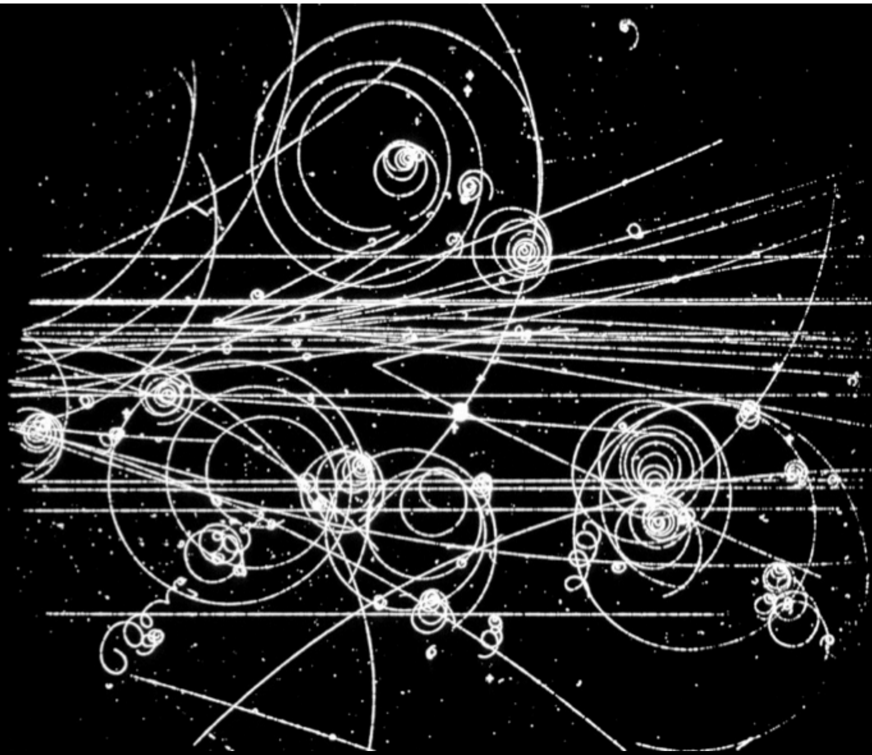
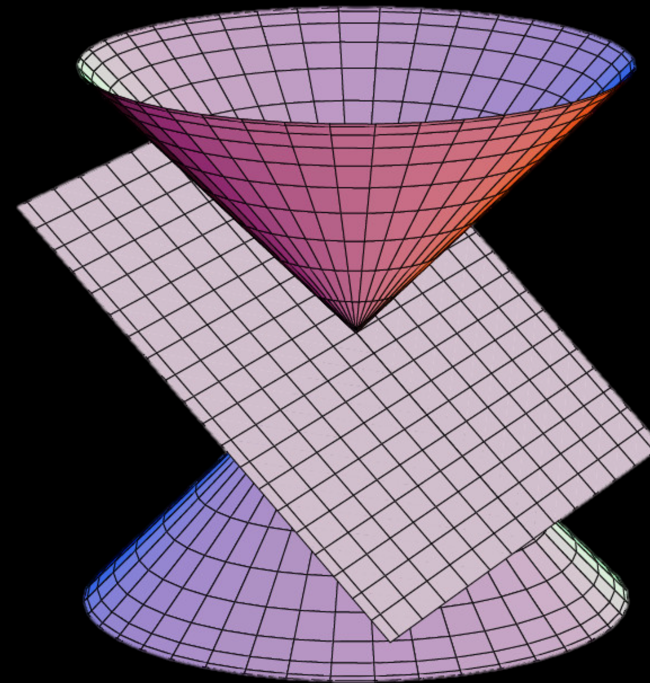




Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Harmonic Oscillator

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{1}{2}kx^2\psi = E\psi$$

- classical solution

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$$

x, v sinusoidal w/ $\omega = \sqrt{k/m}$

QM solution

$$\psi(x) = f(x)\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\psi_0(x) = A\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\partial\psi_0/\partial x = A \cdot -\frac{2m\omega x}{2\hbar}\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= -A\frac{m\omega x}{\hbar}\, e^{-m\omega/2\hbar\, x^2}$$

$$\partial^2\psi_0/\partial x = -A\frac{m\omega}{\hbar}\, e^{-m\omega/2\hbar\, x^2} + A\left(\frac{m\omega x}{\hbar}\right)^2 e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\Rightarrow -\frac{\hbar^2}{2m}\left(\left(\frac{m\omega x}{\hbar}\right)^2 - \left(\frac{m\omega}{\hbar}\right)\right)\cdot A e^{-\frac{m\omega}{2\hbar}x^2}$$

$$+ \frac{1}{2}kx^2 A e^{-\frac{m\omega}{2\hbar}x^2} = E_0 A e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\Rightarrow \frac{-m\omega^2x^2}{2} + \frac{\omega\hbar}{2} + \frac{1}{2}kx^2 = E_0$$

$$\Rightarrow -\frac{kx^2}{2} + \frac{\omega\hbar}{2} + \frac{kx^2}{2} = E_0 \quad //$$

$$\Rightarrow \boxed{E_0 = \frac{\hbar\omega}{2}}$$

$$E_1 = \tfrac{3}{2}\hbar\omega$$

$$\psi_1 = A x \, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$E_2 = \tfrac{5}{2}\hbar\omega$$

$$\psi_2 = A\left(\frac{2m\omega}{\hbar}x^2 - 1\right) e^{-\frac{m\omega}{2\hbar}x^2}$$

$$E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega$$

w/ $\omega = \sqrt{k/m}$

$$\psi_n = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}\, x\right)$$

$$H_n(z) = (-1)^n e^{z^2} \frac{d^n}{dz^n}\left(e^{-z^2}\right)$$

= "Hermite Polynomials"

Quantum Mechanical Harmonic Oscillator

First four harmonic oscillator normalized wavefunctions

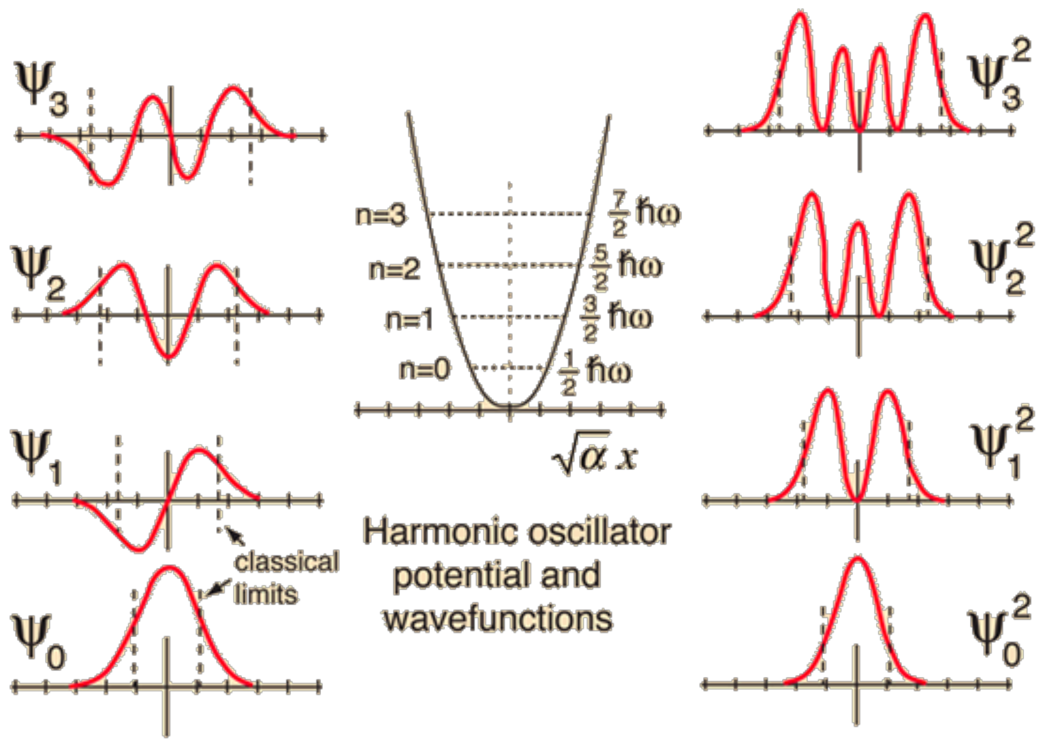
$$\Psi_0 = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-y^2/2}$$

$$\Psi_1 = \left(\frac{\alpha}{\pi}\right)^{1/4} \sqrt{2}y\, e^{-y^2/2}$$

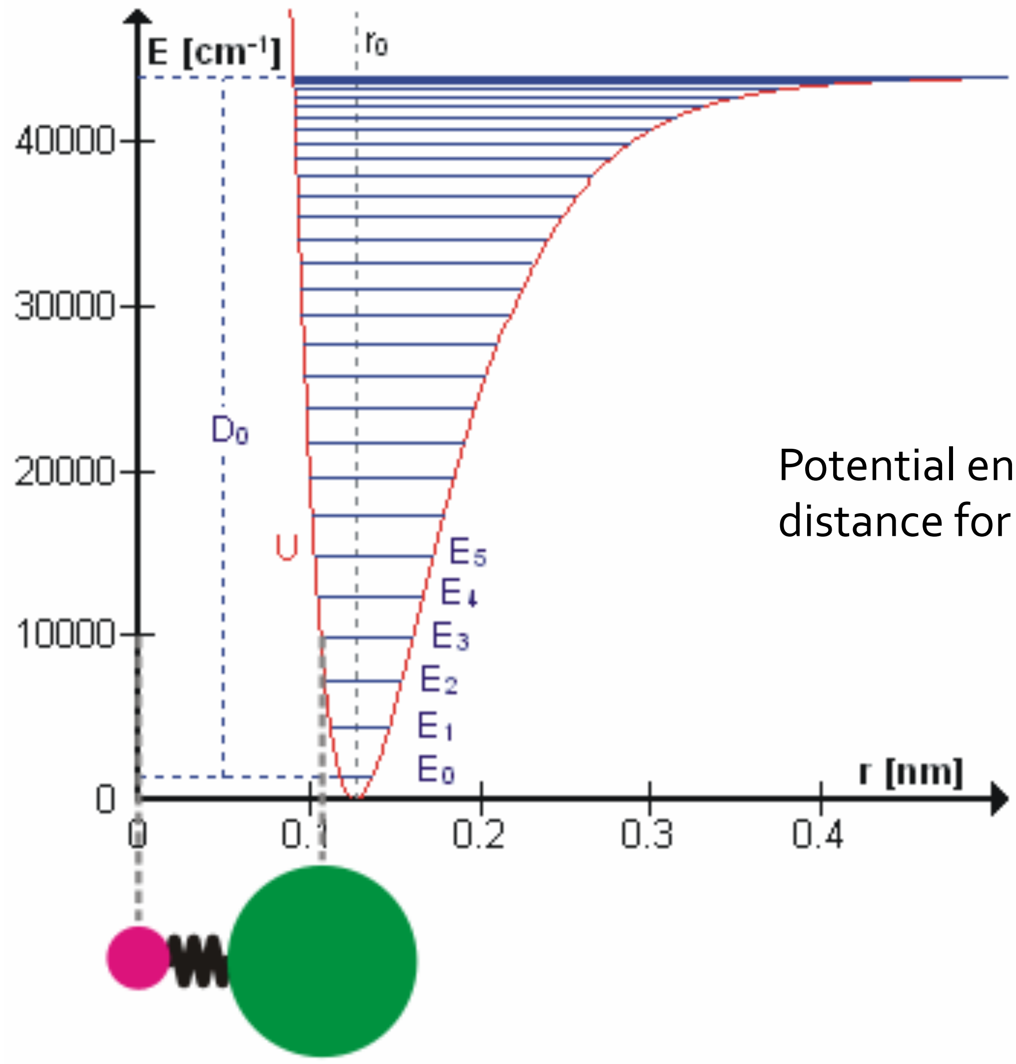
$$\Psi_2 = \left(\frac{\alpha}{\pi}\right)^{1/4} \frac{1}{\sqrt{2}}(2y^2 - 1)\, e^{-y^2/2}$$

$$\Psi_3 = \left(\frac{\alpha}{\pi}\right)^{1/4} \frac{1}{\sqrt{3}}(2y^3 - 3y)\, e^{-y^2/2}$$

$$\alpha = \frac{m\omega}{\hbar} \qquad y = \sqrt{\alpha}\, x$$

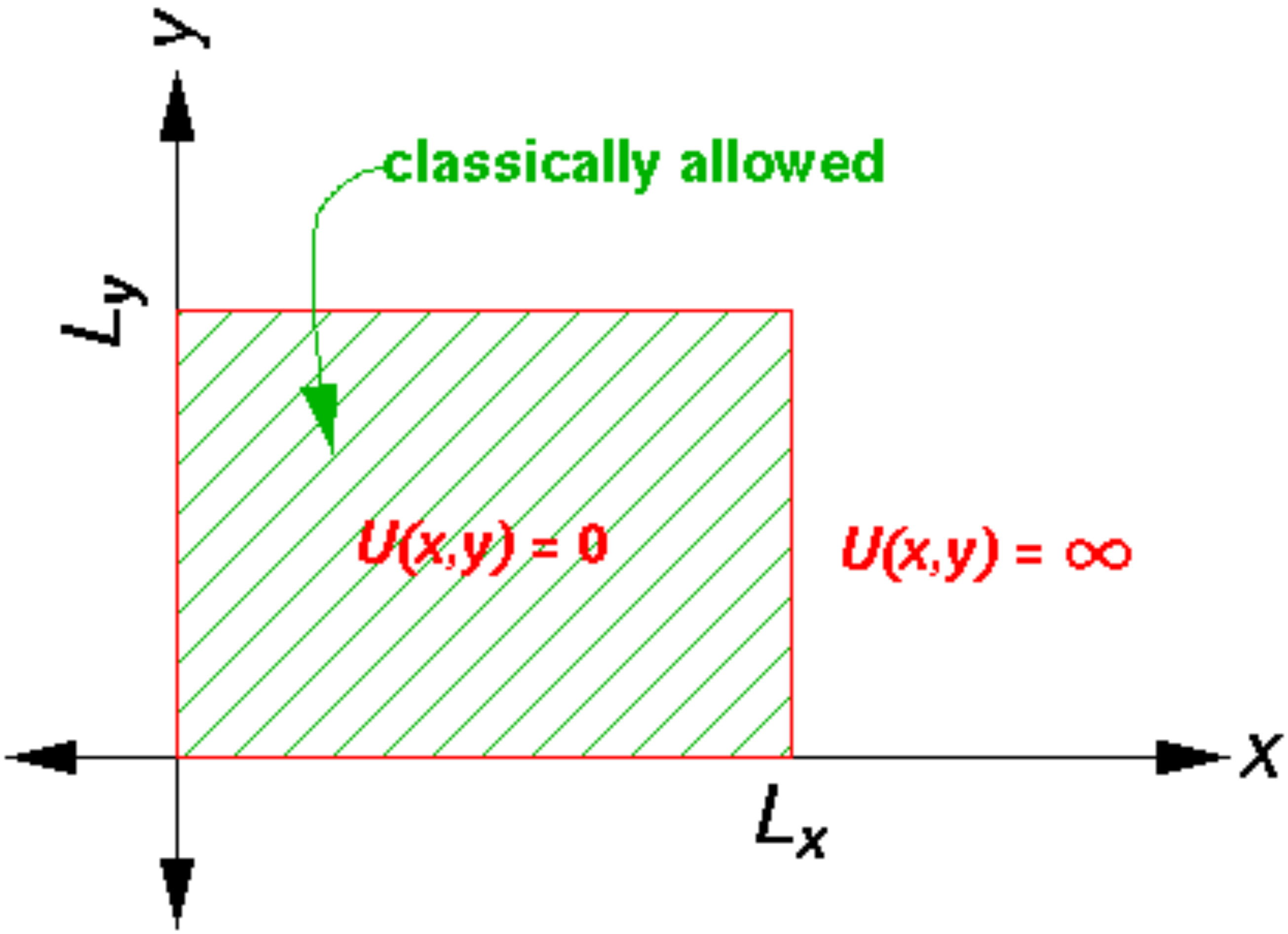


Why Study Harmonic Oscillators?



Potential energy vs. distance for HCl molecule

2d Infinite Square Well



Schrodinger Equation in 2-d or 3-d

3-d: $-\hbar^2/2m \nabla^2 \psi + U\psi = E\psi$

2-d: $-\hbar^2/2m \left(\partial^2\psi/\partial x^2 + \partial^2\psi/\partial y^2\right) + U\psi = E\psi$

Like $p^2 = p_x^2 + p_y^2$

$-\hbar^2/2m \nabla^2 = -\hbar^2/2m\ \partial^2/\partial x^2 - \hbar^2/2m\ \partial^2/\partial y^2$

Separation of Variables

Find $\psi(x,y) = f(x)g(y)$

$$\partial^2\psi/\partial x^2 = \partial^2 f(x)/\partial x^2\ g(y)$$
$$\partial^2\psi/\partial y^2 = f(x)\ \partial^2 g(y)/\partial y^2$$

Square Well

Solve for case $U(x,y) = 0$

w/ boundary conditions

$\psi(x,y) = 0$ $\quad x = 0, L_x$

$\qquad\qquad\qquad\ \ y = 0, L_y$

$$-\hbar^2/2m \left[\partial^2 f/\partial x^2\, g + f\, \partial^2 g/\partial y^2\right] = Efg$$

Try: $f(x) = A\sin(k_x x) + B\cos(k_x x)$

$\qquad g(y) = C\sin(k_y y) + D\cos(k_y y)$

$f(x) = 0 \;@\; x = 0, L_x$

$\Rightarrow f(x) = A \sin\left(\frac{n\pi x}{L_x}\right)$

$g(y) = 0 \;@\; y = 0, L_y$

$\Rightarrow g(y) = C \sin\left(\frac{m\pi y}{L_y}\right)$

$\psi_{nm}(x,y) = AC \sin\left(\frac{n\pi x}{L_x}\right) \sin\left(\frac{m\pi y}{L_y}\right)$

Verify!

LHS: $-\hbar^2/2m \cdot \Big[-AC \cdot \left(\frac{n\pi}{L_x}\right)^2 \sin\left(\frac{n\pi x}{L_x}\right) \sin\left(\frac{m\pi y}{L_y}\right)$
$\qquad - AC \cdot \left(\frac{m\pi}{L_y}\right)^2 \sin\left(\frac{n\pi x}{L_x}\right) \sin\left(\frac{m\pi y}{L_y}\right)$

RHS: $E AC \sin\left(\frac{n\pi x}{L_x}\right) \sin\left(\frac{m\pi y}{L_y}\right)$

$$\Rightarrow E_{nm} = \frac{\hbar^2 n^2 \pi^2}{2m L_x^2} + \frac{\hbar^2 m^2 \pi^2}{2m L_y^2}$$

$$= p_x^2/2m + p_y^2/2m$$

$$= \frac{\hbar^2 \pi^2}{2m} \left[\frac{n^2}{L_x^2} + \frac{m^2}{L_y^2} \right]$$

Two quantum numbers!

$$\Psi_{nm}(x,y,t) = \psi_{nm}(x,y)\, e^{-iE_{nm}t/\hbar}$$

Concept Check

- What normalization condition should the 2-d square well functions satisfy?

A. $\int |\psi|^2 dx = 1$

B. $\iint |\psi|^2 dxdy = 1$

C. $\iint |\psi|^2 dxdy = L^2$

D. $\iiint |\psi|^2 dxdydz = 1$

Concept Check

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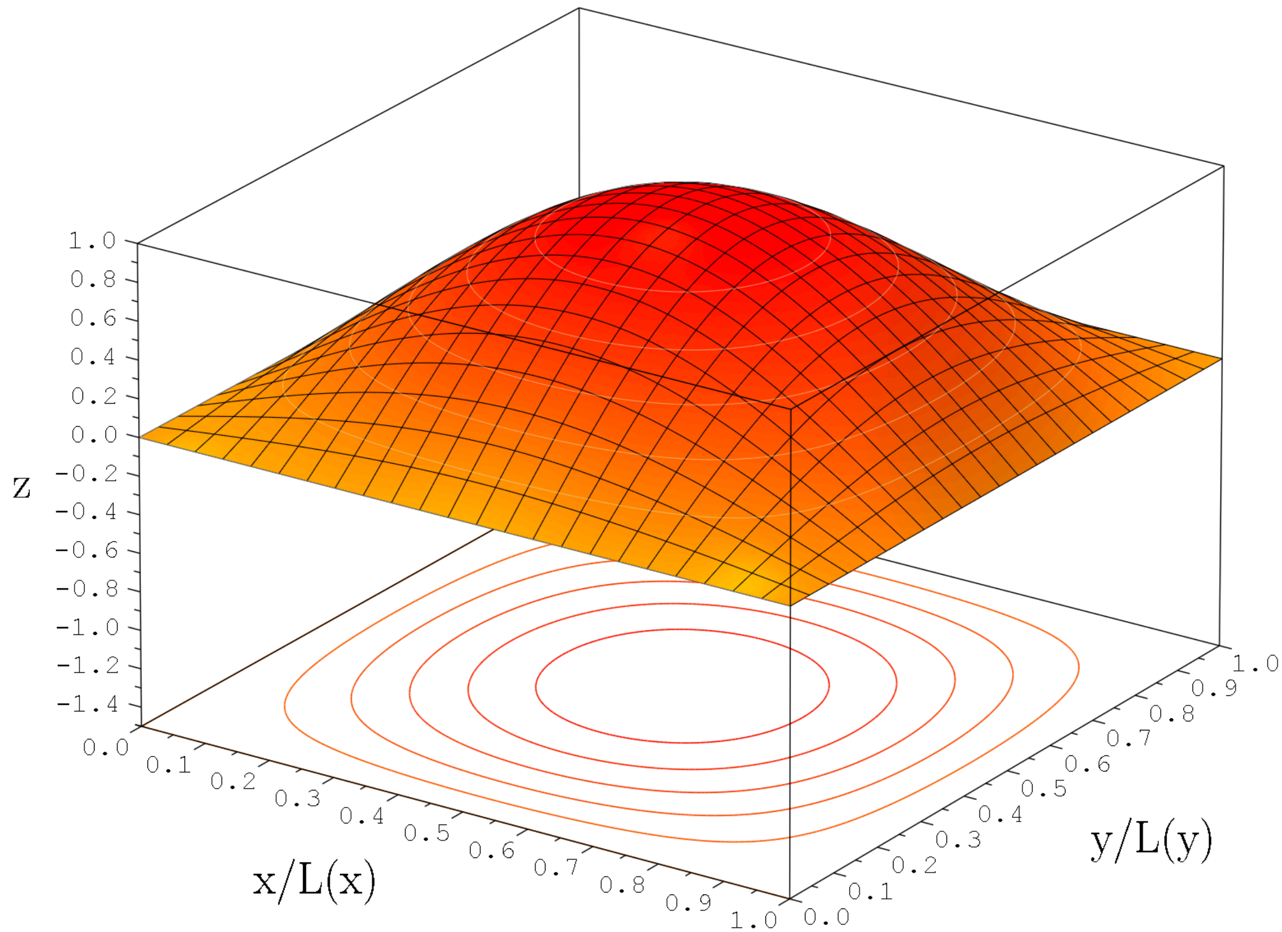
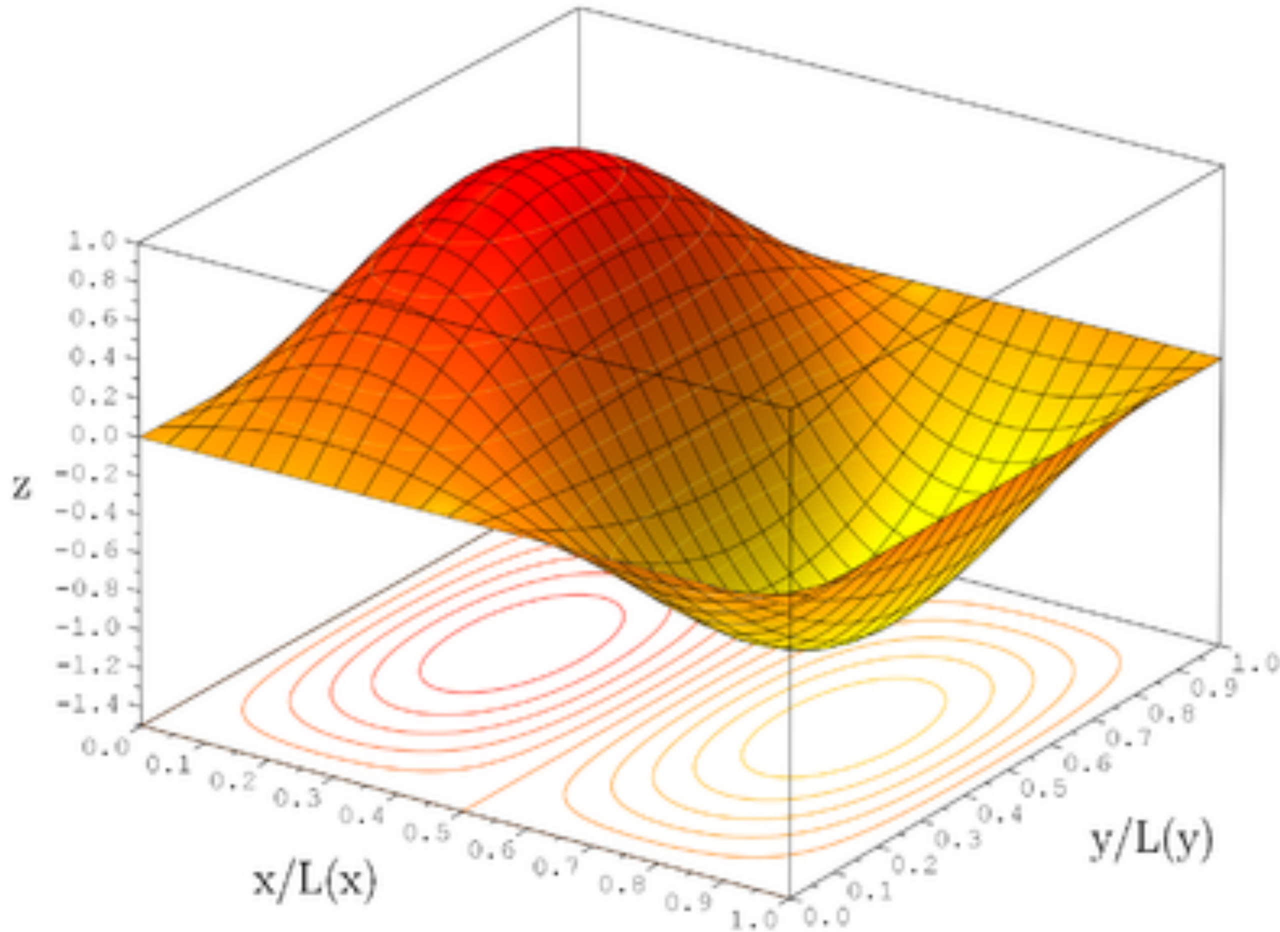
D. $\iiint |\psi|^2 dxdydz = 1$

n = 1, m = 1

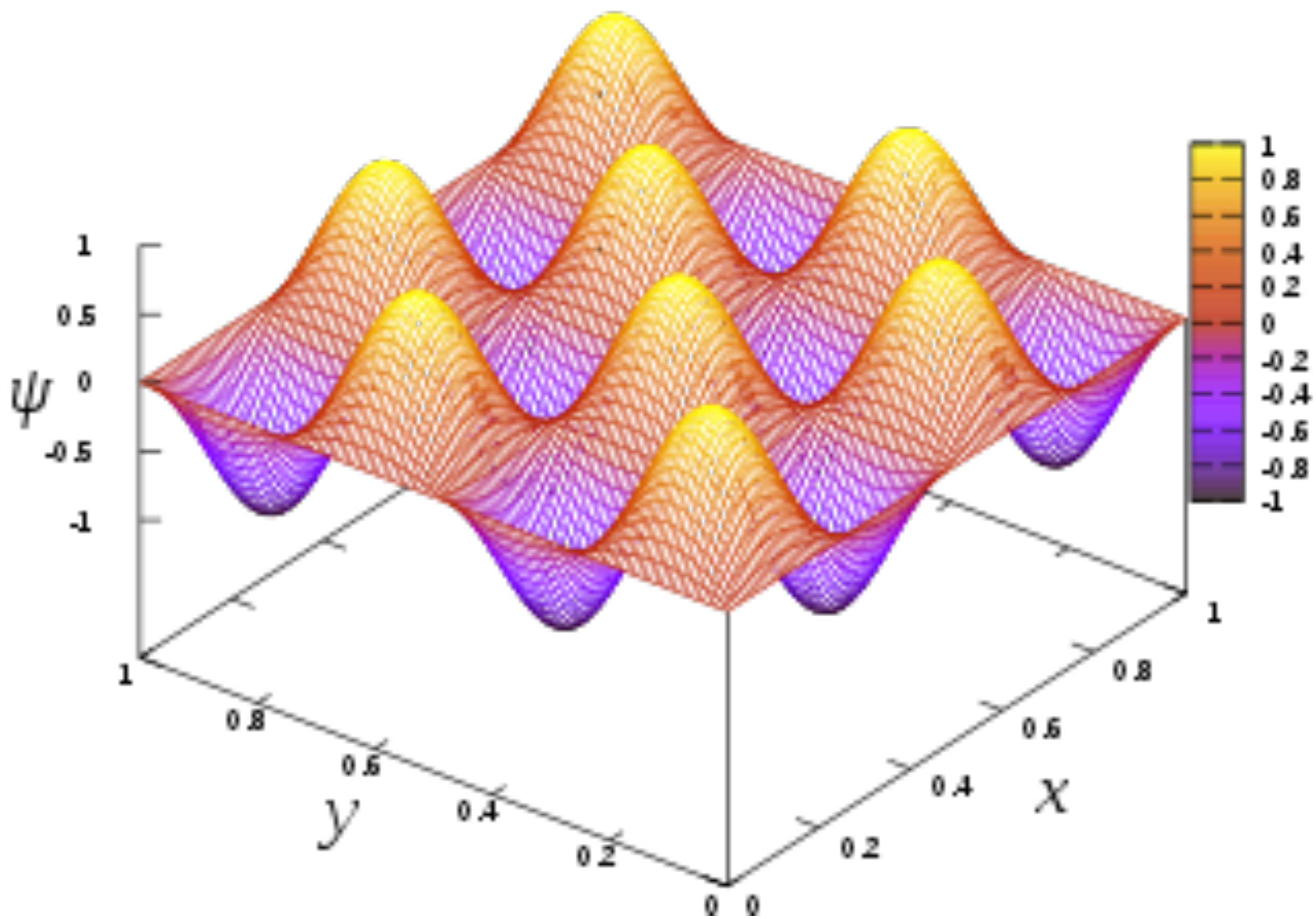


1.0
0.8
0.6
0.4
0.2
0.0
z
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
x/L(x)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
y/L(y)

n = 2, m = 1


z
x/L(x)
y/L(y)
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

n = 4, m = 4



Time Dependence