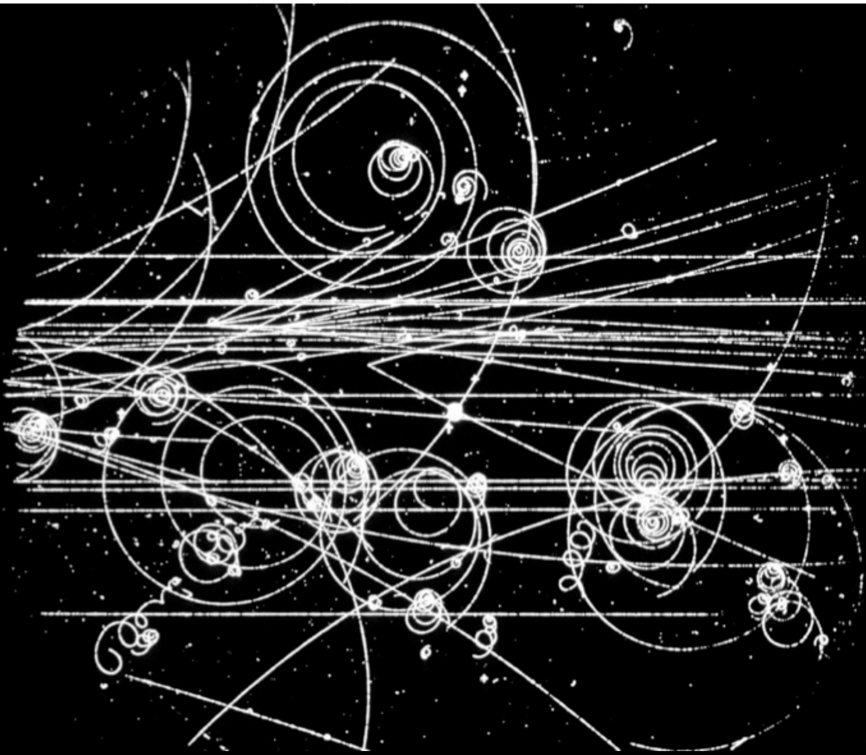
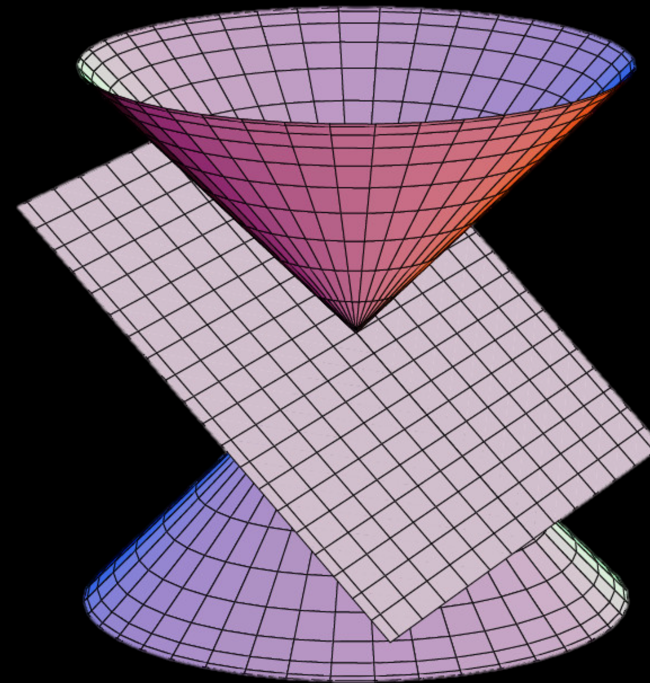




Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Last Few Weeks

- Light sometimes behaves like a wave, sometimes like a particle
- Particles also sometimes behave like waves

- $p = h/\lambda$

- $E^2 = (pc)^2 + (mc^2)^2$

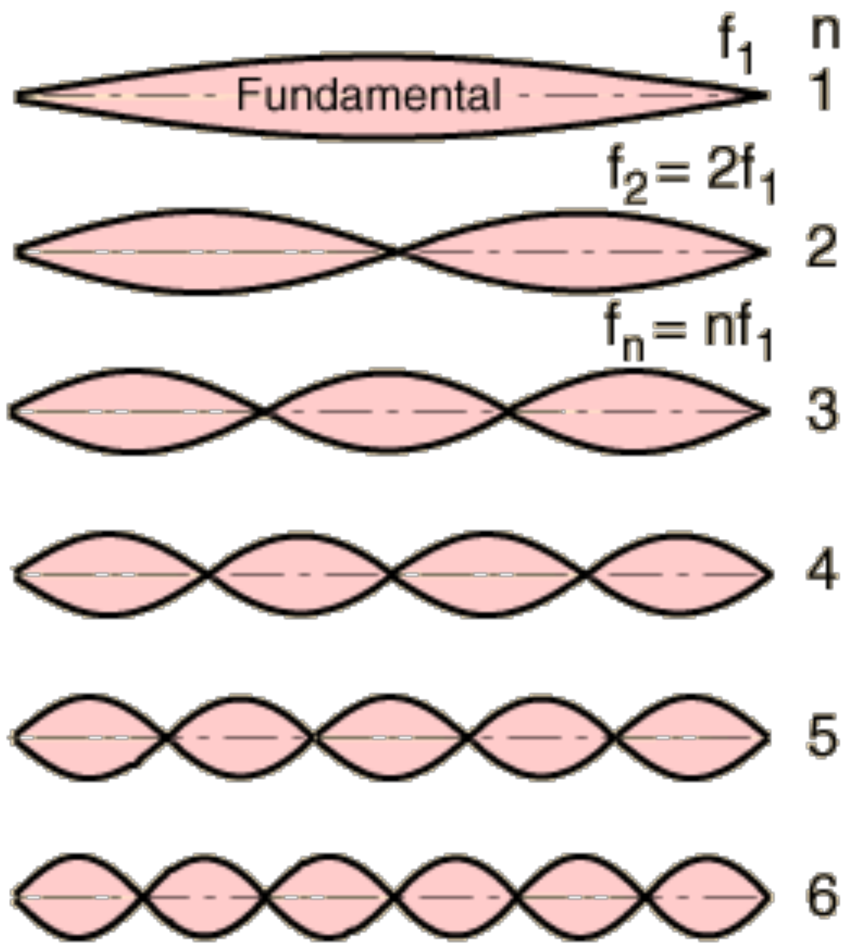
- $E_{m=0} = hc/\lambda = h\nu$

How do we think about waves?

Standing Waves

Confined

Not Moving

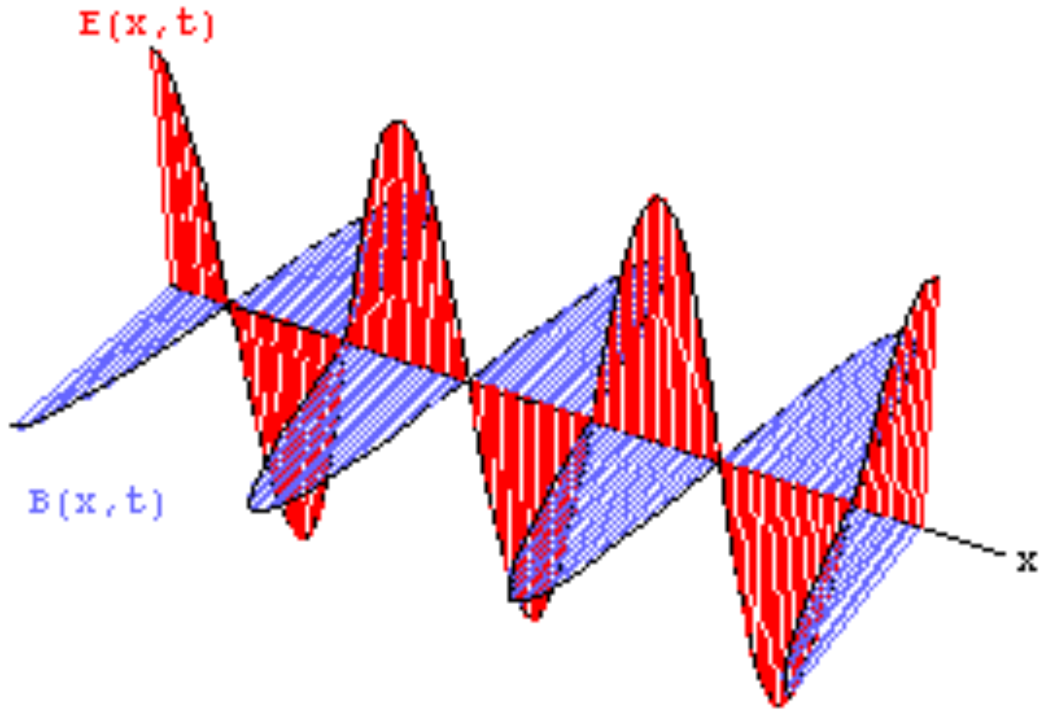


How do we think about waves?

Traveling Waves

Moving

Semi-Infinite Extent

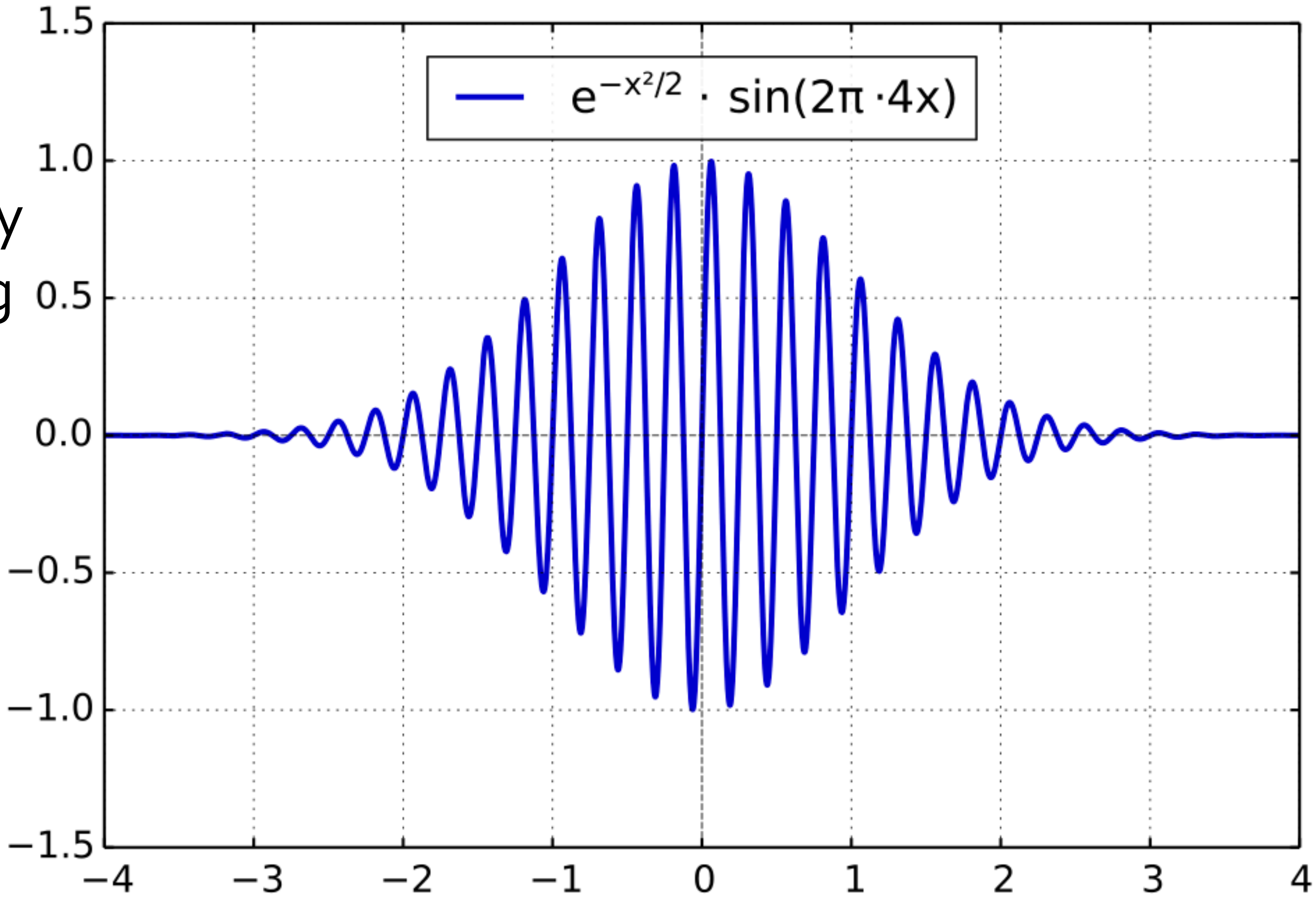


Wave Packet

Can be Stationary or Moving

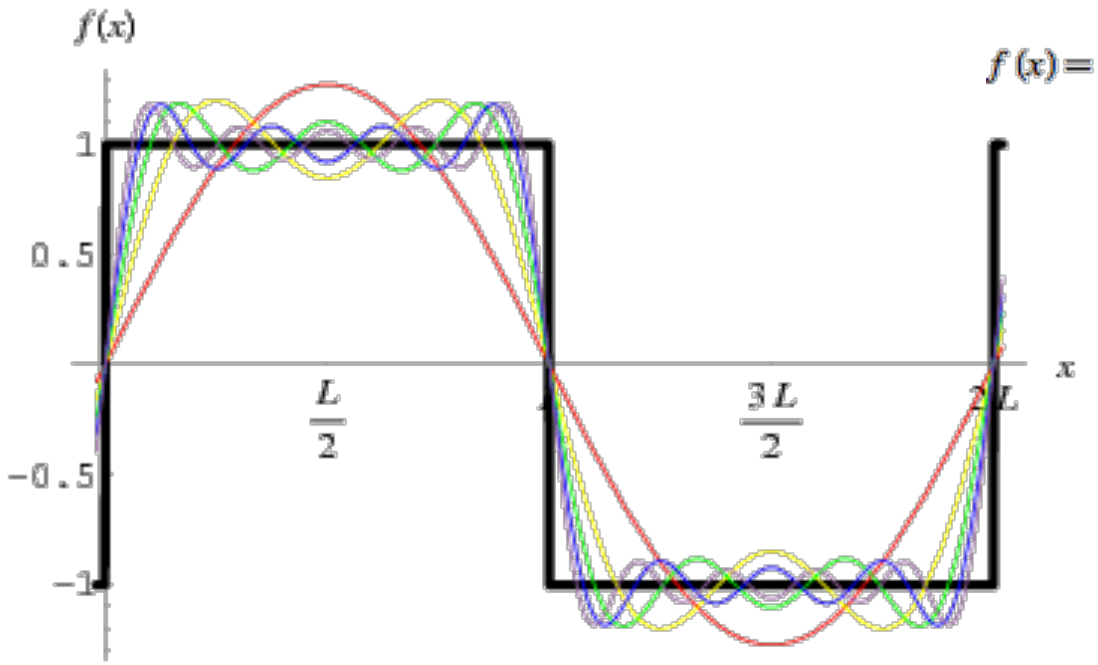
Localized

Almost Like a Particle



Fourier Analysis

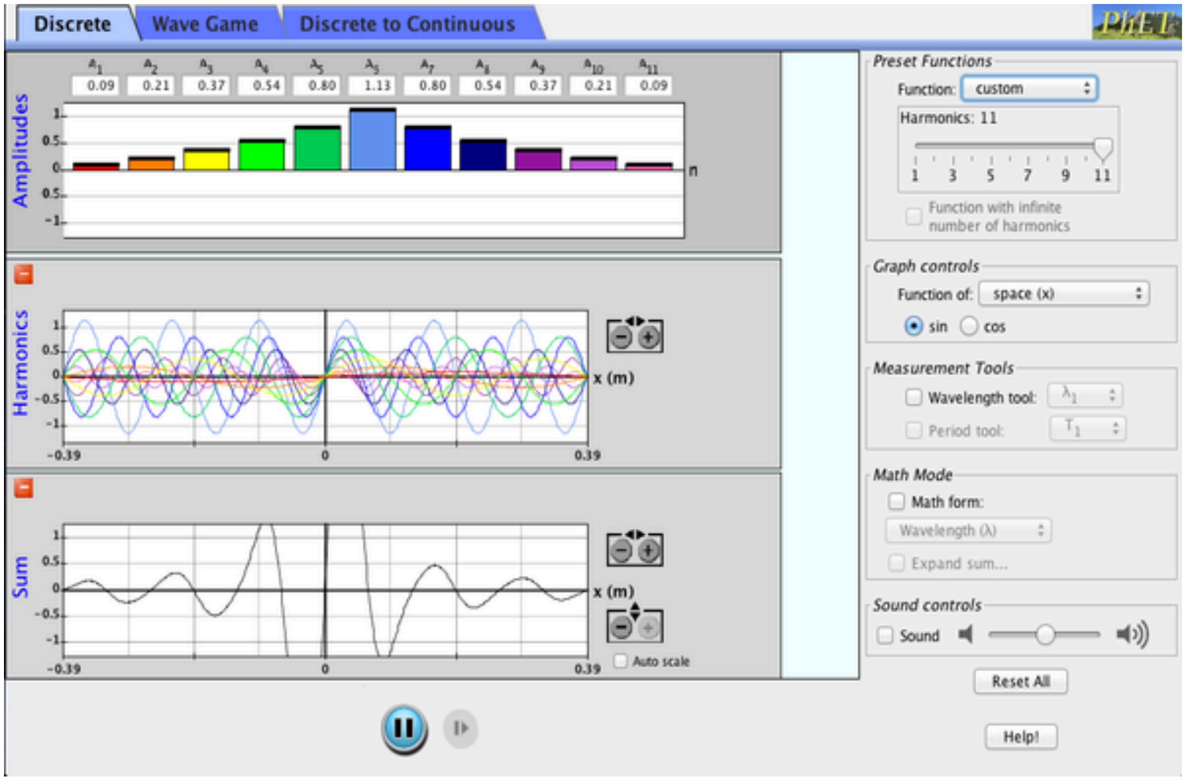
- Any wave form can be built up from a sum of sinusoidal waves of various wavelengths



$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right).$$

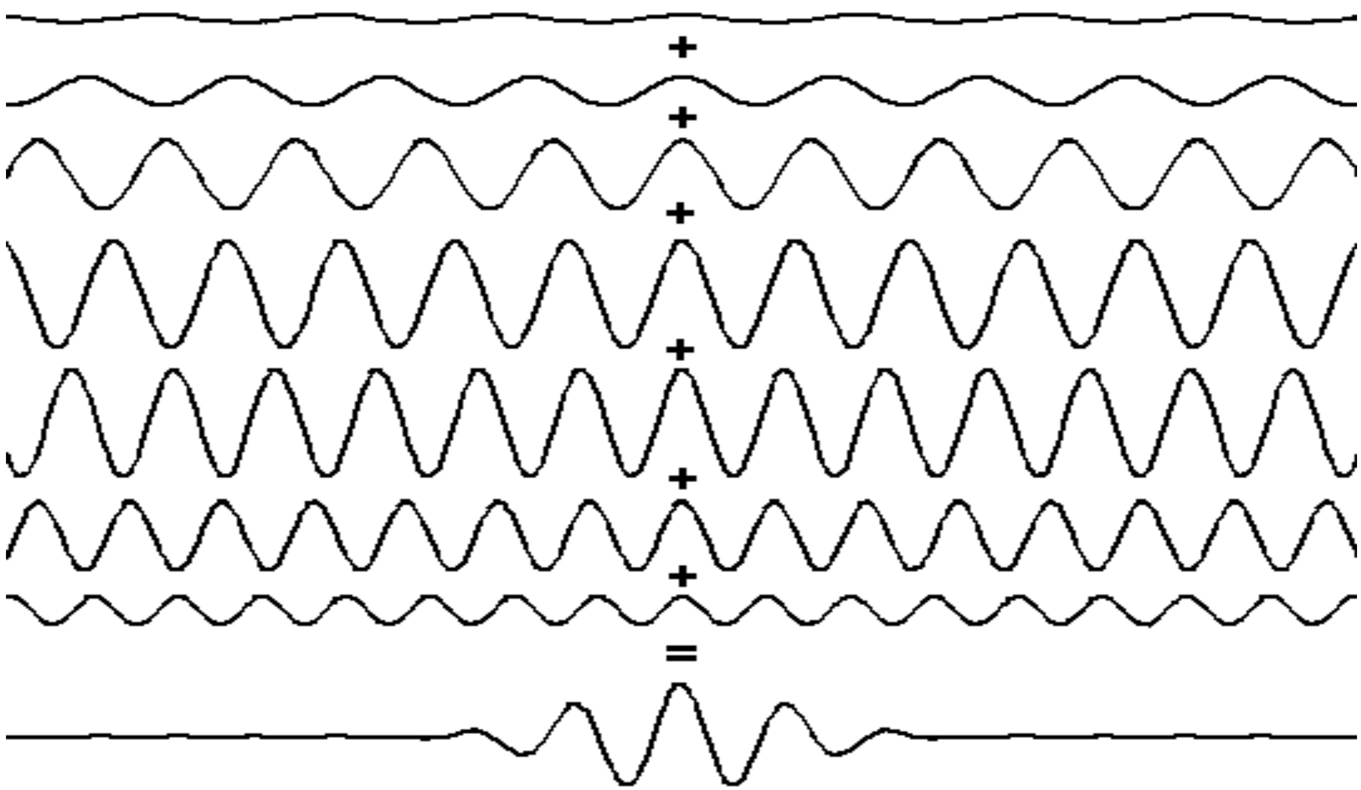
Making Waves

- https://phet.colorado.edu/en/simulation/fourier



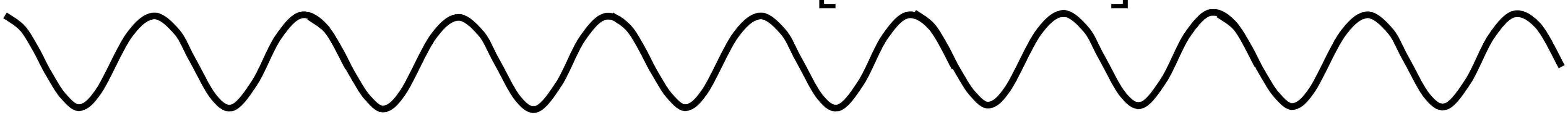
Constructing a Wave Packet

$$\Psi(x,t) = \sum_n A_n \exp\left[i\left(k_n x - \omega_n t\right)\right]$$



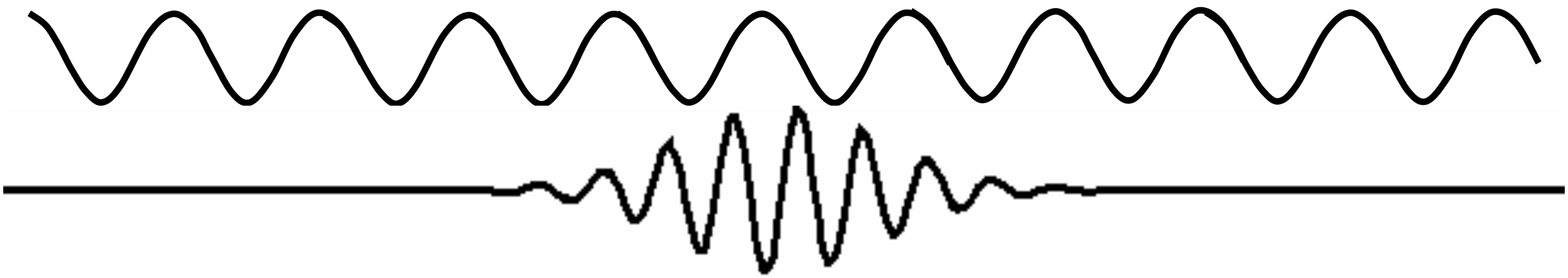
Plane Waves Vs. Wave Packets

$$\Psi(x,t) = A\exp\left[i\left(kx-\omega t\right)\right]$$



$$\Psi(x,t) = \sum_n A_n \exp\left[i\left(k_n x-\omega_n t\right)\right]$$

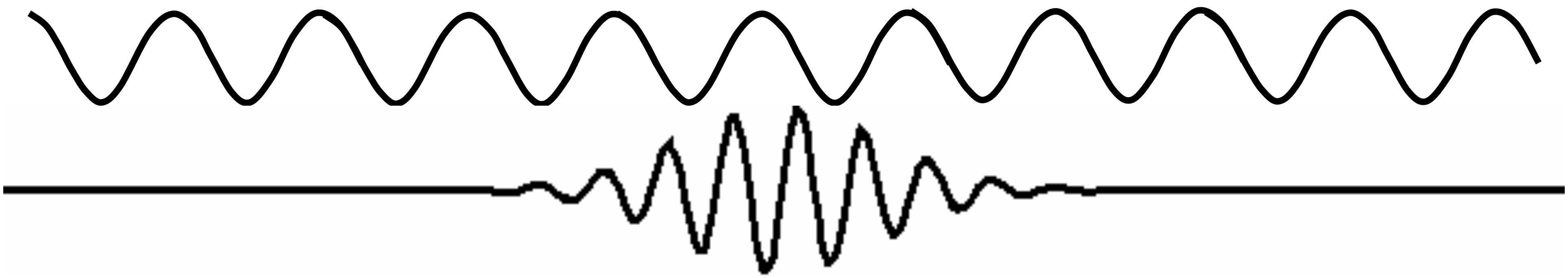
Concept Check



For which type of wave are the position (x) and momentum (p) most well-defined?

A) x most well-defined for plane wave, p most well-defined for wave packet.
B) p most well-defined for plane wave, x most well-defined for wave packet.
C) p most well-defined for plane wave, x equally well-defined for both.
D) x most well-defined for wave packet, p equally well-defined for both.
E) p and x are equally well-defined for both.

Concept Check



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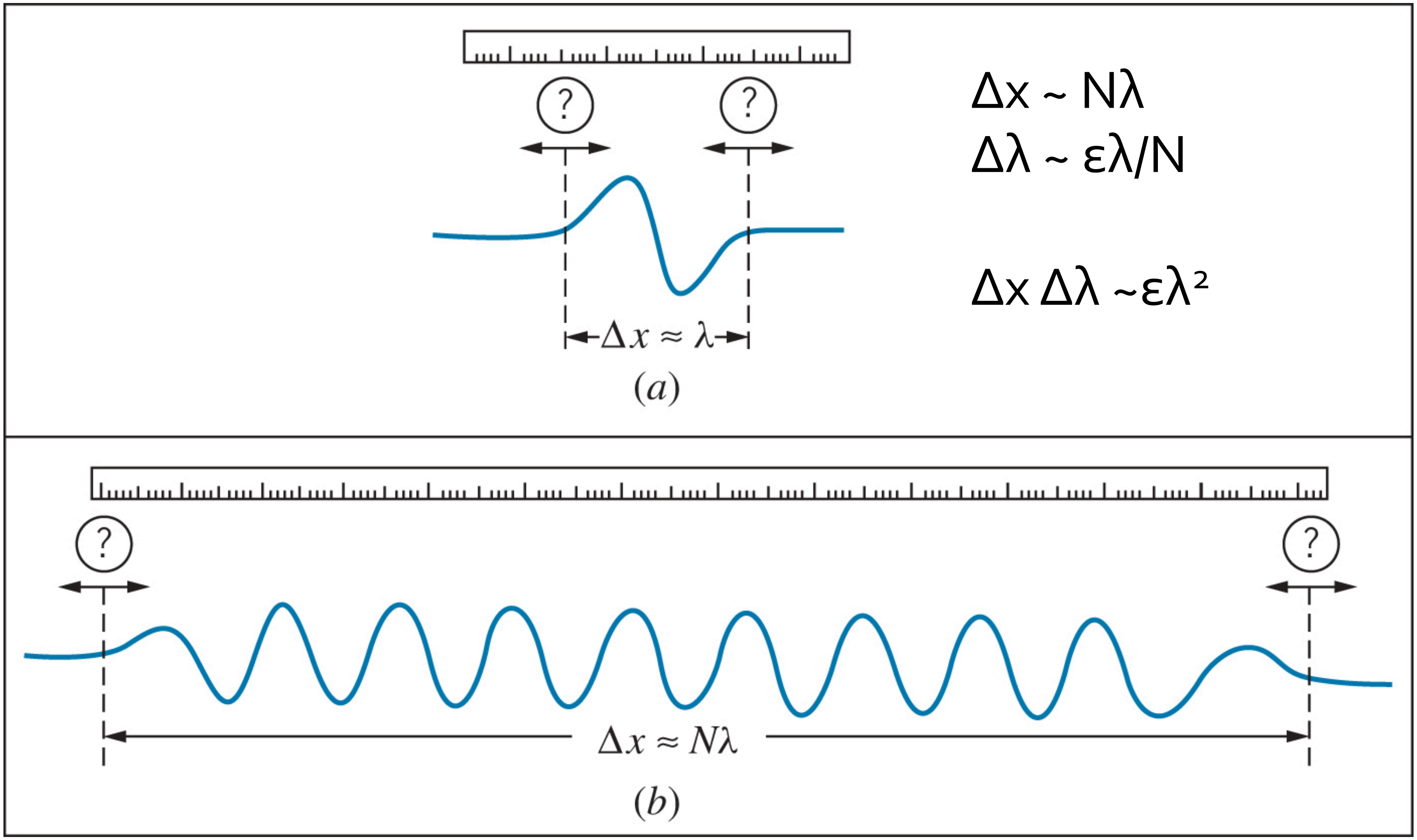
Classical Uncertainty Relations



$\Delta x \sim N\lambda$

$\Delta\lambda \sim \varepsilon\lambda/N$

$\Delta x\, \Delta\lambda \sim \varepsilon\lambda^2$

Uncertainty Principle

$$\Delta x \, \Delta\lambda \sim \varepsilon \lambda^2$$

$$\lambda = h/p \Rightarrow d\lambda/dp = -h/p^2$$

$$\Rightarrow d\lambda = -h/p^2 \, dp$$

$$\text{or } |d\lambda| = h/p^2 \, |dp|$$

$$\Delta x \, \Delta\lambda = \Delta x \cdot h/p^2 \, \Delta p$$
$$\sim \varepsilon \lambda^2$$
$$= \varepsilon \, h^2/p^2$$

$$\Rightarrow \Delta x \, \Delta p \sim \varepsilon h$$

Exact formula

$$\Delta x \, \Delta p \geq h/4\pi$$

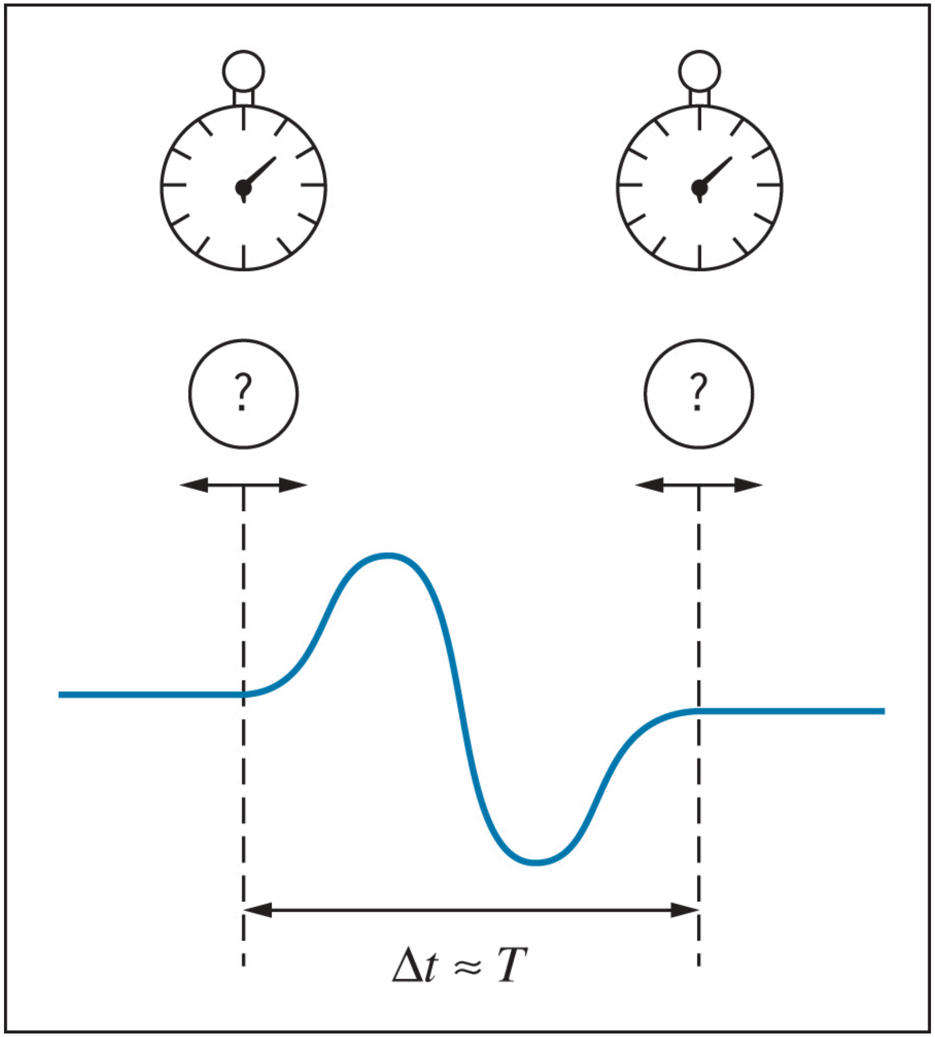
$$\text{define } \hbar = h/2\pi$$

$$\boxed{\Delta x \, \Delta p \geq \hbar/2}$$

Best case

Reasonable approximation $\Delta x \Delta p \sim \hbar$

Classical Uncertainty Relations



$\Delta t \, \Delta T \sim \varepsilon T^2$

$\Delta t \, \Delta T \sim \varepsilon T^2$

$\nu = 1/T$

$E = h\nu = h/T$

$\Delta E = h/T^2 \, \Delta T$

$\Delta t \, \Delta T = \Delta E \Delta T \cdot T^2/h$

$\sim \varepsilon T^2$

$\Delta E \Delta t \sim \varepsilon h$

Exact $\boxed{\Delta E \Delta t \geq \hbar/2}$

often $\Delta E \Delta t \sim \hbar$

Heisenberg Uncertainty Principle(s)

$$\Delta p \, \Delta x \geq \frac{1}{2} \hbar$$

$$\Delta E \, \Delta t \geq \frac{1}{2} \hbar$$

Ehhh... not that one...

At Home With the Heisenbergs

Concept Check

- My car has a mass of ~1000 kg. If I know its position to 1 nm (10^{-9} m) accuracy, what is my uncertainty as to its speed?

A. ~0.1 m/s
B. ~10^{-12} m/s
C. ~10^{-22} m/s
D. ~10^{-28} m/s

$\hbar \sim 10^{-34}$

Concept Check

- My car has a mass of ~1000 kg. If I know its position to 1 nm (10^{-9} m) accuracy, what is my uncertainty as to its speed?

A. ~0.1 m/s
B. ~10^{-12} m/s
C. ~10^{-22} m/s
D. ~10^{-28} m/s

My Car

$$m \sim 1000 \text{ kg}$$
$$p \sim 1000 \cdot v$$

$$\Delta v = \Delta p / 1000$$

$$\Delta p \sim \hbar / \Delta x$$

$$\Delta v \sim \hbar / (1000 \, \Delta x)$$

$$\sim 10^{-34} / (10^{3} \cdot 10^{-9})$$

$$\sim 10^{-28} \text{ m/s}$$