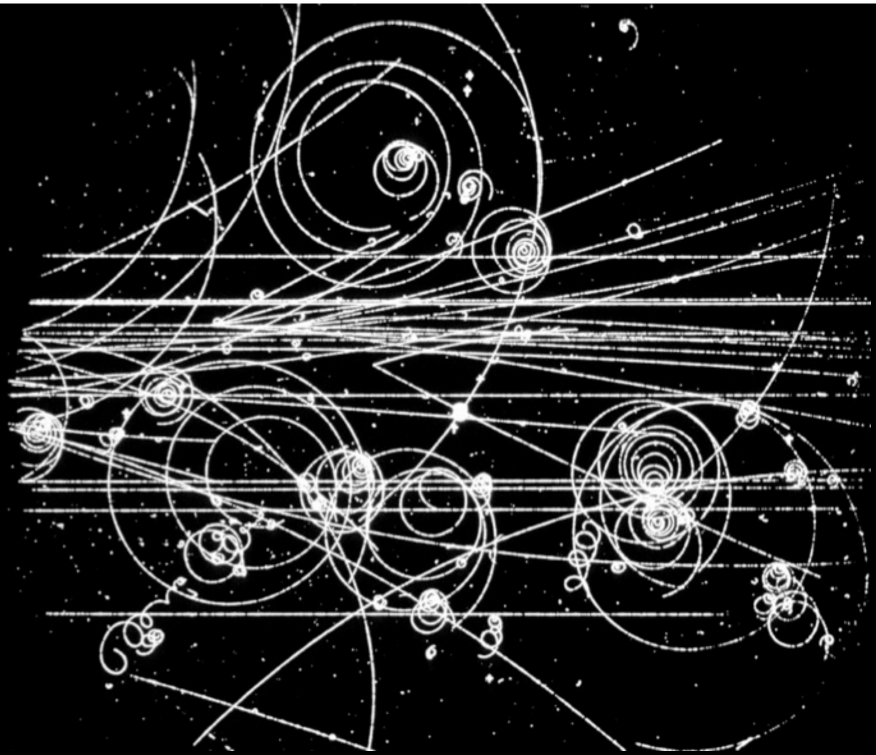
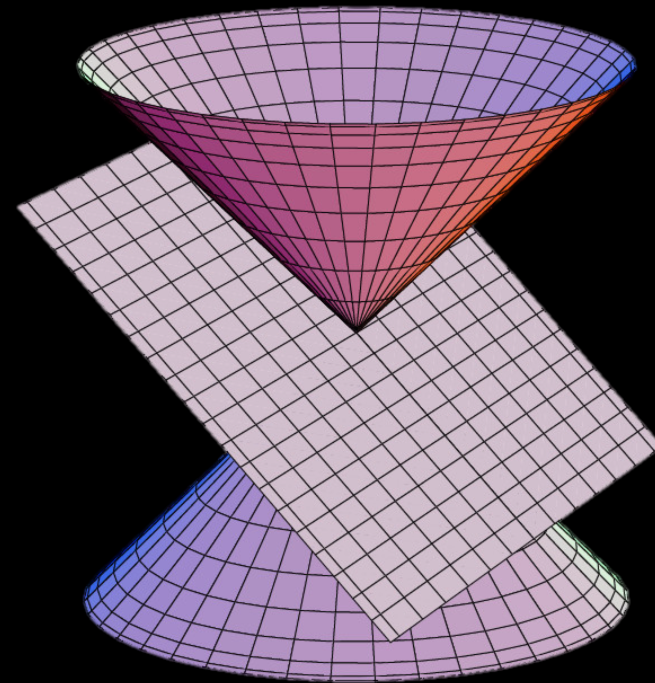




Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70

MWF 12:30-1:20 Lecture

Announcements

- Midterm II in class Wednesday
  - Covers Ch. 5-7 (minus exclusions listed on Friday)
  - Same policies as Midterm I
  - Practice Midterm II solutions now posted

Schrödinger Equation

Time-Dependent

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+U(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}$$

U(x) constant in time

Time-Independent

$$\frac{-\hbar^2}{2m}\frac{d^2\Psi(x)}{dx^2}+U(x)\Psi(x)=E\Psi(x)$$

Valid Wave Functions

- $\Psi(x,t) = \psi(x)e^{-i\omega t}$ with $\omega = E/\hbar$ (Time-independent U(x))

$\Psi^*(x,t)\Psi(x,t) =$ probability of finding particle at x at time t provided the wavefunction is normalized.

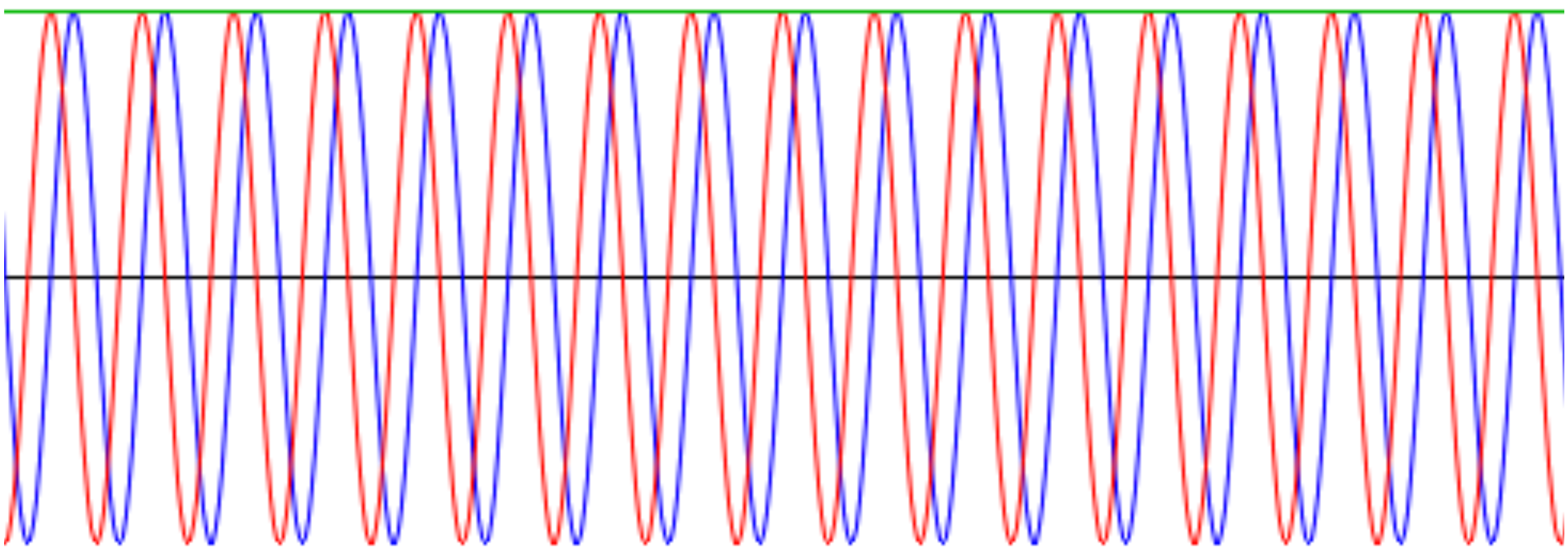
$$\int \Psi^*\Psi \, dr = 1$$

Wave functions must be continuous in value (always) and in slope (unless the potential energy is infinite).

Traveling Wave: Constant Potential ($E>U_0$)

$\psi(x) = A_1 \sin(kx) + B_1 \cos(kx)$

or $\psi(x) = A_2 e^{ikx} + B_2 e^{-ikx}$ $\qquad k = \sqrt{[2m(E-U_0)]}/\hbar$

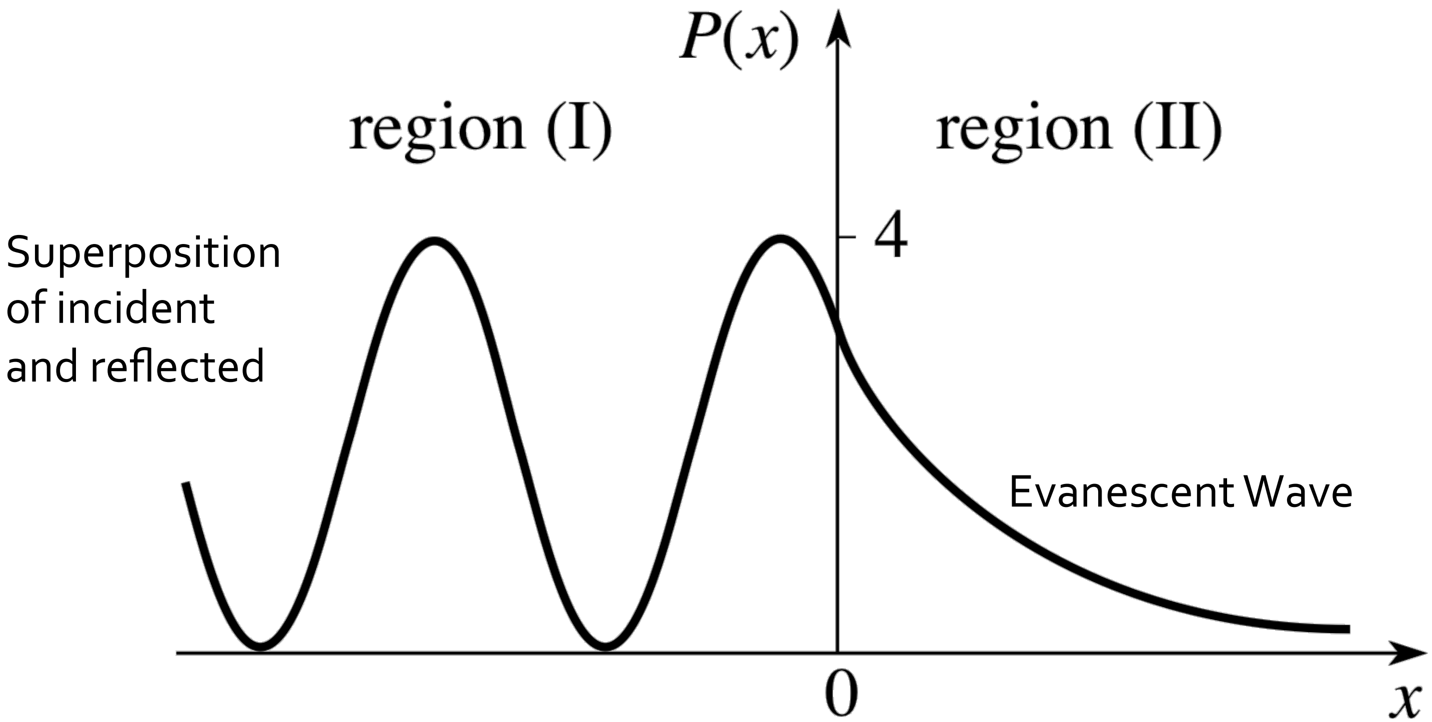


Real, Imaginary, Magnitude

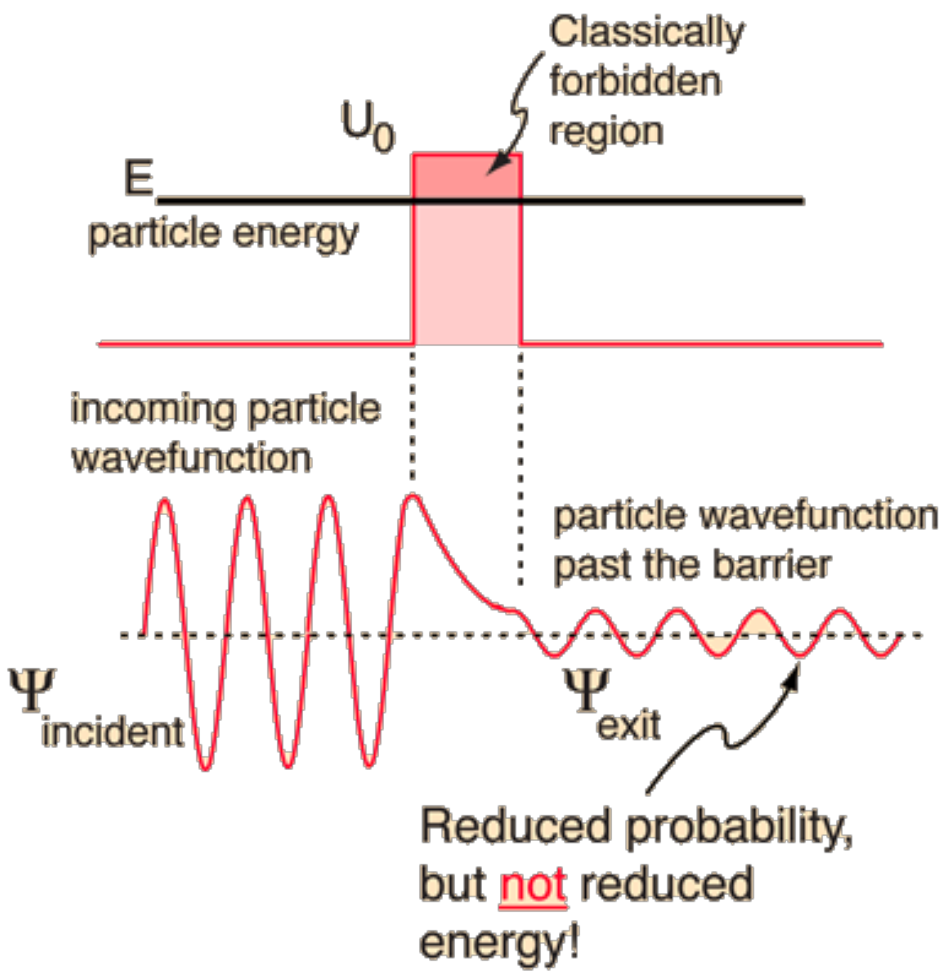
Evanescent Wave: Constant Potential ($E<U_o$)

$\psi(x) = Ce^{kx}$ or De^{-kx}

$k = \sqrt{[2m(U_0-E)]}/\hbar$

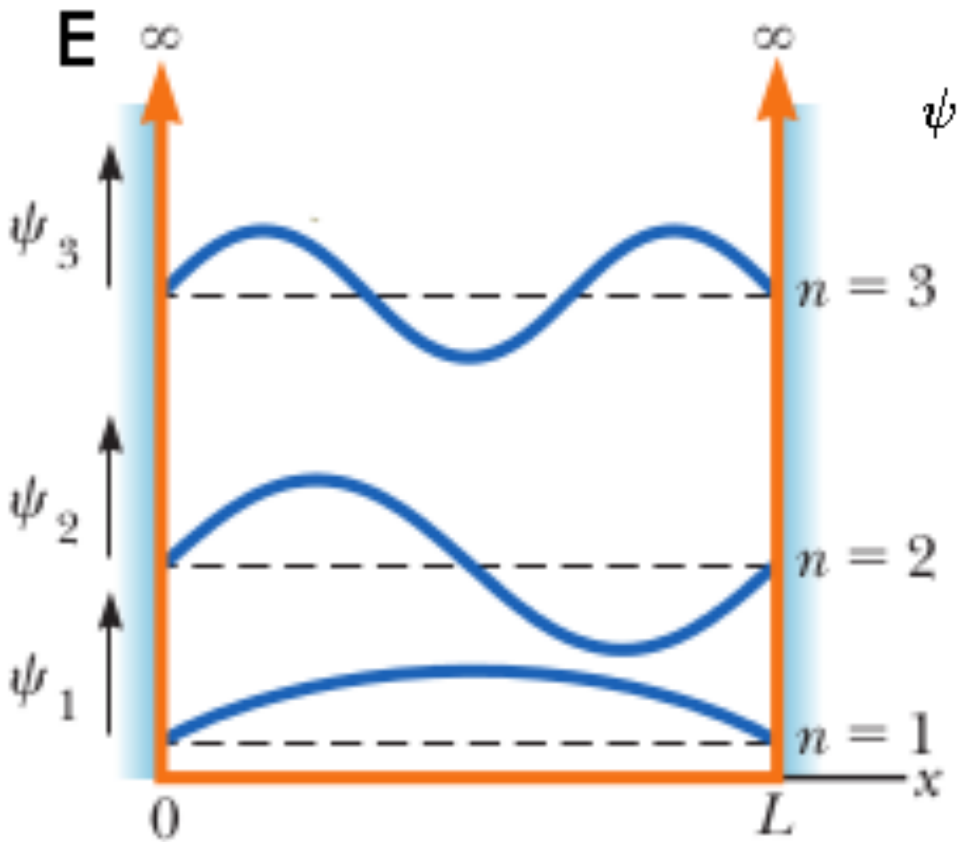


Tunneling



Classically
forbidden
region
U0
E
particle energy
incoming particle
wavefunction
particle wavefunction
past the barrier
Ψ
incident
Ψ
exit
Reduced probability,
but not reduced
energy!

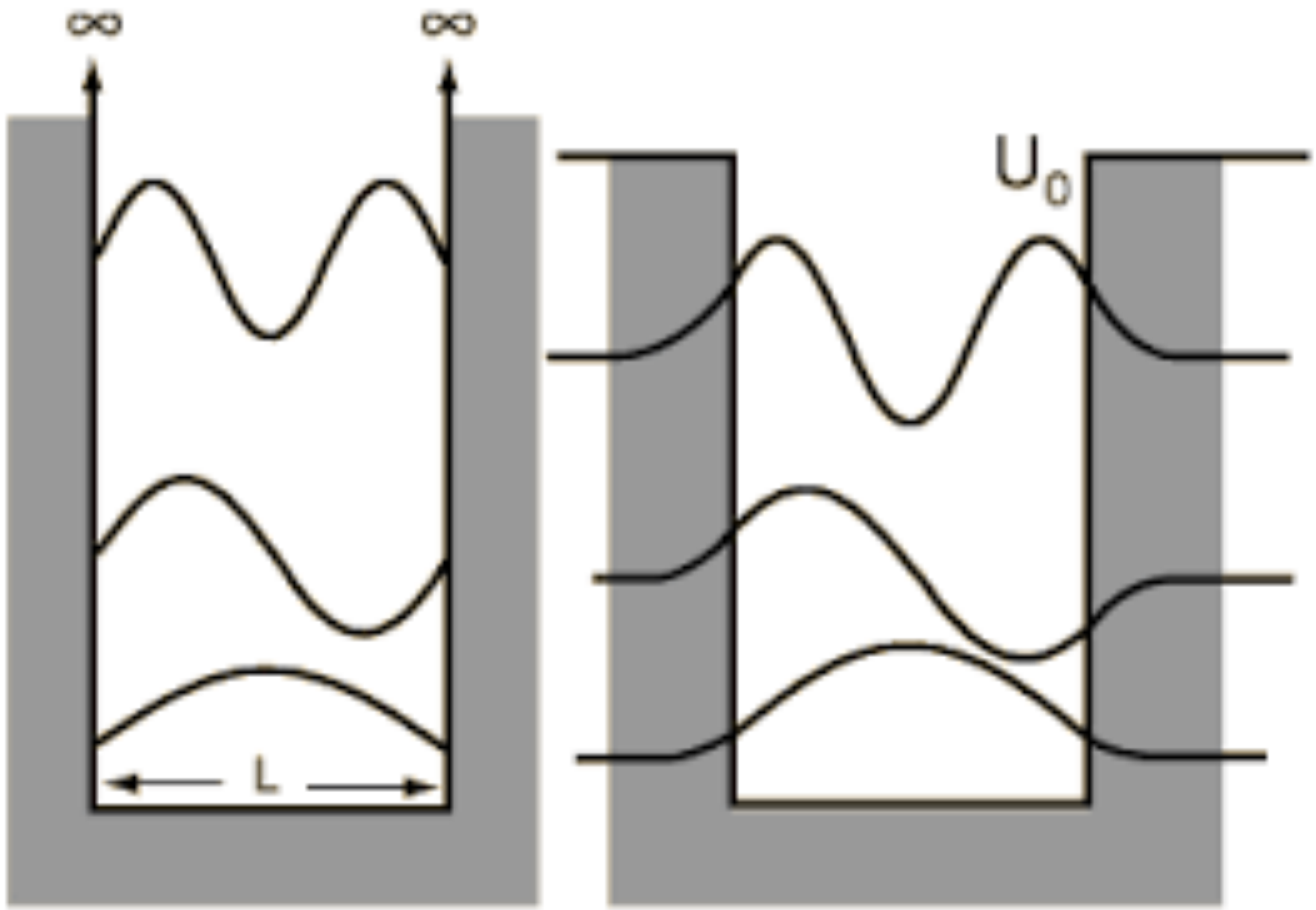
Infinite Square Well



$$\psi(x) = \sqrt{\frac{2}{L}}\,\sin\left(k_n x\right) \qquad k_n = \frac{\pi n}{L}$$

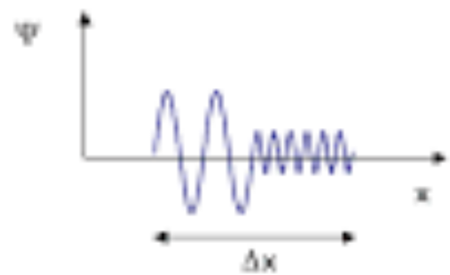
$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

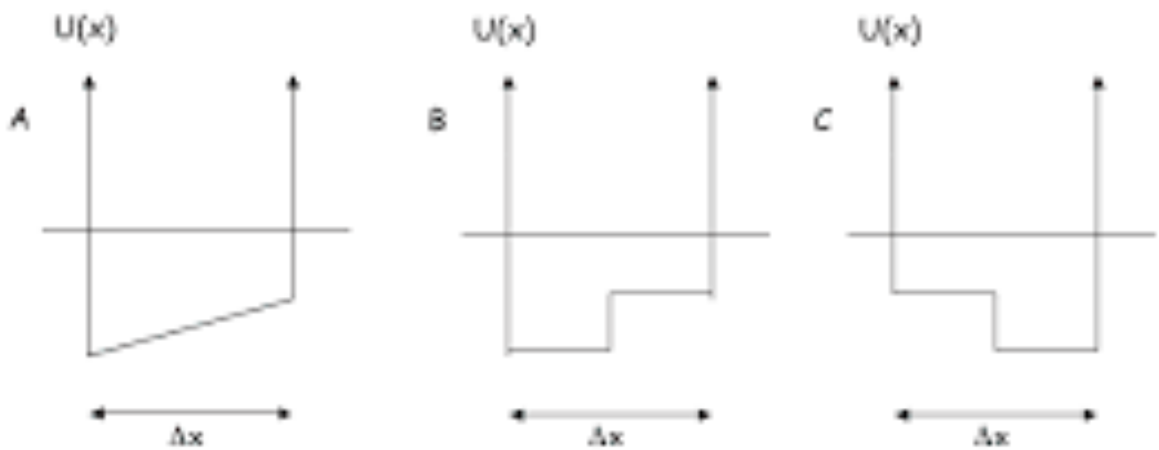
Infinite -> Finite Square Well



Concept Check

Which potential well would this wave function make sense for?





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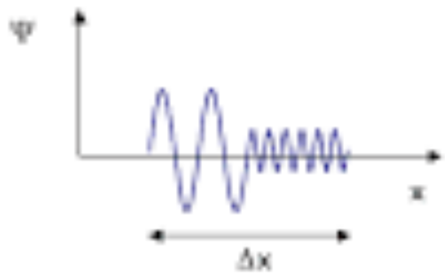
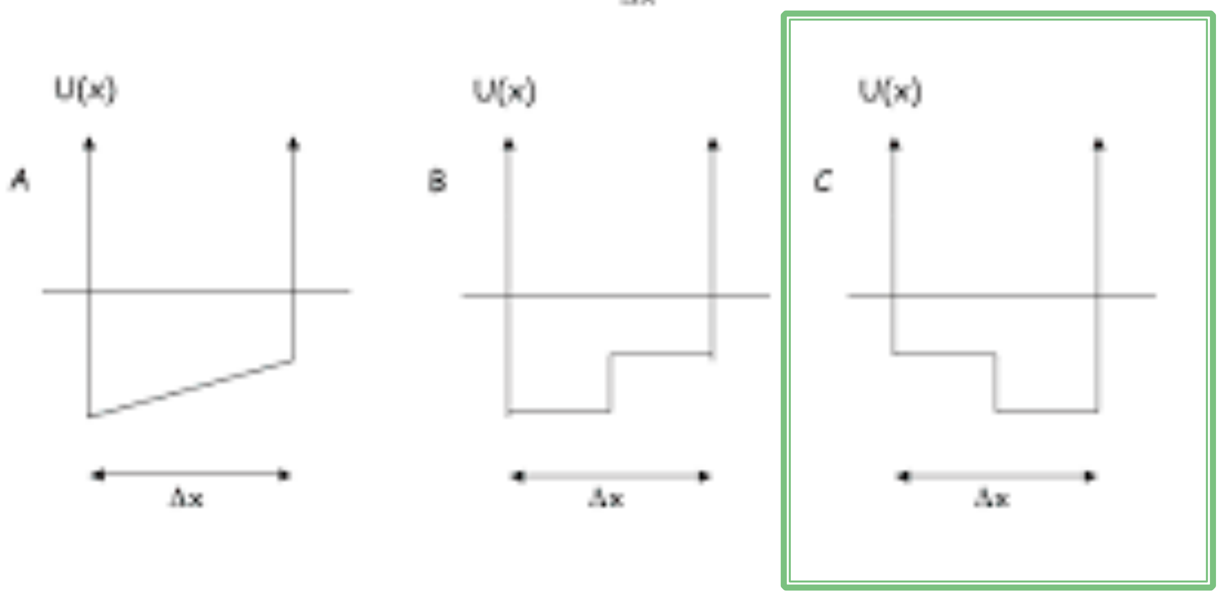
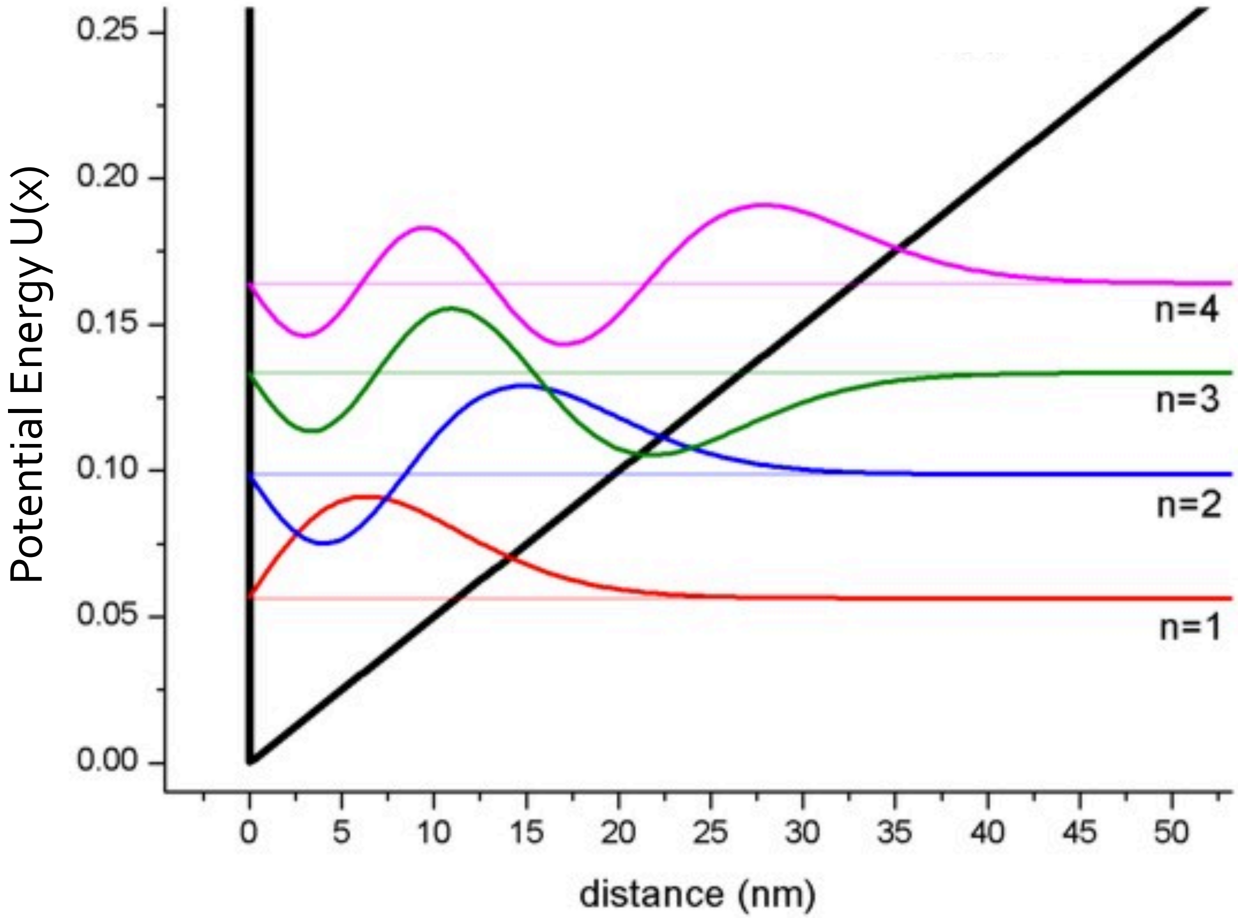
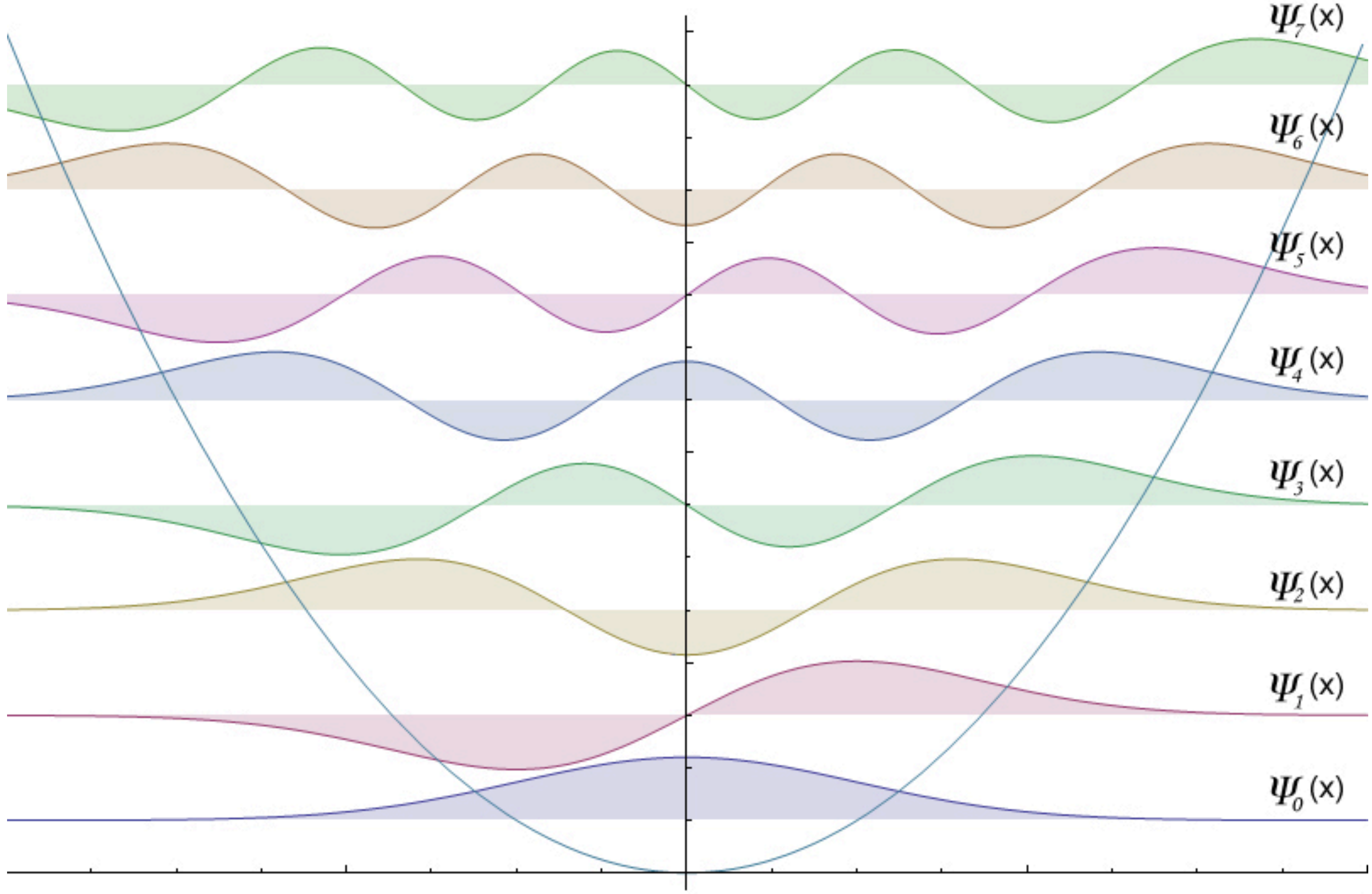




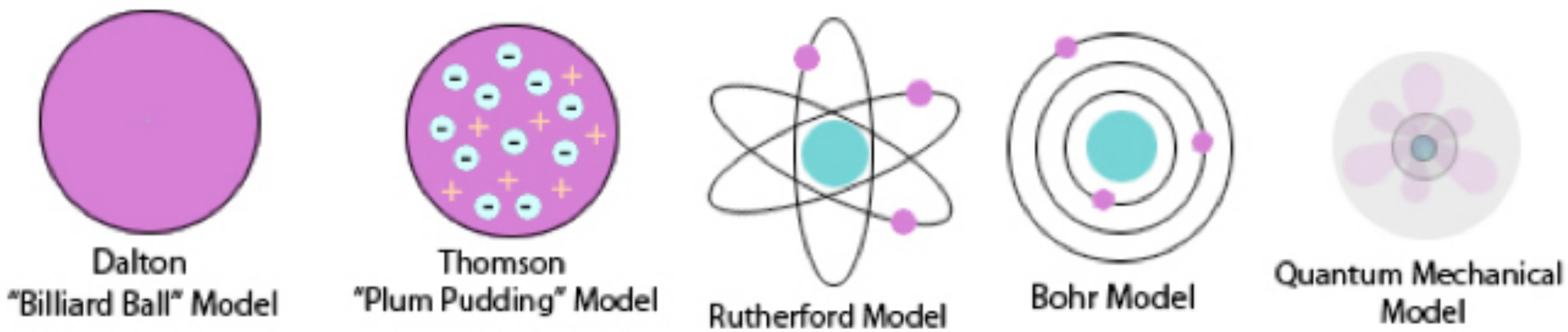
Asymmetric Well



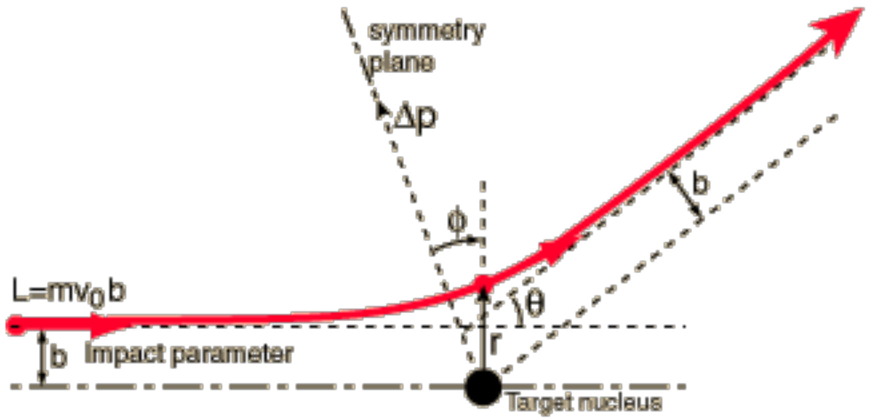
Harmonic Oscillator



Atomic Models

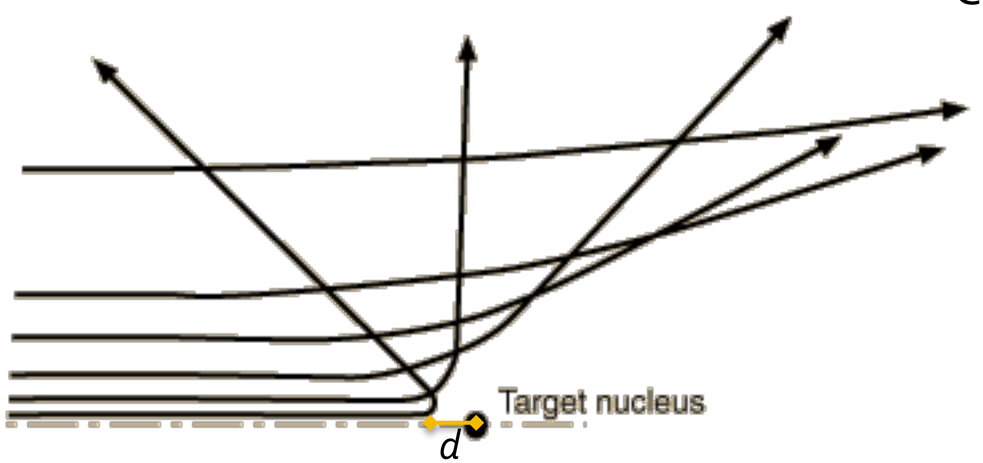


Rutherford Scattering



Impact Parameter

$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{E}\quad\frac{z}{2}$$

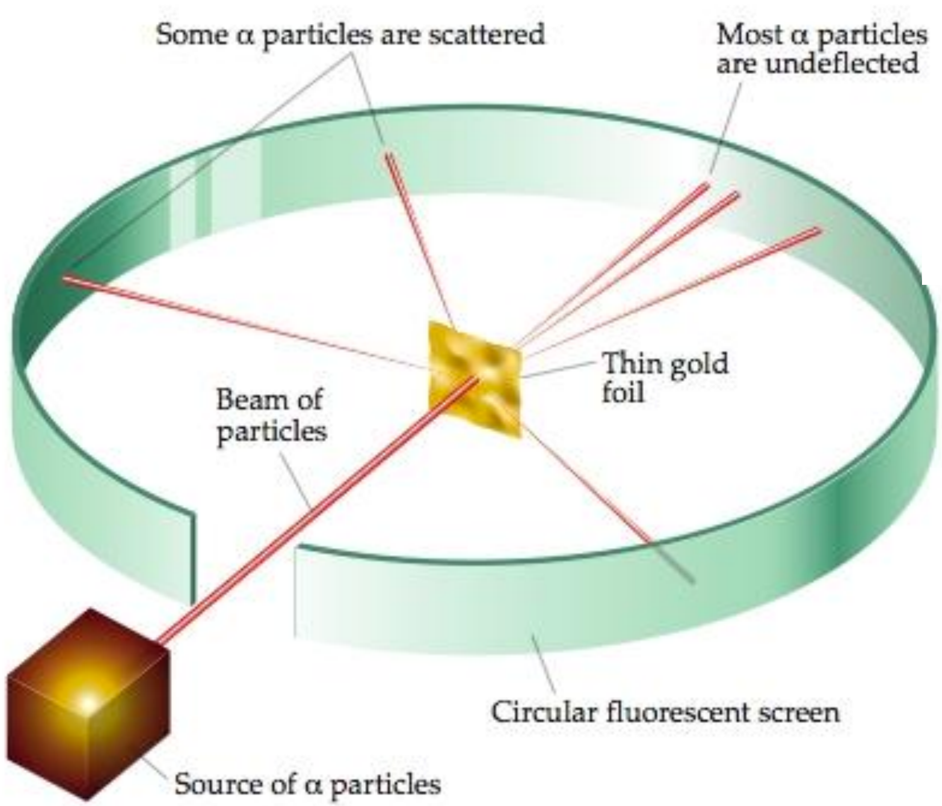


Closest Approach (b=o)

$$E = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\,\frac{z\ eZe}{d}$$

$$d = \frac{z\ Ze^2}{4\pi\varepsilon_0 K}$$

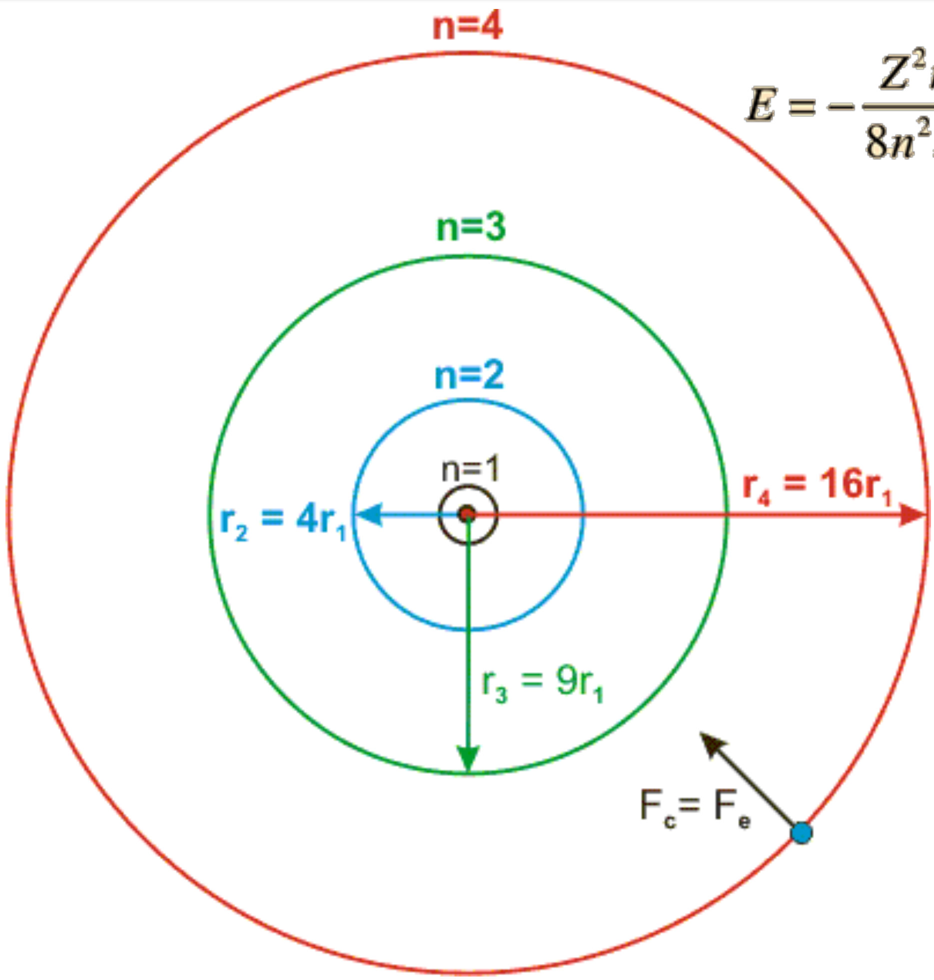
Rutherford Scattering



$$N(\theta) = \frac{nt}{4r^2}\left(\frac{zZ}{2K}\right)^2\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{1}{\sin^4(\frac{1}{2}\theta)}$$

N = scattered flux
n = target density
t = thickness of target
r = detector distance
K = projectile energy
θ = scattering angle

Bohr Model for Hydrogenic Atoms



$$E = -\frac{Z^2 me^4}{8n^2h^2\varepsilon_0^2} = \frac{-13.6Z^2}{n^2} eV \qquad r = \frac{n^2h^2\varepsilon_0}{Z\pi me^2} = \frac{n^2 a_0}{Z}$$

$$a_0 = 0.0529nm = Bohr\ \ radius$$

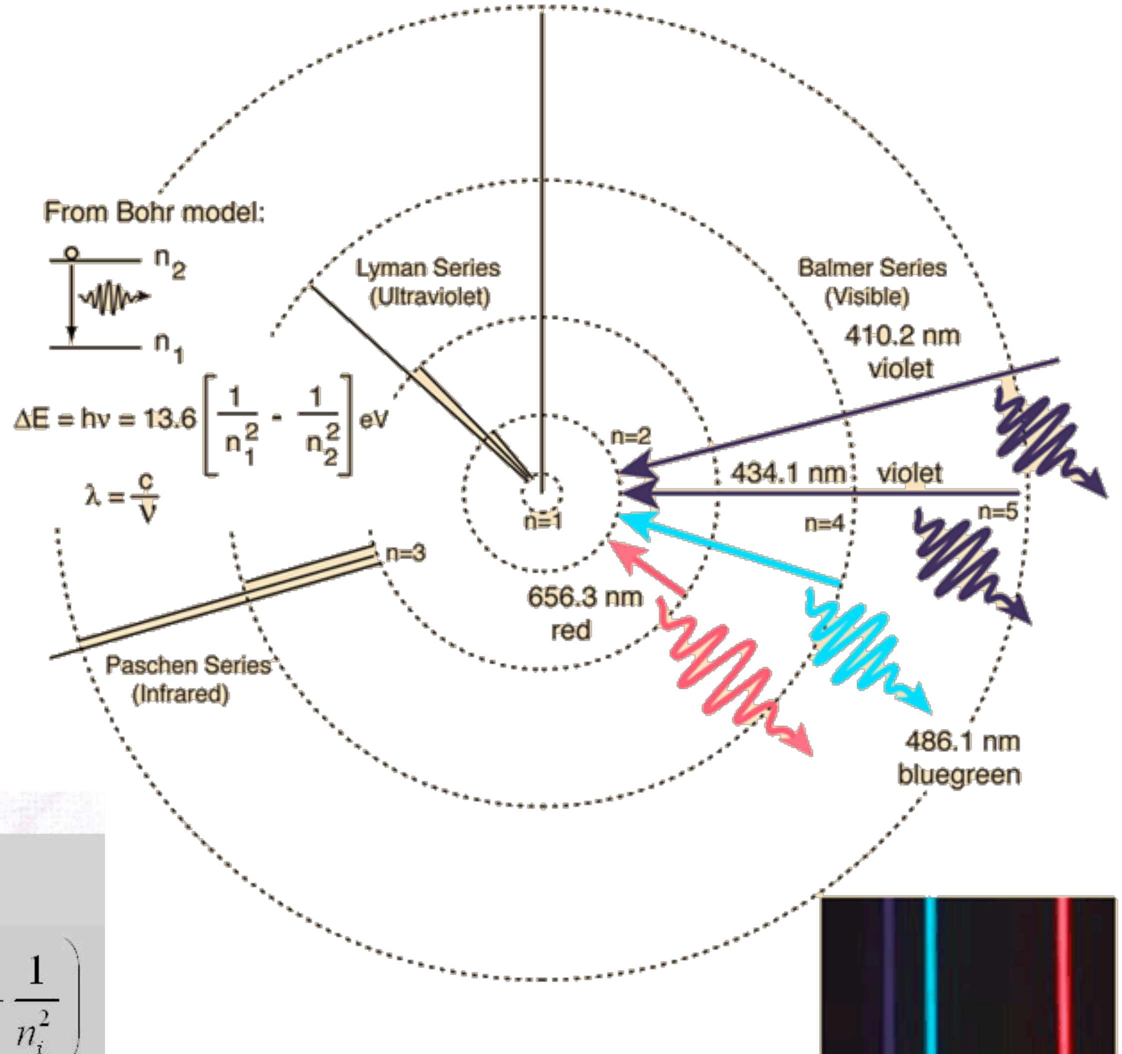
Bohr model works for atoms (or ions) with only one electron

Assumes circular orbits, with angular momentum L = nħ

Not consistent with wavelike nature of electrons

Violates uncertainty principle

Emission/Absorption Spectrum

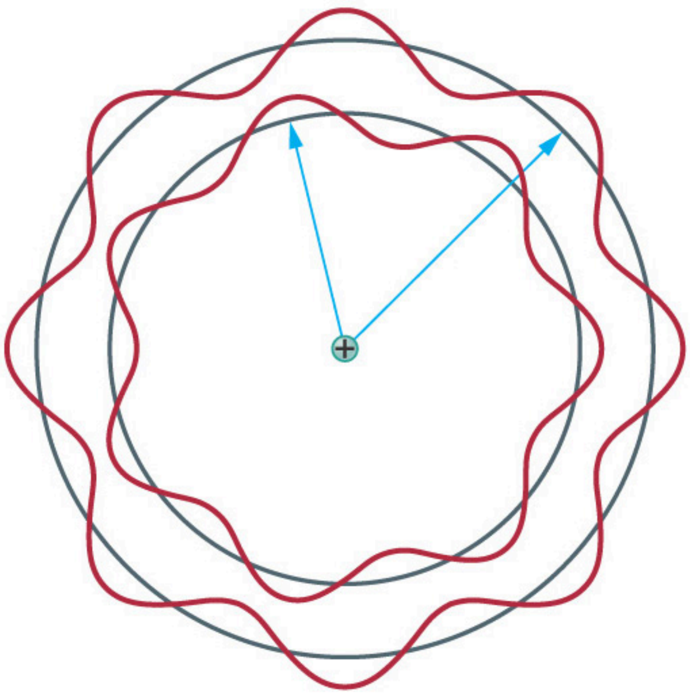


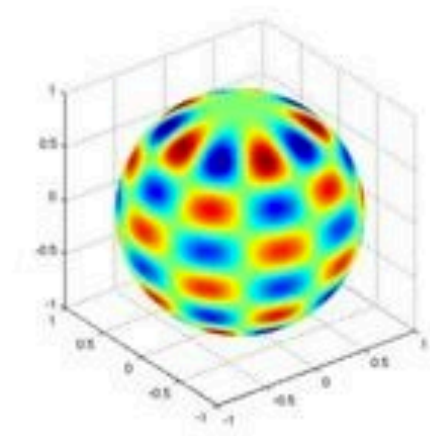
$$h\upsilon = \frac{Z^2 m e^4}{8h^2\varepsilon_0^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = \frac{\nu}{c} = \frac{E_i - E_f}{ch} = \frac{E_0}{ch}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

$$= \frac{m_e e^4}{4c\pi\hbar^3\left(4\pi\varepsilon_0\right)^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \equiv R_\infty\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

From 1-d to 3-d Standing Waves!

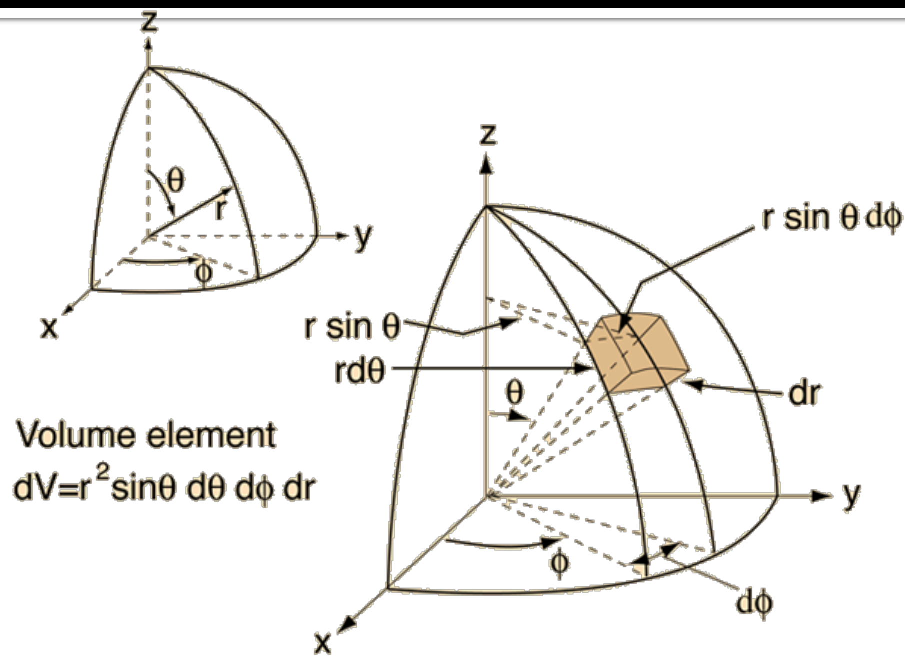




$$\ell = 10$$

$$m = 5$$

Spherical Schrödinger Equation



r (radial): 0 to ∞
θ (polar): 0 to π
ϕ (azimuthal): 0 to 2π

$$\frac{-\hbar^2}{2\mu}\frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right)+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right)+\frac{1}{\sin\theta}\frac{\partial^2\Psi}{\partial\phi^2}\right]$$

$$+U(r)\Psi(r,\theta,\phi)=E\,\Psi(r,\theta,\phi)$$

Hydrogen Atom: Separation of Variables Solution

$$U(r) = \frac{-e^2}{4\pi\varepsilon_0 r}$$

$$\Psi(r,\theta,\phi) = R(r)\,\Theta(\theta)\Phi(\phi)$$

n	ℓ	m_ℓ
principal quantum number	orbital quantum number	magnetic quantum number

$$\Phi(\phi) = Ae^{im_\ell \phi} \qquad m_\ell = -\ell, -\ell+1, \ldots +\ell$$

$$\Theta_{\ell m}(\theta) = N_{\ell m} P_n^m(\cos\theta) \qquad \ell = 0,1,2,3,\ldots n-1$$

$$R_{n,l} = r^l L_{n,l} e^{-r/na_0} \qquad n = 1,2,3,\ldots$$

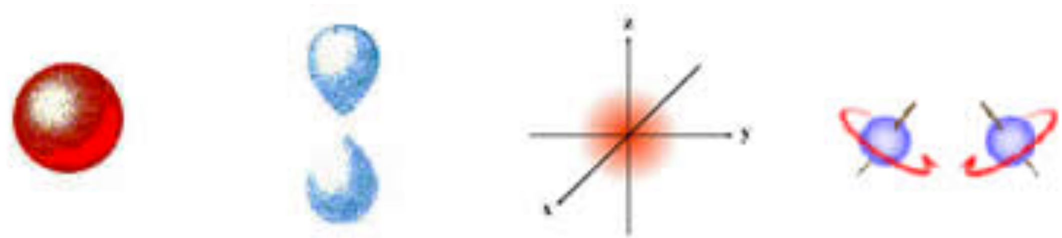
Quantum Numbers and Electron Orbital Properties

$$E_n = \frac{-me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = \frac{-13.6eV}{n^2} \qquad n = 1,2,3,\ldots$$

$$L^2 = \ell(\ell+1)\hbar^2$$

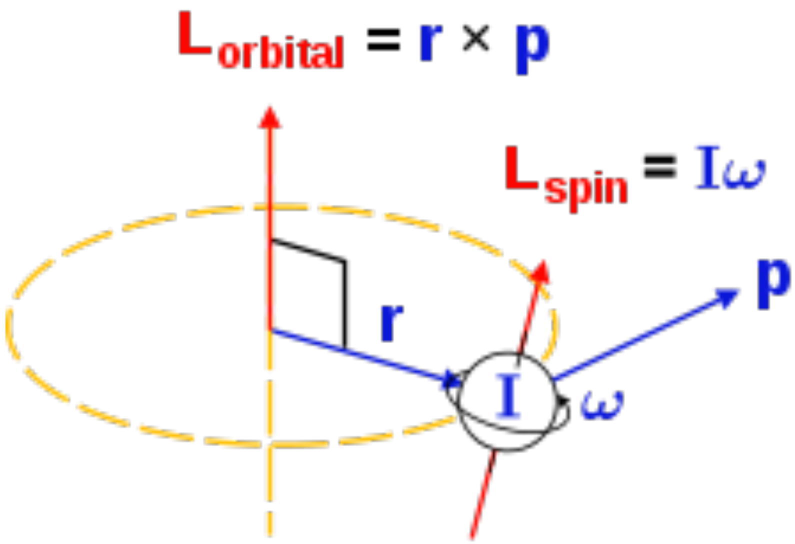
$$L_z = m_\ell \hbar$$

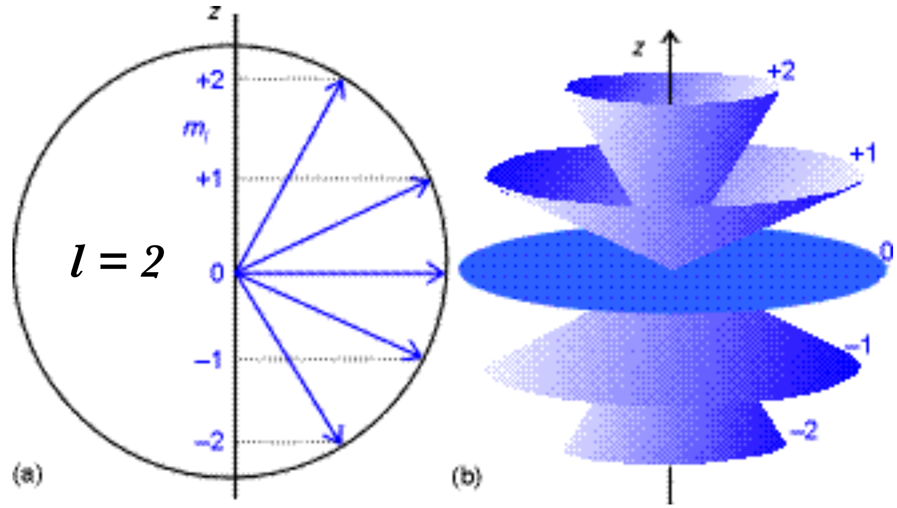
$$S_z = m_s \hbar\,, \quad m_s = \pm\frac{1}{2}$$

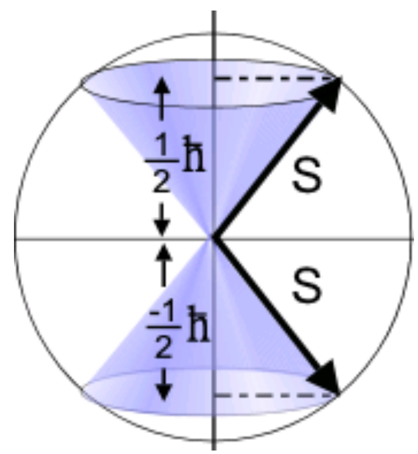


n **= principal**	***l*** **= angular**	***m*** **= magnetic**	***s*** **= spin**
distance from nucleus	shape of orbital	orientation in space	electron spin
Energy	Orbital Angular Momentum	Direction of Orbital Angular Momentum	Rotational Angular Momentum

Angular Momentum







Concept Check

- Consider an electron in the $l = 2$, $m_l = 2$ orbital. Where would this electron be most likely to be found?

A. Near the z-axis
B. Near the x-y plane
C. Equally likely to be found anywhere
D. Not enough information

Concept Check

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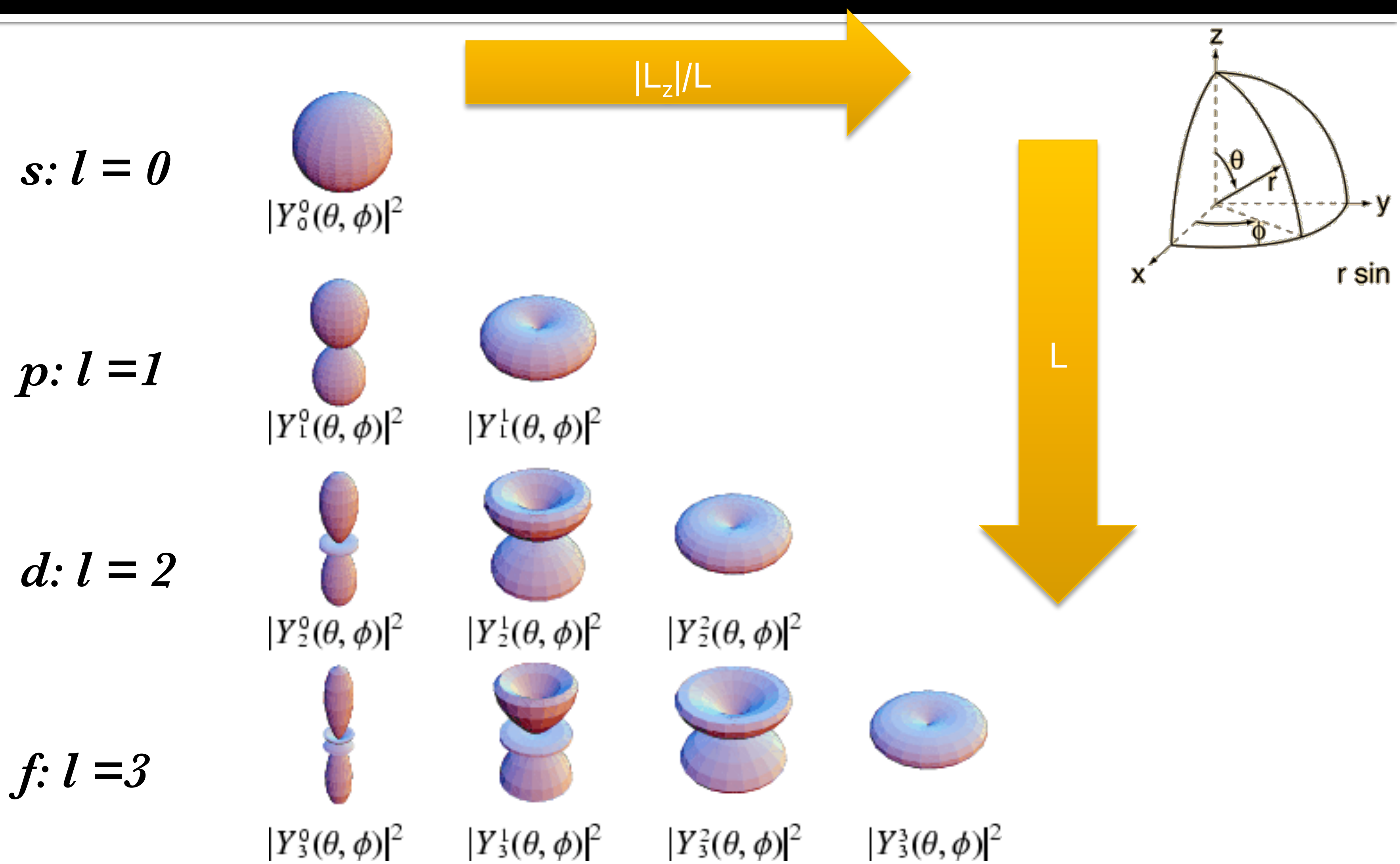
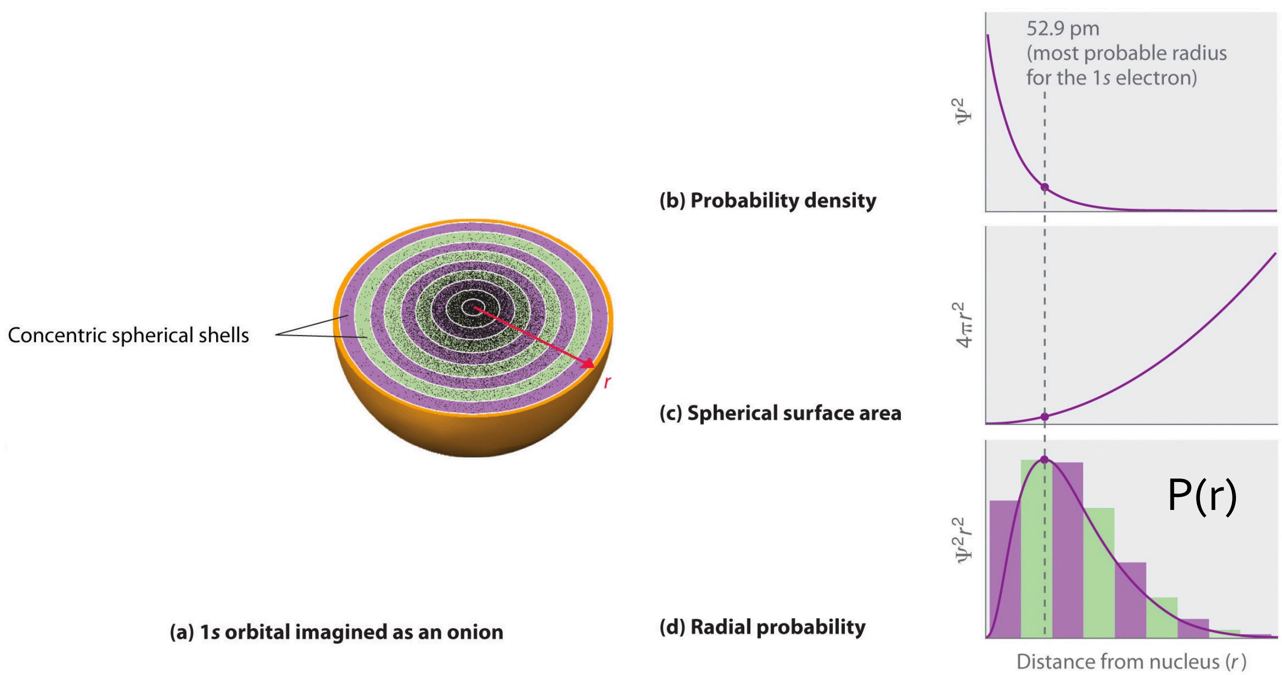
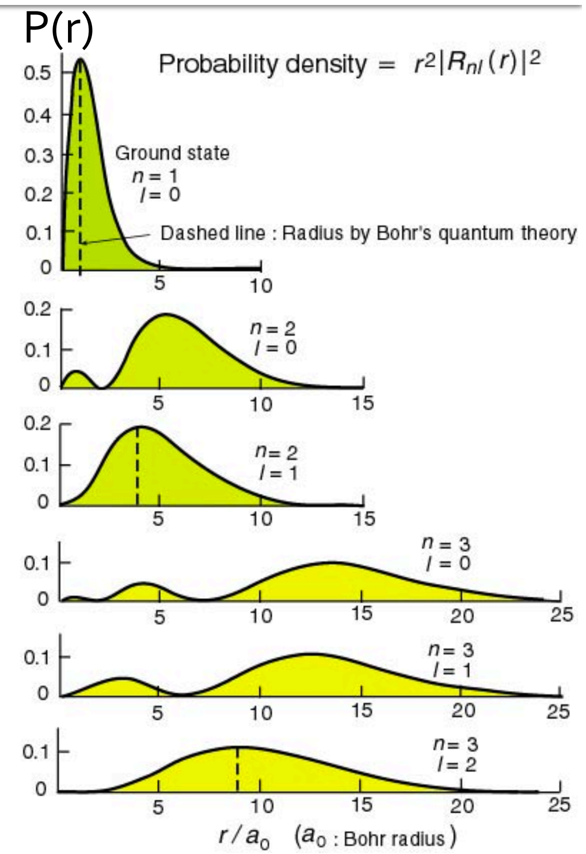
Angular Probability Distribution



Radial Probability Distribution





Maximum probability radius where d/dr (P(r)) = 0

Average radius: $\langle r \rangle = \int_0^\infty rP(r)dr$