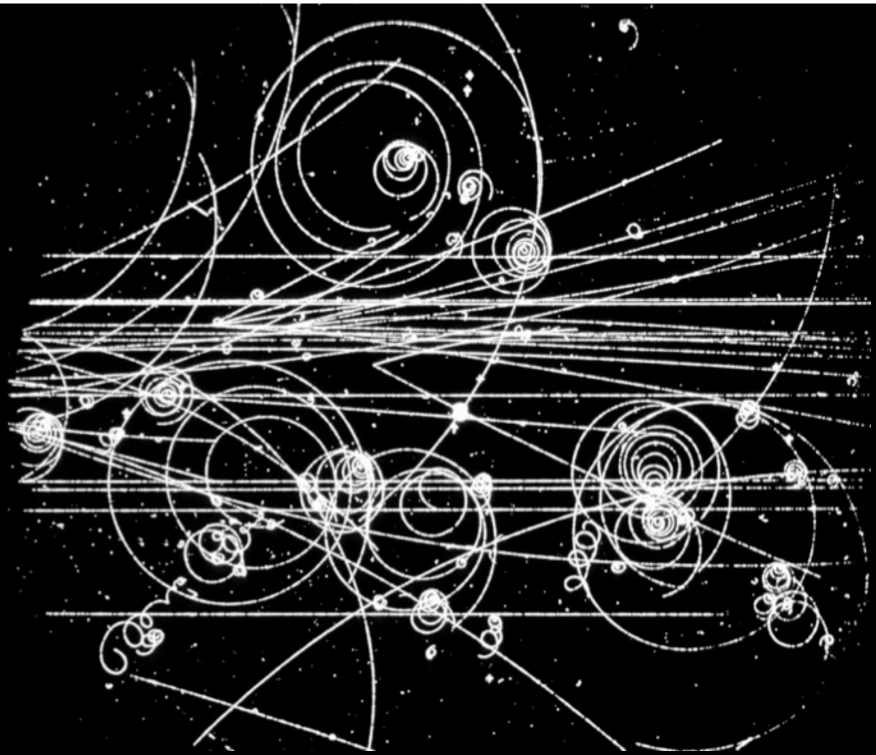
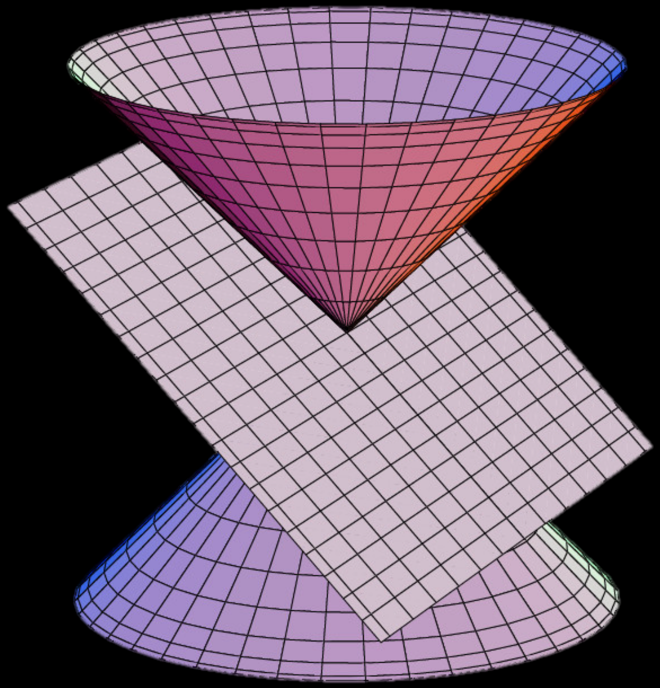


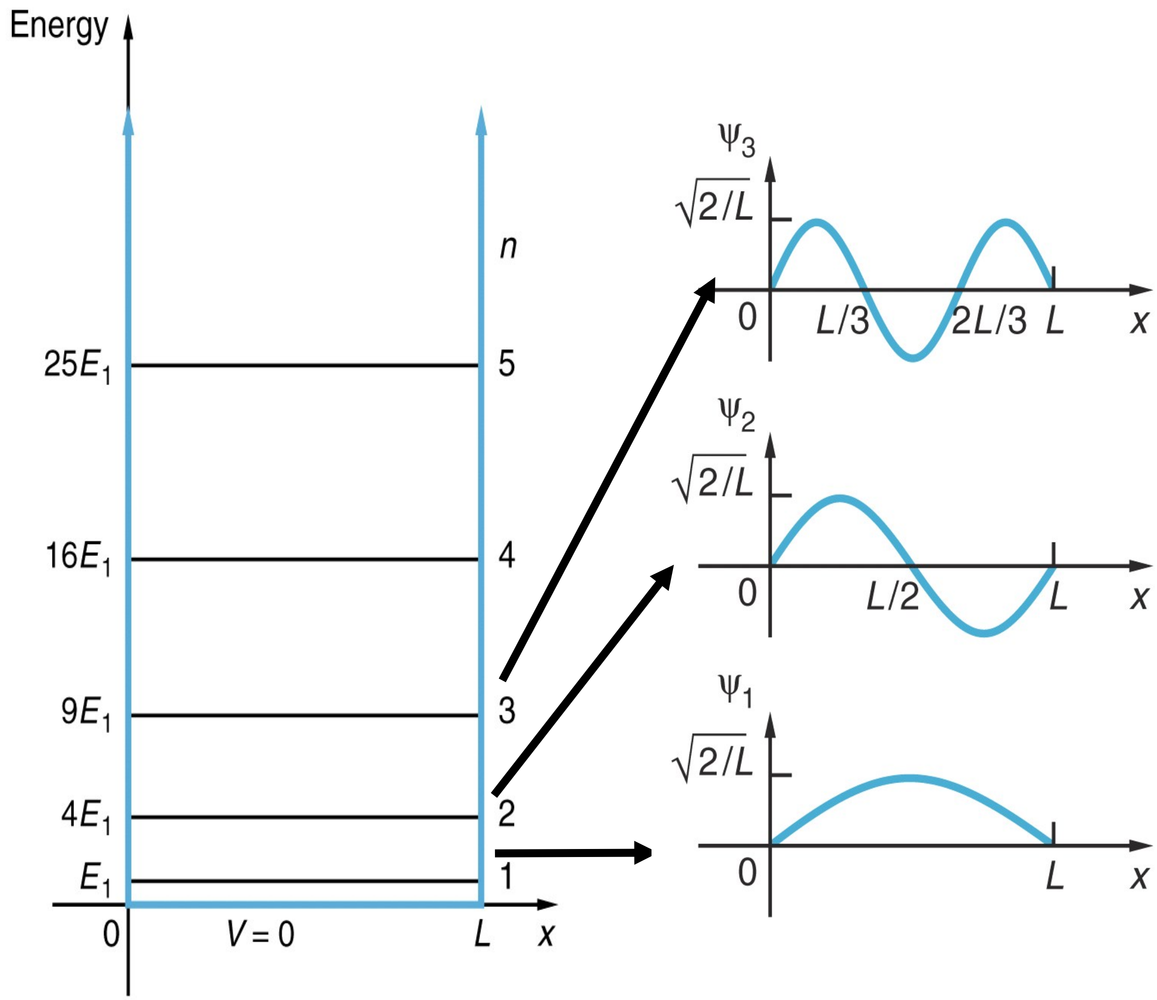


Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70

MWF 12:30-1:20 Lecture

Square Well Wave Functions



Superposition of Wave Functions

$$\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

$$\psi_2(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$$

both solutions to square well

$A\psi_1(x) + B\psi_2(x)$ also a solution

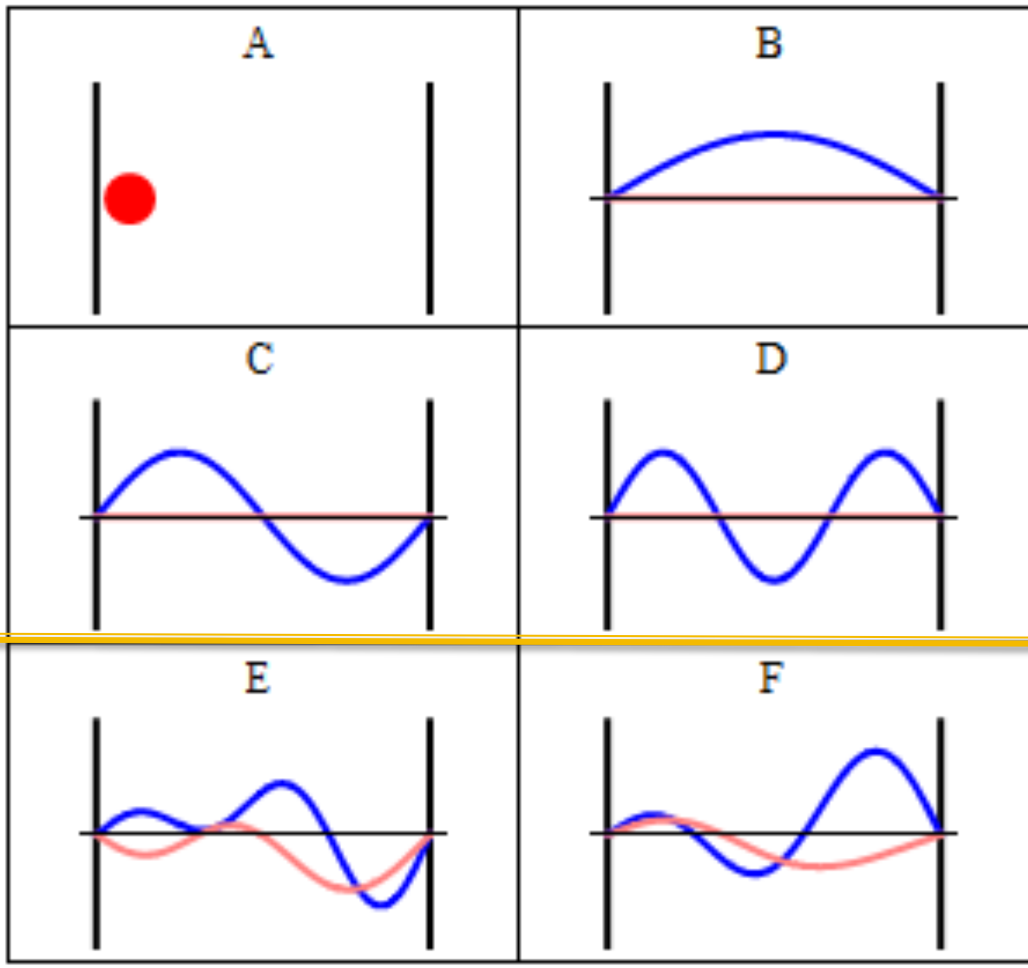
provided that $|A|^2 + |B|^2 = 1$

$$\Psi_1(x,t) = \sqrt{\frac{2}{2}} \sin\left(\frac{\pi x}{2}\right) e^{-iE_1 t/\hbar}$$

$$\Psi_2(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right) e^{-iE_2 t/\hbar}$$

- $A\Psi_1(x,t) + B\Psi_2(x,t)$
  full time-dependent solution
- ψ_1 & ψ_2 oscillate @ different frequency
  $\Rightarrow$ beat patterns

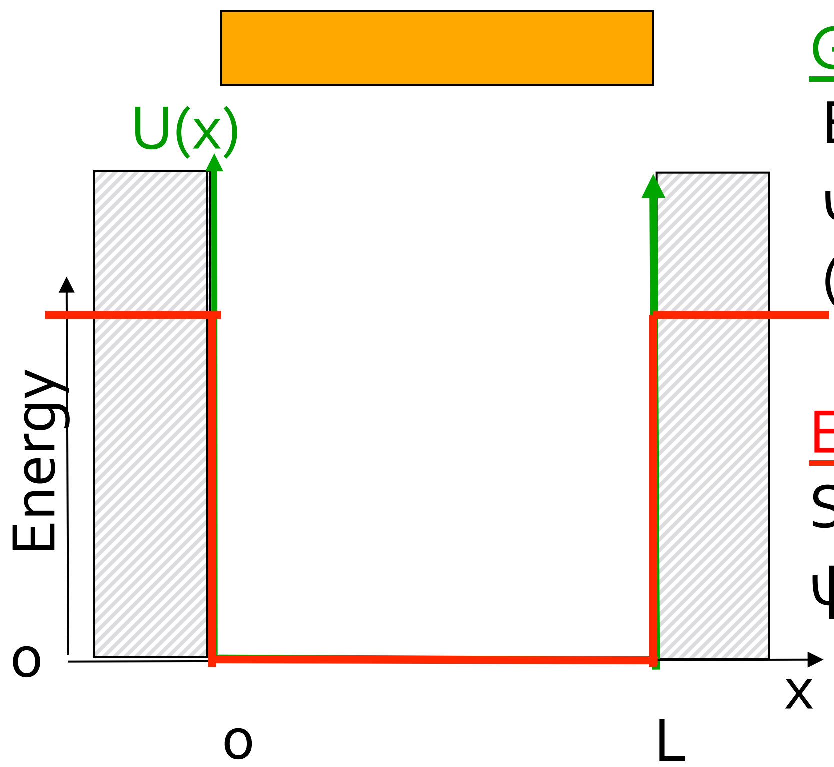
Square Well Wave Functions



"Pure Wave Functions"
a.k.a. Eigenstates

"Mixed Wave Functions"
Superpositions of eigenstates

Finite Square Well



Good Approximation:

Electrons never leave wire

ψ(x<o or x>L) =o.

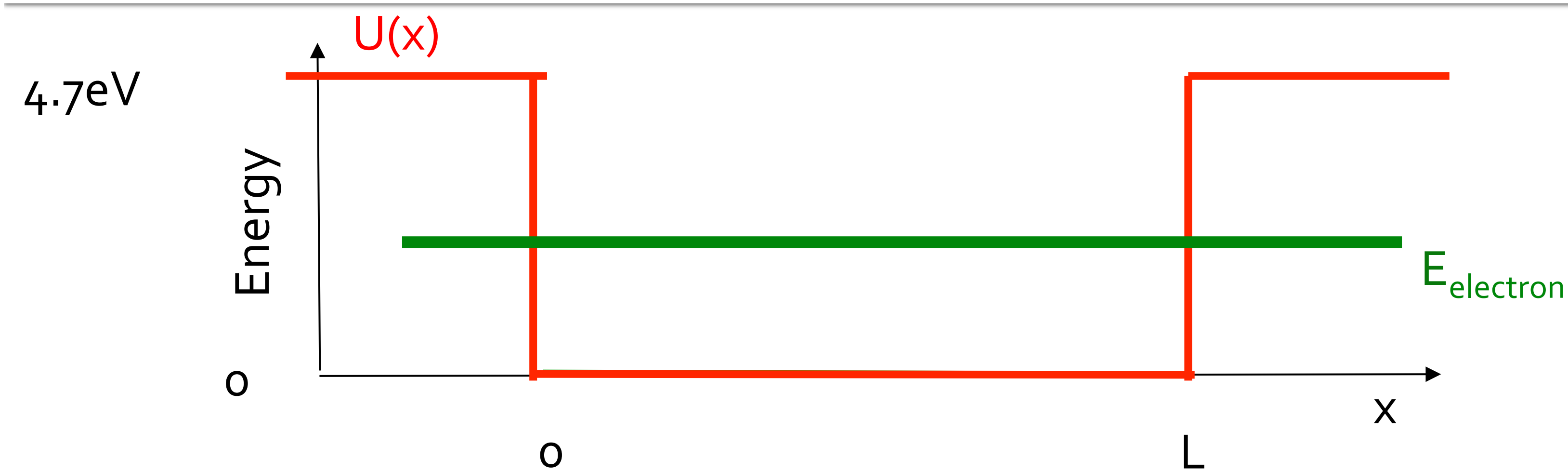
(OK when Energy << work function)

Exact Potential Energy curve (U):

Small chance electrons get out!

ψ(x<o or x>L)~o, but not exactly o!

Finite Square Well



Region I

Region II

Region III

$$\psi_I(x) = Ee^{\alpha x} + Fe^{-\alpha x}$$

$$\psi_{II}(x) = C\sin(kx) + D\cos(kx)$$

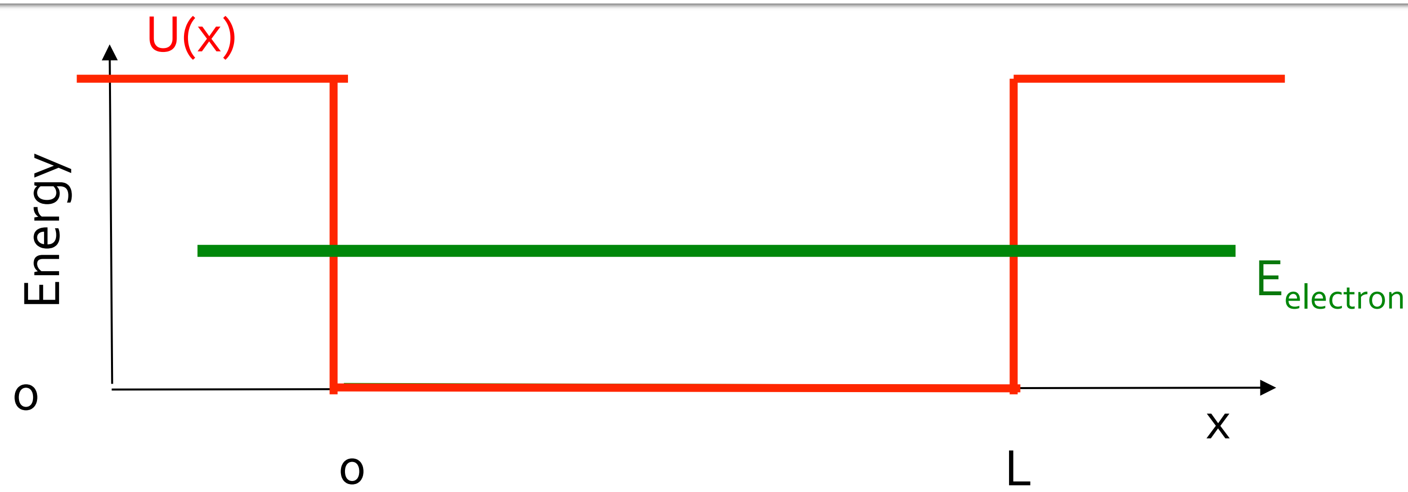
$$\psi_{III}(x) = Ae^{\alpha x} + Be^{-\alpha x}$$

What constraints can we set on the wave function in region III?

a. A must be 0 b. B must be 0 c. A and B must be equal

d. A=0 and B=0 e. A and B can be anything, need more info.

Finite Square Well



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$$\psi_{III}(x) = Ae^{\alpha x} + Be^{-\alpha x}$$

What constraints can we set on the wave function in region III?

a. A must be 0 b. B must be 0 c. A and B must be equal

d. A=0 and B=0 e. A and B can be anything, need more info.

Boundary Conditions

Outside well (E<U):
(Region I)

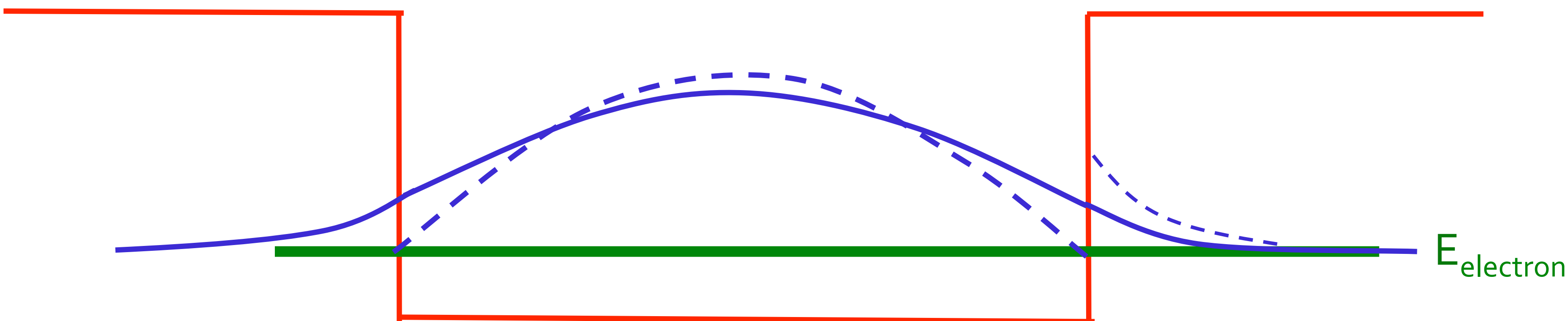
$$\psi_I(x) = Ae^{+\alpha x}$$

Inside well (E>U):
(Region II)

$$\psi_{II}(x) = C\sin(kx) + D\cos(kx)$$

Outside well (E<U):
(Region III)

$$\psi_{III}(x) = Be^{-\alpha x}$$



Boundary Conditions:

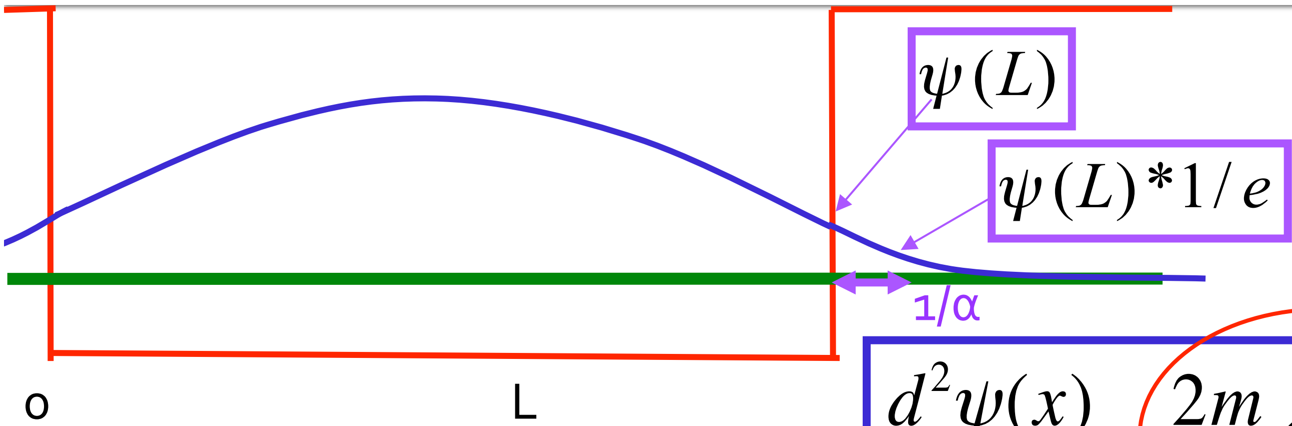
$$\psi(L) = \text{continuous}$$

$$\psi_{II}(L) = \psi_{III}(L)$$

$$\frac{d\psi(L)}{dx} = \text{continuous}$$

$$\frac{d\psi_{II}(L)}{dx} = \frac{d\psi_{III}(L)}{dx}$$

Penetration Depth



$$\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2}(U-E)\psi(x) = \alpha^2\psi(x)$$

How far does wave extend into this "classically forbidden" region?

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(U-E)}$$

$$\psi(x) = Be^{-\alpha x}$$

α big -> quick decay
α small -> slow decay

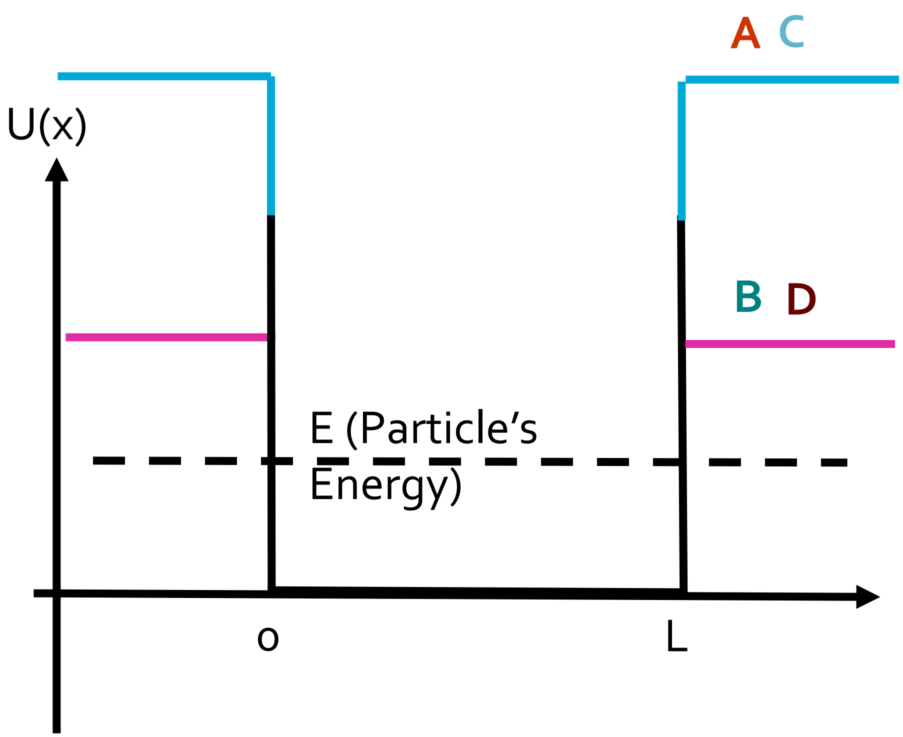
Measure of penetration depth = 1/α
ψ(x) decreases by factor of 1/e

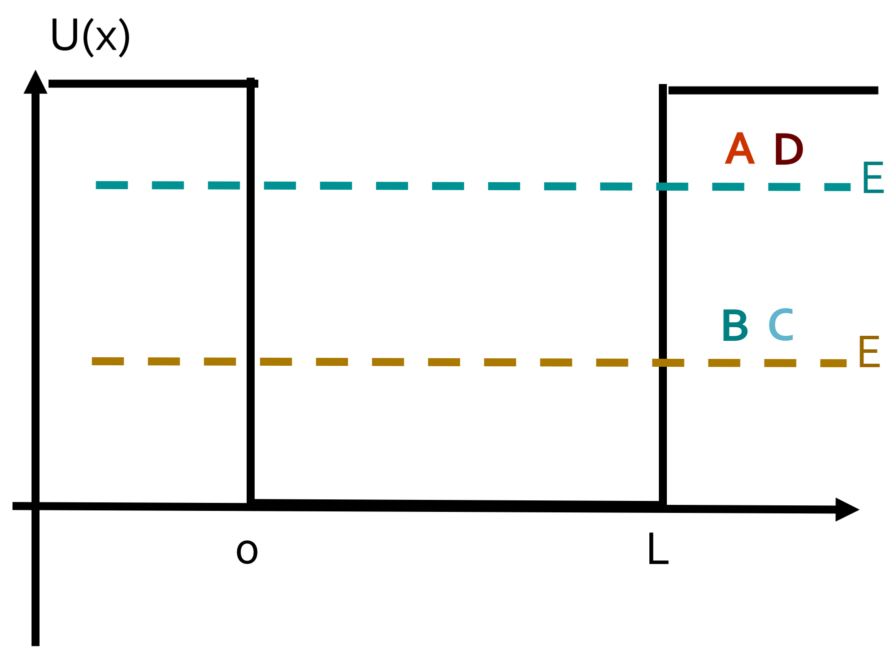
For U-E = 4.7eV, 1/α ..9x10^{-11} meters
(very small ~ an atom!!!)

Concept Check

Which case corresponds to the smallest penetration?

$$\psi(x) = Be^{-\alpha x} \qquad \alpha = \sqrt{\frac{2m}{\hbar^2}(U-E)}$$





Concept Check

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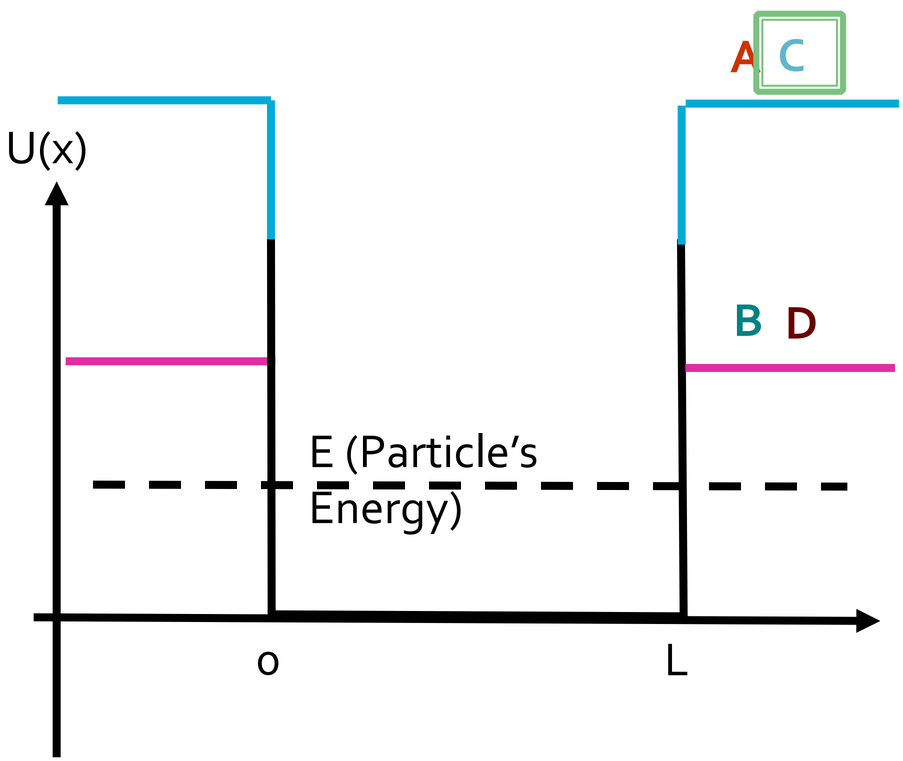
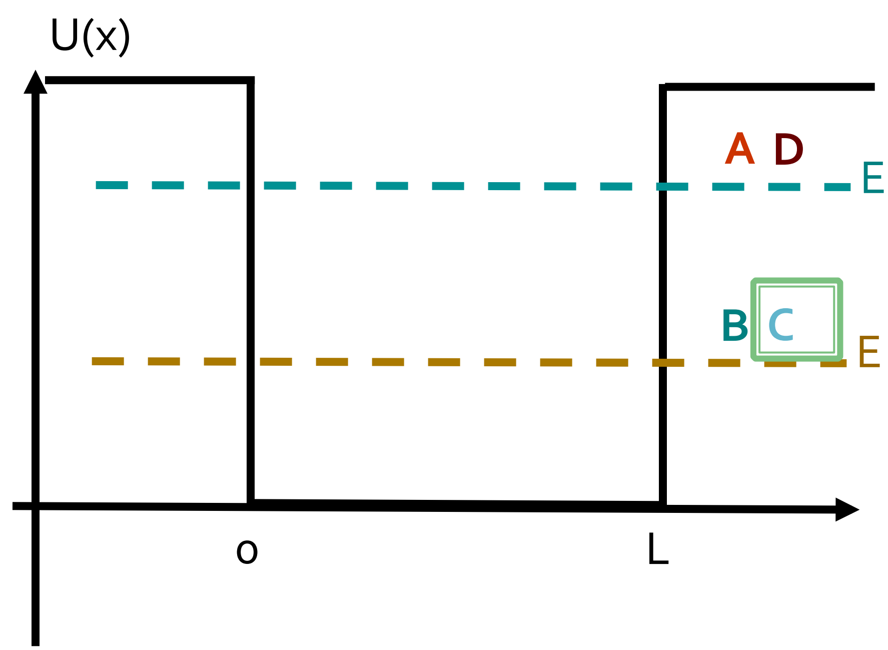
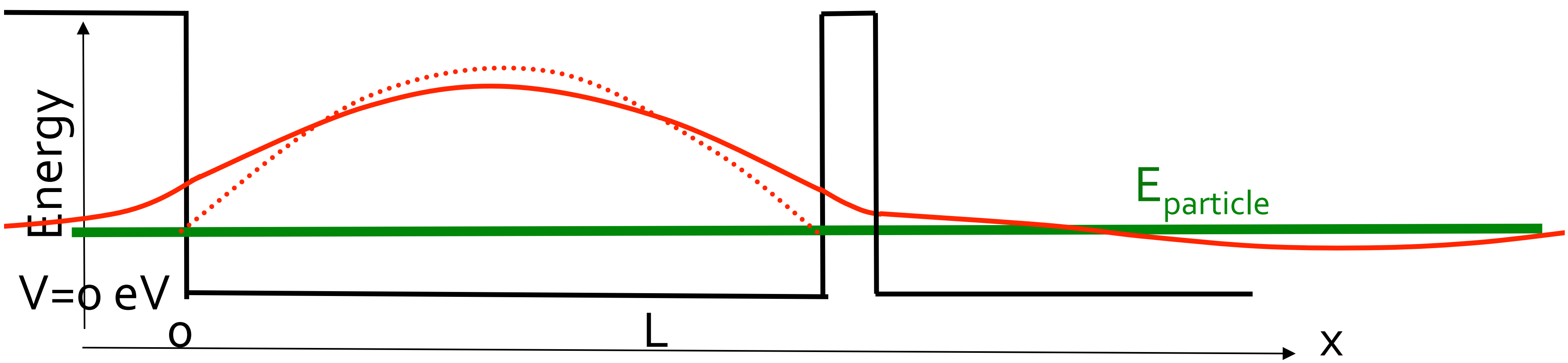
$$\psi(x) = Be^{-\alpha x} \qquad \alpha = \sqrt{\frac{2m}{\hbar^2}(U-E)}$$





Tunneling

The thinner and shorter the barrier, the easier it is to tunnel …



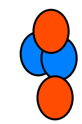
Radioactive Decay

Nucleus is unstable → emits a particle

Typically found for large atoms with lots of protons and neutrons.

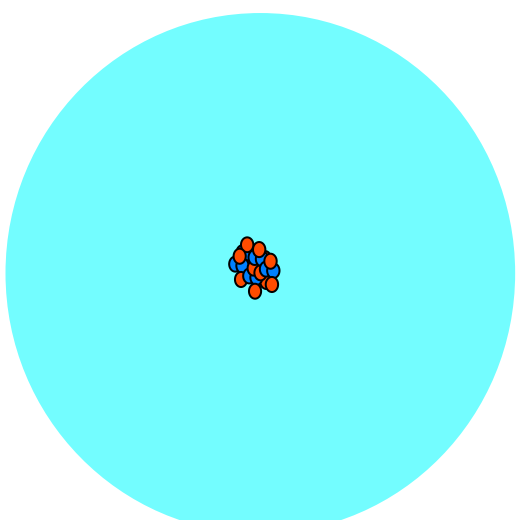
Alpha Decay → Nucleus emits an alpha particle

An alpha particle is 2 neutrons and 2 protons.



Radon-222
86 protons,
136 neutrons

- Proton (**positive charge**)
- Neutron (**no charge**)



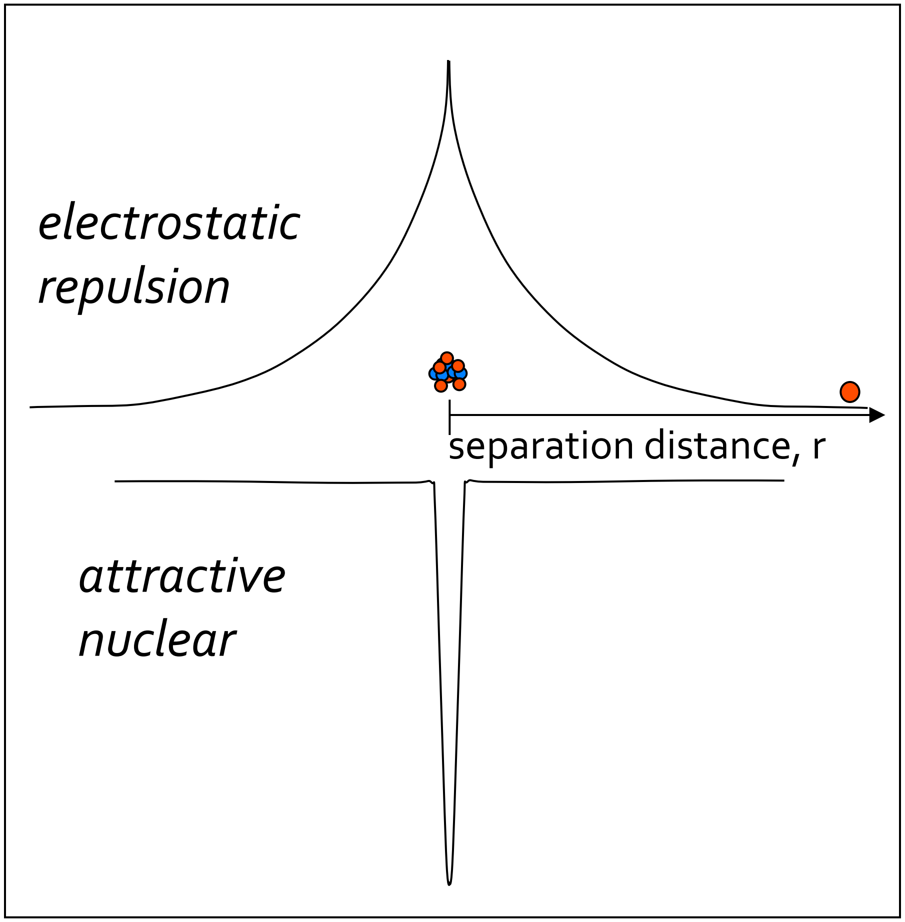
Nucleus has lots of protons and lots of neutrons.

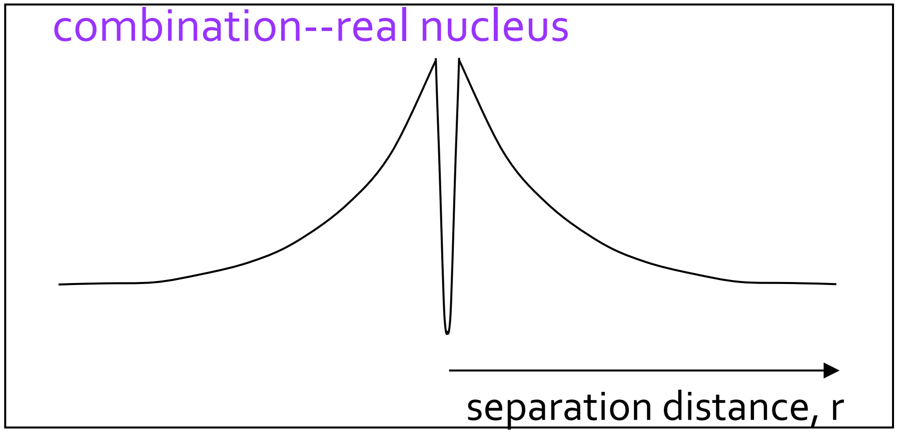
Two forces acting in nucleus:

- Coulomb force .. Protons really close together, so very big repulsion from coulomb force
- Nuclear force (attraction between nuclear particles is very strong if very close together) … called the STRONG Force.

Atomic + Nuclear Potential Energy

Potential energy curve for proton or alpha particle near nucleus

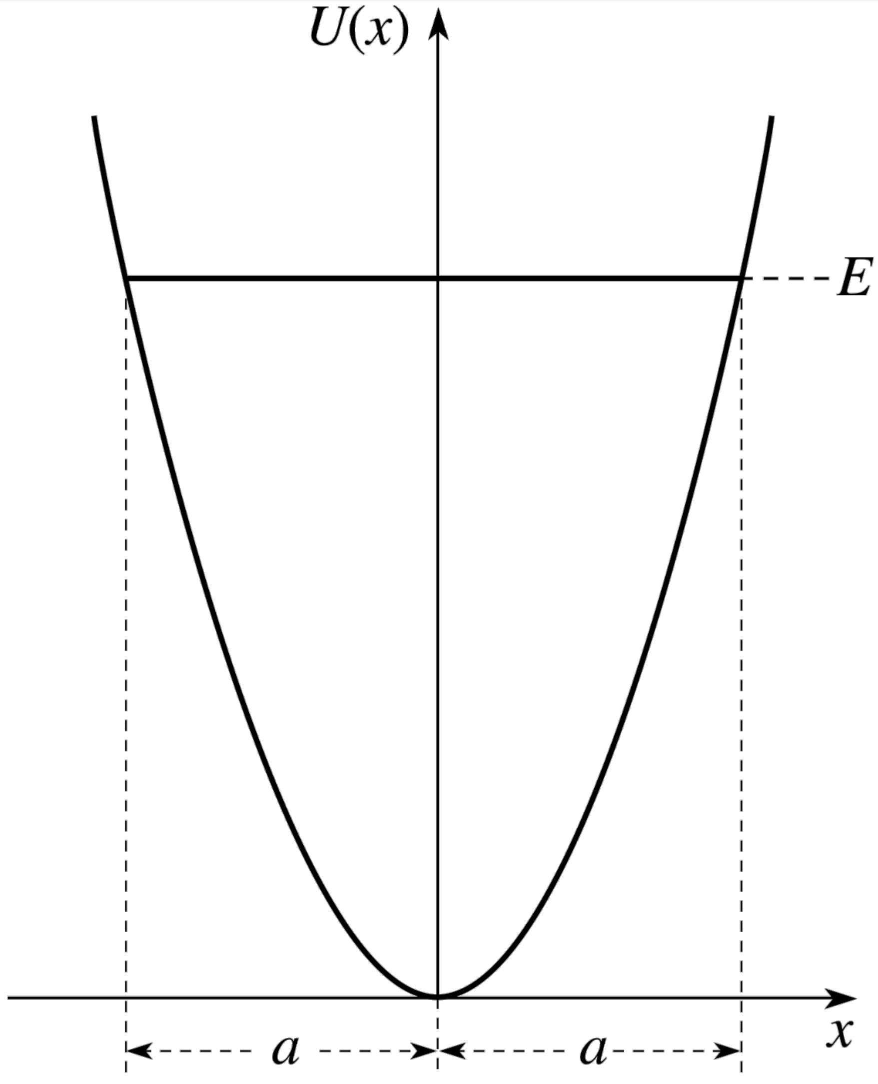




Concept Check: Harmonic Oscillator

- Where should the amplitude of the wave function be largest?

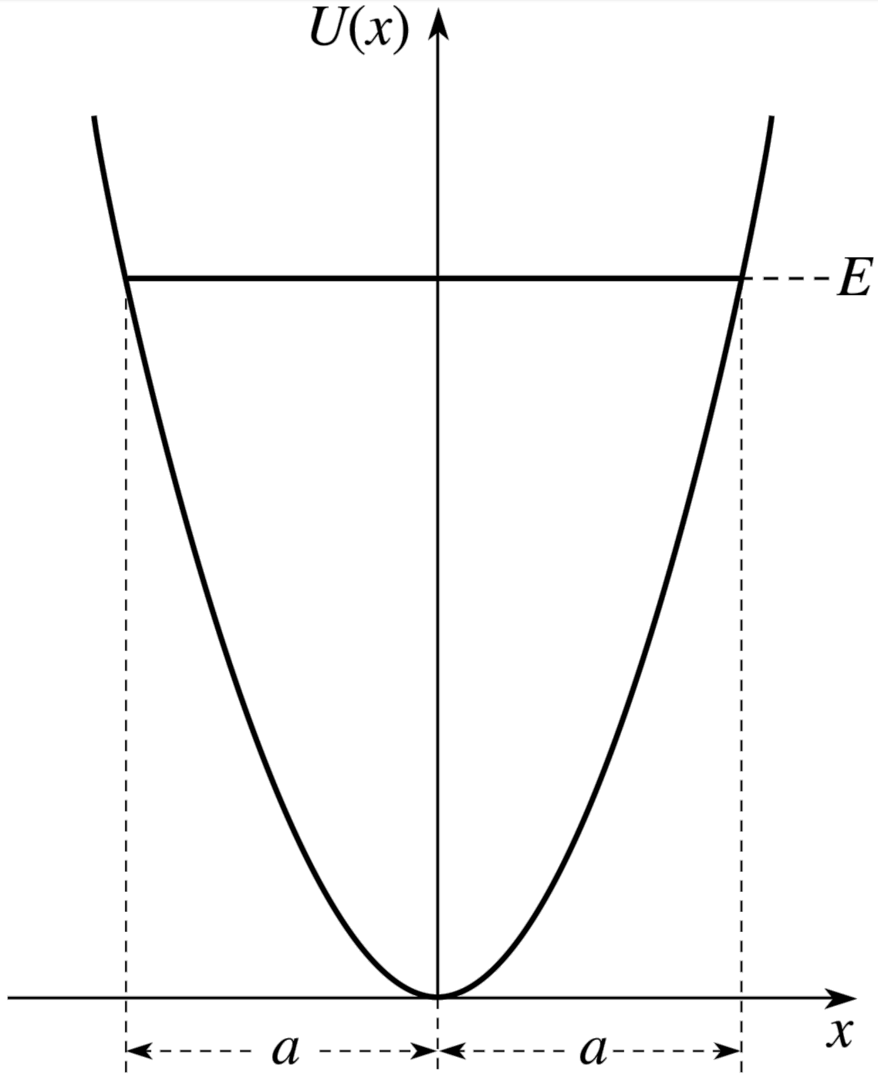
A. Near the center
B. Near the edges
C. Same or similar everywhere



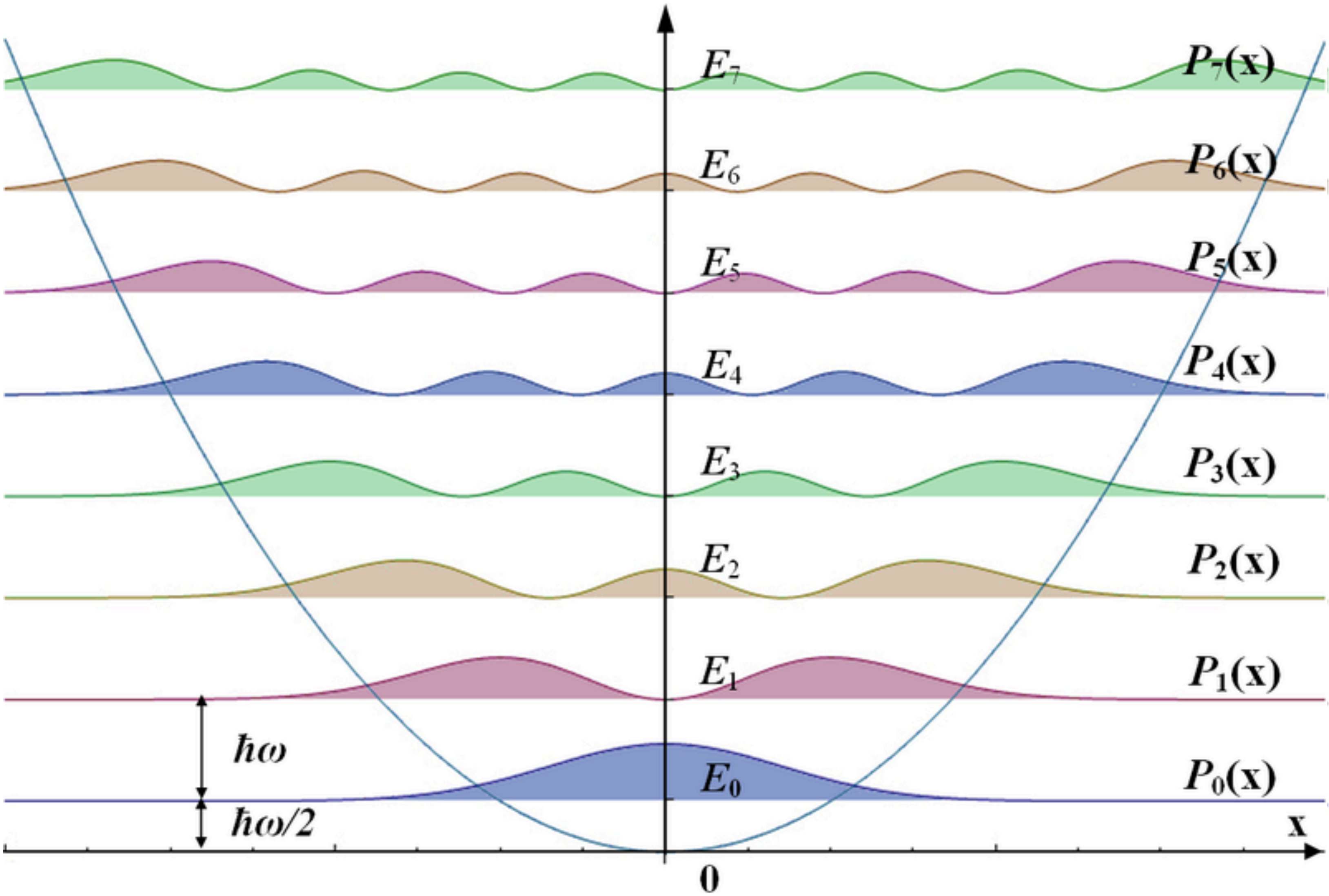
Concept Check: Harmonic Oscillator

- Where should the wavelength of the wave function be longest?

A. Near the center
B. Near the edges
C. Same or similar everywhere



Harmonic Oscillator Wavefunctions



Classical Vs. Quantum Probability