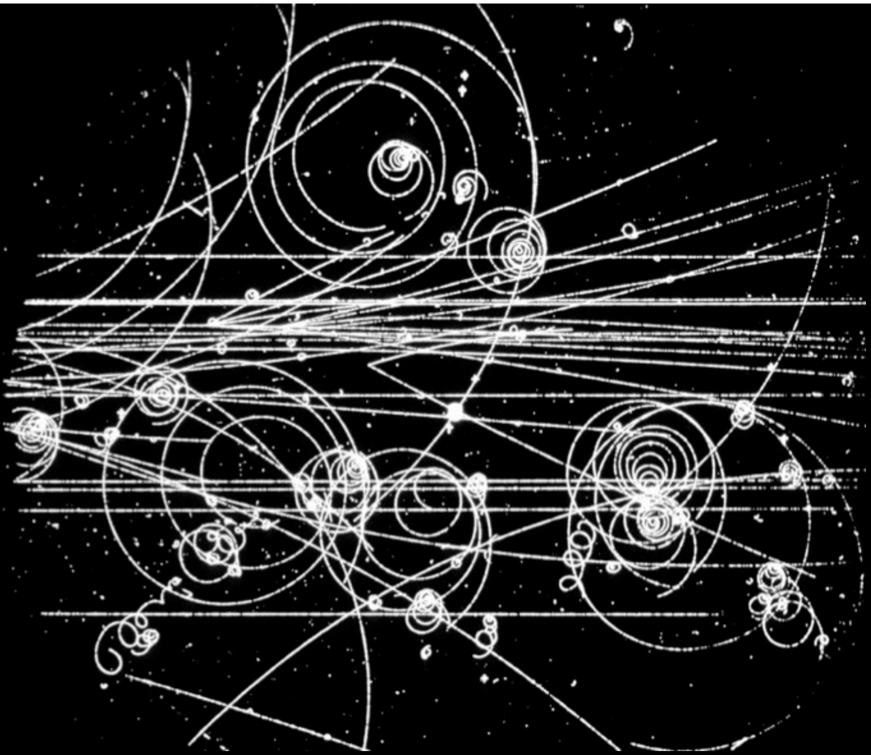

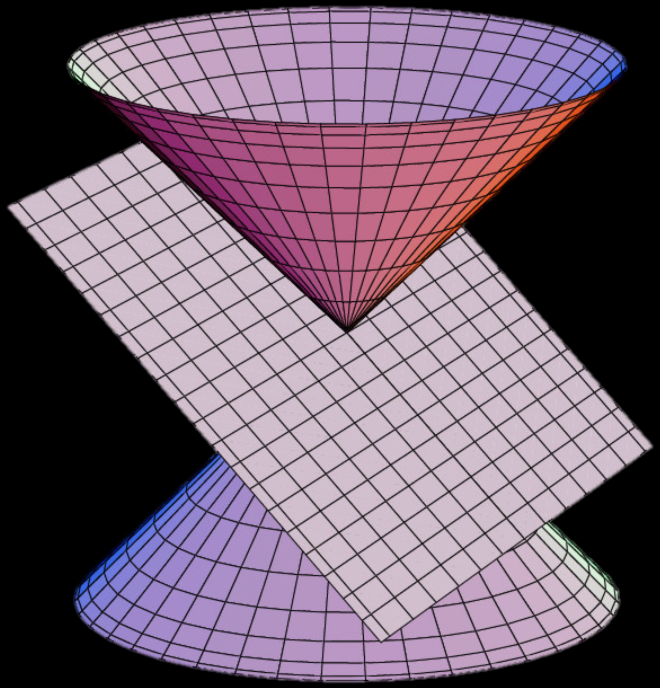

Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Light Wave

$$E(x,t) = E_0 \sin(kx - \omega t)$$

$$\partial^2 E / \partial x^2 = -k^2 E$$
$$\partial^2 E / \partial t^2 = -\omega^2 E$$

photon; $\omega = 2\pi\nu = 2\pi E / h$

$k = 2\pi / \lambda = 2\pi p / h$

$$\omega / k = \nu \lambda = c$$
$$\Rightarrow E / p = c$$

So $\partial^2 E / \partial x^2 = 1/c^2 \, \partial^2 E / \partial t^2$

$$\Updownarrow$$

$$-k^2 = -\omega^2 / c^2$$

or $\omega / k = \pm c$

Schrödinger Equation

Assume $\Psi(x,t) = A e^{i(kx - \omega t)}$

$$\partial^2 \Psi / \partial x^2 = -k^2 \Psi$$
$$\partial^2 \Psi / \partial t^2 = -\omega^2 \Psi$$
$$\partial \Psi / \partial x = ik \Psi$$
$$\partial \Psi / \partial t = -i\omega \Psi$$

Classical energy Eq.

$$E = \tfrac{1}{2} m v^2 + U$$
$$= p^2 / 2m + U$$

$$p = h/\lambda = hk/2\pi = \hbar k$$
$$E = h\nu = h\omega/2\pi = \hbar\omega$$

$$\hbar^2 k^2 / 2m + U = \hbar\omega$$

$$\Downarrow$$

$$-\hbar^2/2m \, \partial^2\Psi/\partial x^2 + U\Psi = i\hbar \, \partial\Psi/\partial t$$

1-D Schrodinger Equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+U(x,t)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

We want to use this to calculate electron waves. What's the first step?

A. Figure out how many electrons will be interacting
B. Figure out what general solutions will be by plugging in trial solutions and seeing if can solve.
C. Figure out what the forces will be on the electron in that physical situation.
D. Figure out what the boundary conditions must be on the electron wave.
E. Figure out what potential energy is at different x and t for the physical situation.

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Time-Independent Schrödinger Eq.

$E = \hbar\omega = \text{const.}$

(true if $U(x,t) = U(x)$)

$\partial\Psi/\partial t = -i\omega\Psi$

$-\hbar^2/2m \; \partial^2\Psi/\partial x^2 + U(x)\Psi = \hbar\omega\Psi$
$= E\Psi$

Factor out $e^{-i\omega t}$

$$-\hbar^2/2m \; \partial^2\psi/\partial x^2 + U(x)\psi(x) = E\psi(x)$$

$$\Psi(x,t) = \psi(x)\, e^{-i\omega t}$$

For physical solution

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$$

Time-Independent Schrodinger Eq.

- Most physical situations, like H atom, no time dependence in U!
  - e.g. $U(r) = -ke^2/r$ for H atom…

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$$

with $\Psi(x,t) = \psi(x)\, e^{-iEt/\hbar}$

Steps to Solving Time-Independent Schrodinger Equation

- 1. Figure out what $U(x)$ is, for situation given.
- 2. Guess or look up functional form of solution.
- 3. Plug in to check if ψ's and all x's drop out, leaving an equation involving only a bunch of constants.
- 4. Figure out what boundary conditions must be to make sense physically.
- 5. Figure out values of constants to meet boundary conditions and normalization:

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

- 6. Multiply by time dependence $\varphi(t) = \exp(-iEt/\hbar)$ to obtain full time-dependent solution if needed.

Simplest Case of Schrodinger Eq

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)=E\psi(x)$$

Electron in free space, no electric fields or gravity around.

1. Where does it want to be?
2. What is U(x)?
3. What are boundary conditions on ψ(x)?

1. No preference- all x the same.
2. Constant.
3. None, could be anywhere.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}=E\psi(x)$$

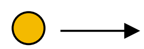
Concept Check

What does this equation describe?

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)$$

A. Nothing physical, just a math exercise.

B. Only an electron in free space along the x-axis with no electric fields around.



C. An electron flying along the x-axis between two metal plates with a voltage between them (as in photoelectric effect)

D. An electron in an enormously long wire not hooked to any voltages.

E. More than one of the above.

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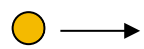

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Correct Eq for B or D.

Solution for Free Electron

A solution to this differential equation is:

(A) A cos(kx)
(B) A e^{-kx}
(C) A e^{ikx}
(D) (B & C)
(E) (A & C)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)$$

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Check Solution

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x) \qquad \psi(x) = A\exp(ikx)$$

$$\frac{\hbar^2 k^2}{2m} = E$$

...makes sense, because $p = \hbar k$

Condition on k is just saying that $(p^2)/2m = E$.
$U(x)=0$, so $E = KE = \frac{1}{2}mv^2 = p^2/2m$

Concept Check

The total energy of the free electron is:

A. Quantized according to E_n = (constant) x n^2, n= 1,2, 3,…
B. Quantized according to E_n = const. x (n)
C. Quantized according to E_n = const. x ($1/n^2$)
D. Quantized according to some other condition but don't know what it is.
E. Not quantized, energy can take on any value.

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Time Dependence

$$\psi(x) = A\exp(ikx) \quad \frac{\hbar^2 k^2}{2m} = E$$

k (and therefore E) can take on any value.

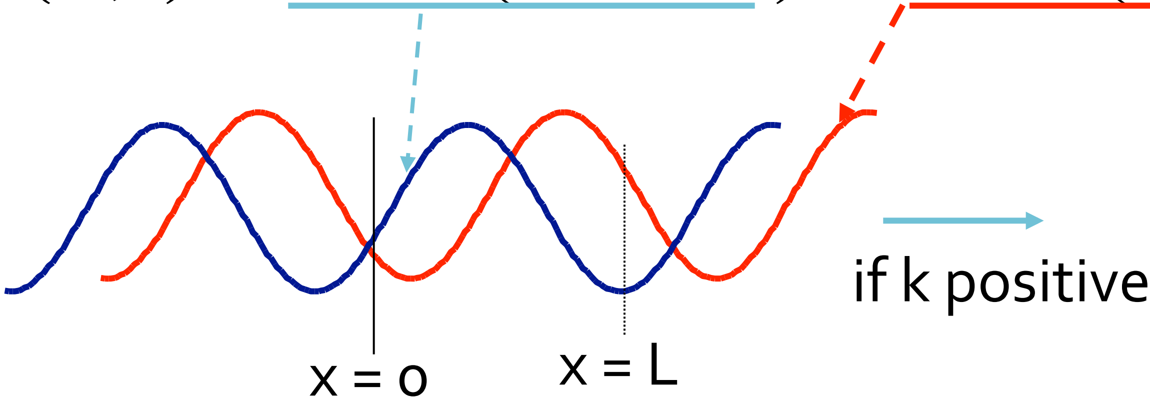
Almost have a solution, but remember we still have to include time dependence:

$$\Psi(x,t) = \psi(x)\phi(t) \qquad \boxed{\phi(t) = e^{-iEt/\hbar}}$$

$$\Psi(x,t) = A\exp[i(kx - \omega t)]$$

Concept Check: Probabilities

$$\Psi(x,t) = A\cos(kx - \omega t) + Ai\sin(kx - \omega t)$$



Using equation, probability $|\Psi|^2$ of electron being in dx at x = L is ________ probability of being in dx at x = 0.

A. always bigger than B. always same as
C. always smaller than
D. oscillates up and down in time between bigger and smaller
E. Without being given k, can't figure out

Concept Check: Probabilities

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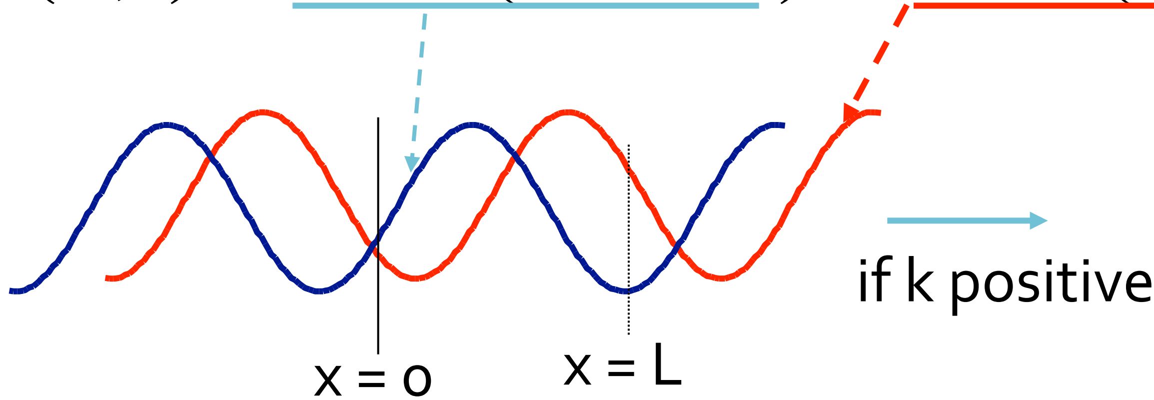

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