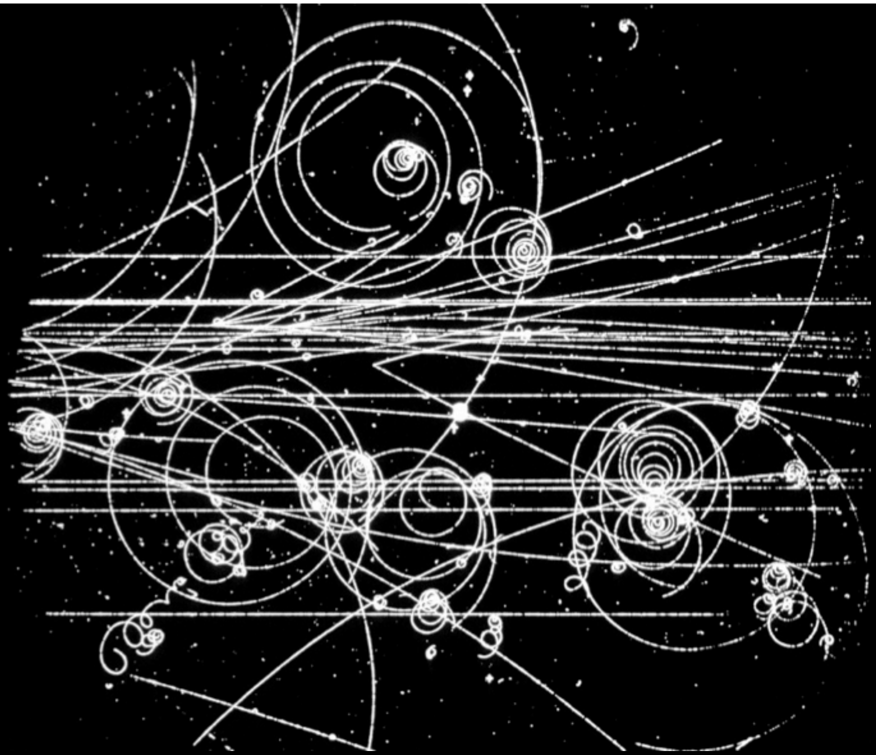
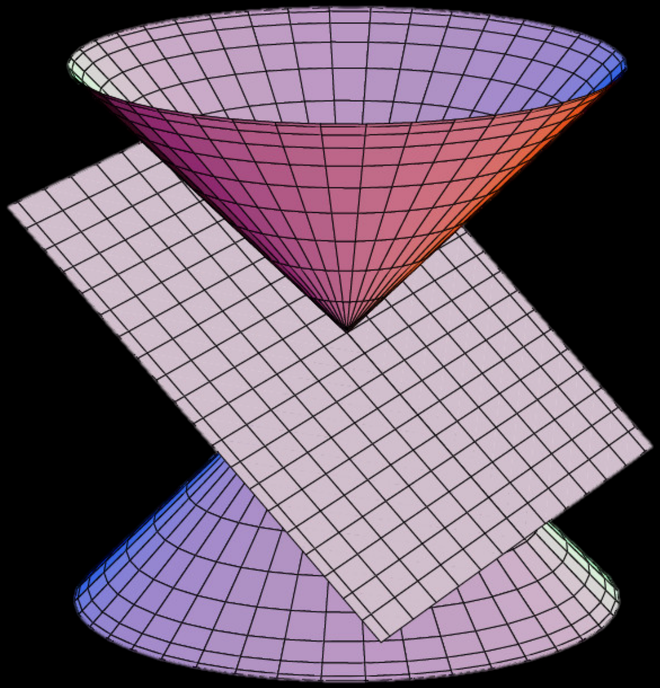


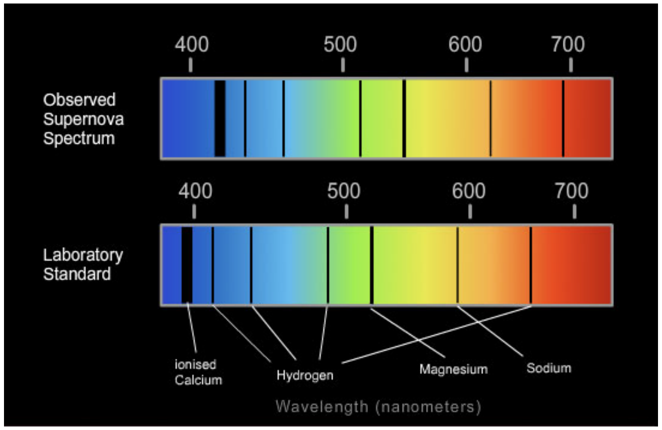


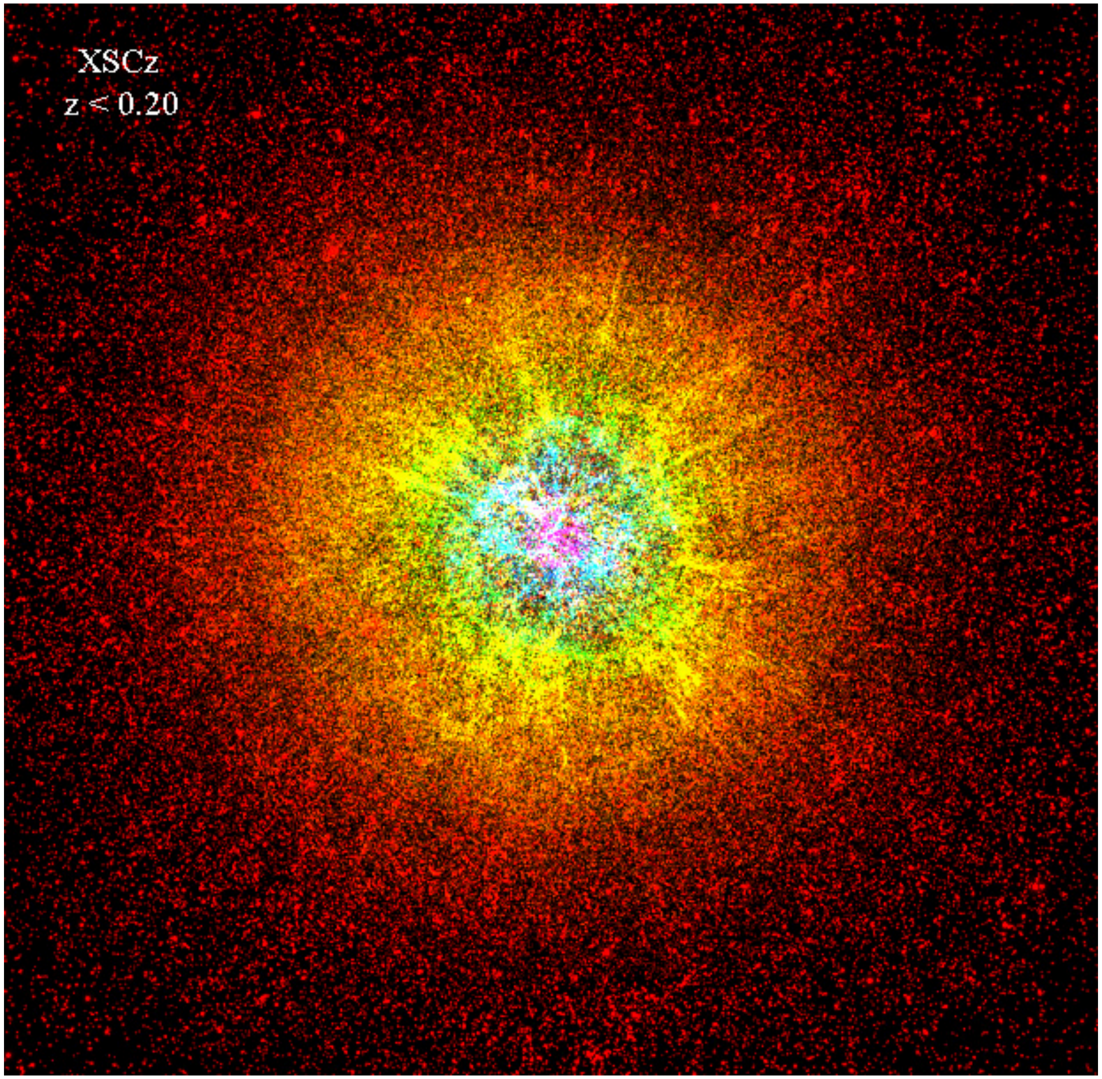
Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Doppler Shift and Expansion of the Universe

$$\nu' = \nu * \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}$$





Transformations

If S′ is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

Galilean transformation
(classical)

$$x' = x - ut$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

Lorentz transformation
(relativistic)

$$x' = \gamma(x - ut)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \frac{u}{c^2}x)$$

Note: This assumes (0,0,0,0) is the same event in both frames.

Velocity transformation (3D)

Classical:

$$v'_x = v_x - u$$

$$v'_y = v_y$$

$$v'_z = v_z$$

Relativistic:

$$v'_x = \frac{v_x - u}{1 - v_x u / c^2}$$

$$v'_y = \frac{v_y}{\gamma\left(1 - v_x u / c^2\right)}$$

$$v'_z = \frac{v_z}{\gamma\left(1 - v_x u / c^2\right)}$$

Velocity Addition

$$v_x' = \frac{v_x - u}{1 - v_x u / c^2}$$

what if $v_x = c$?

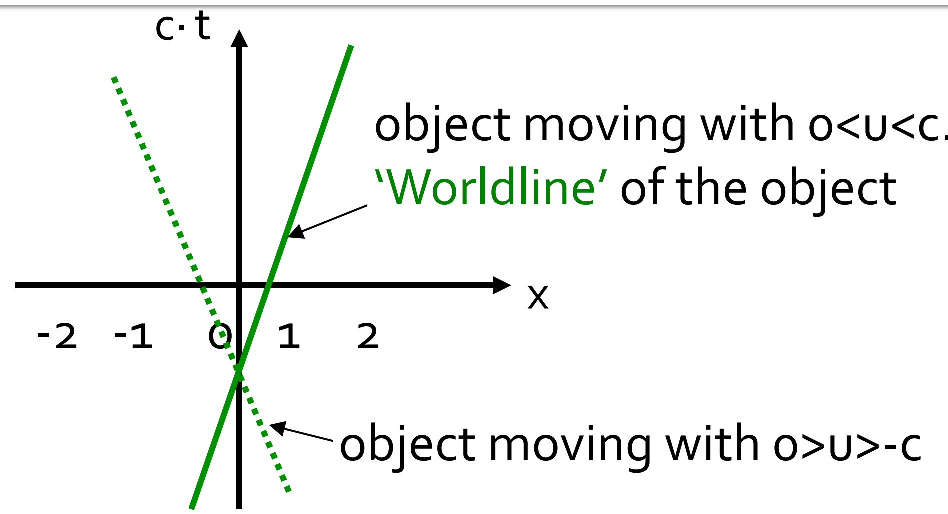
$$\frac{c - u}{1 - cu/c^2} = \frac{c - u}{1 - u/c}$$

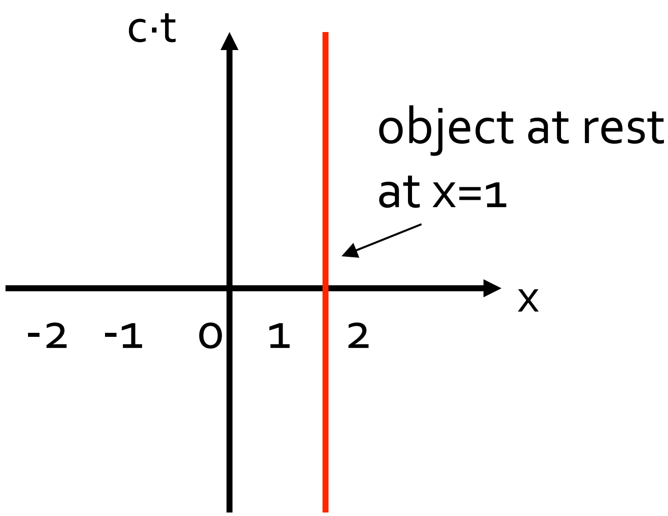
$$= c(1 - u/c) / (1 - u/c) = c$$

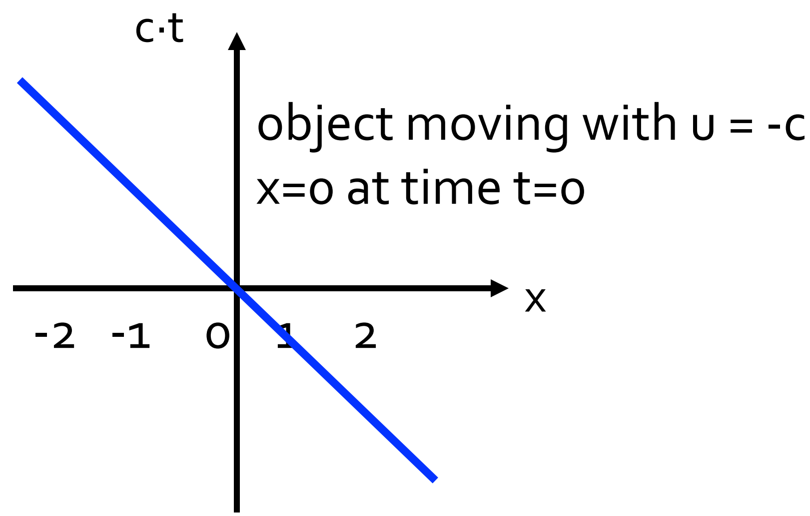
so $c + u = c$

c is the "speed limit"

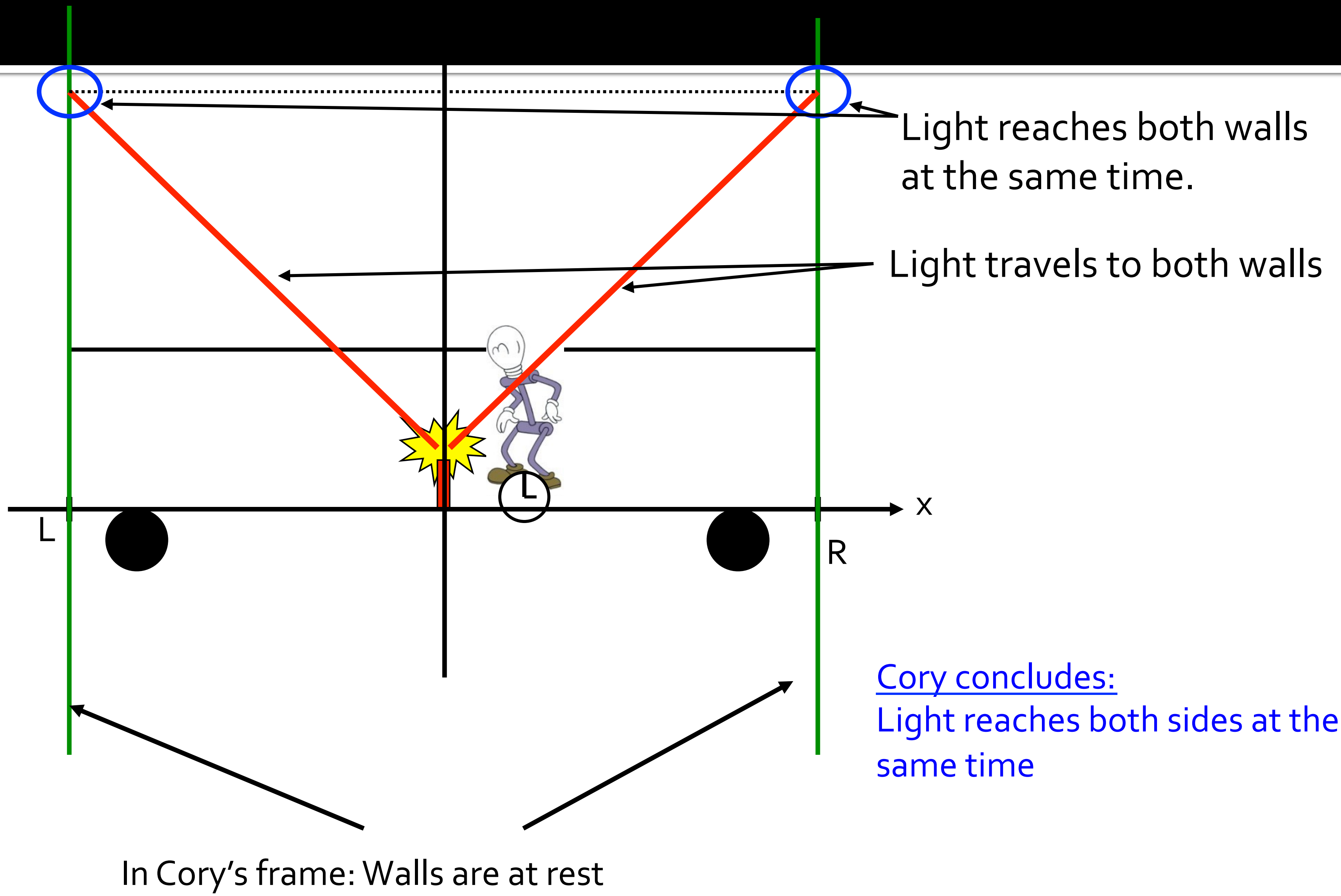
Spacetime Diagrams (1D in space)



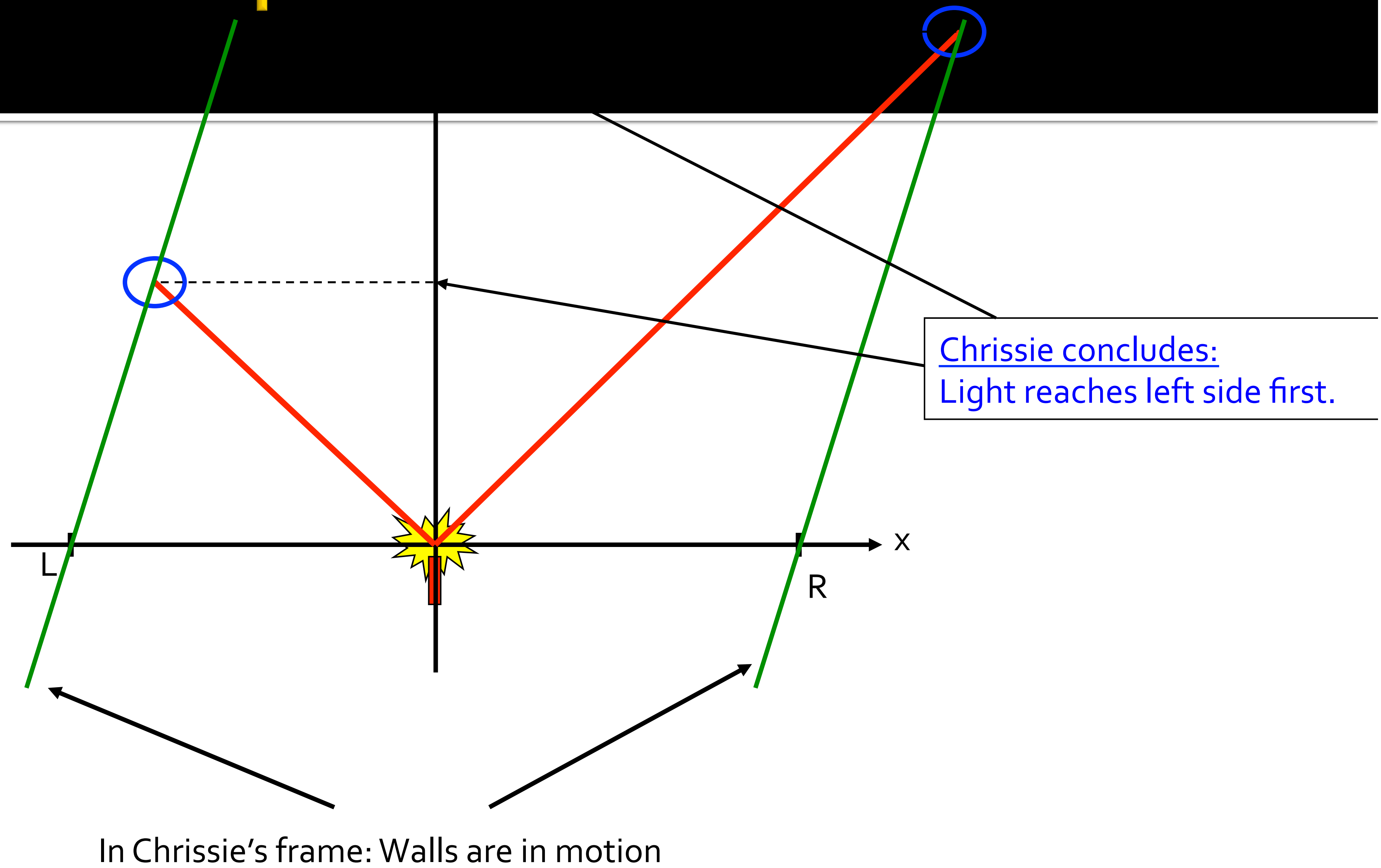




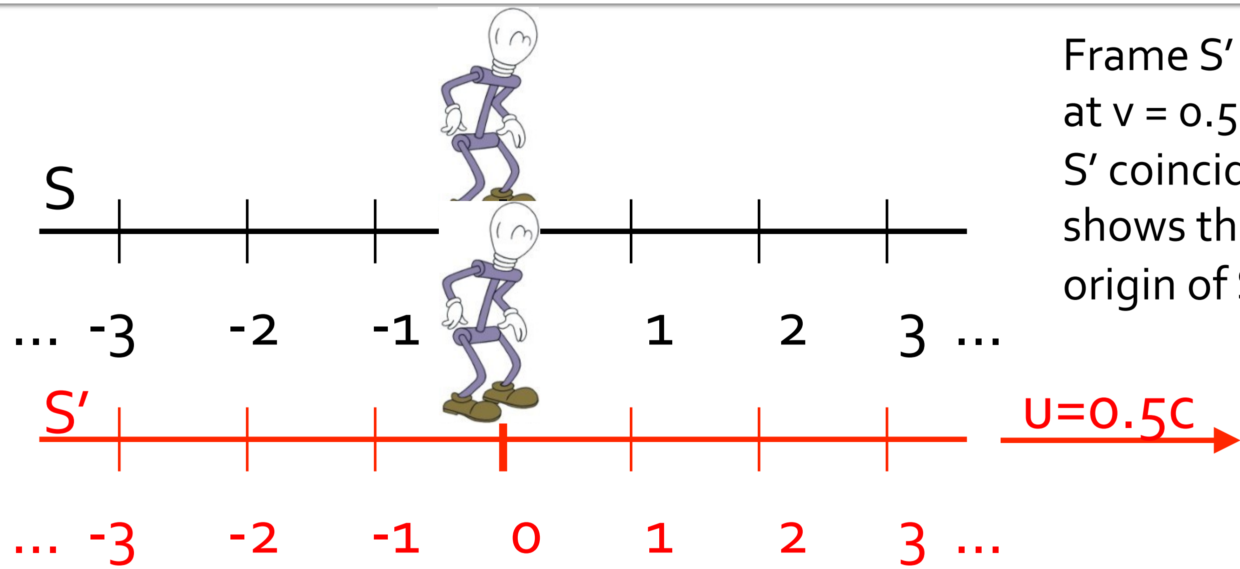
Example: Cory in the train



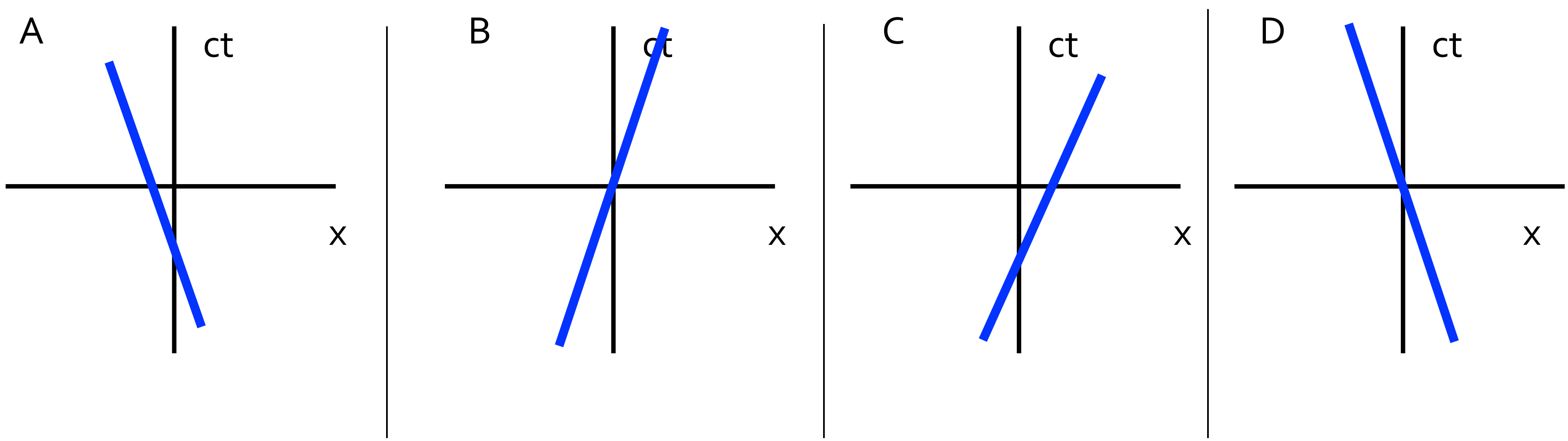
Example: Chrissie on the tracks



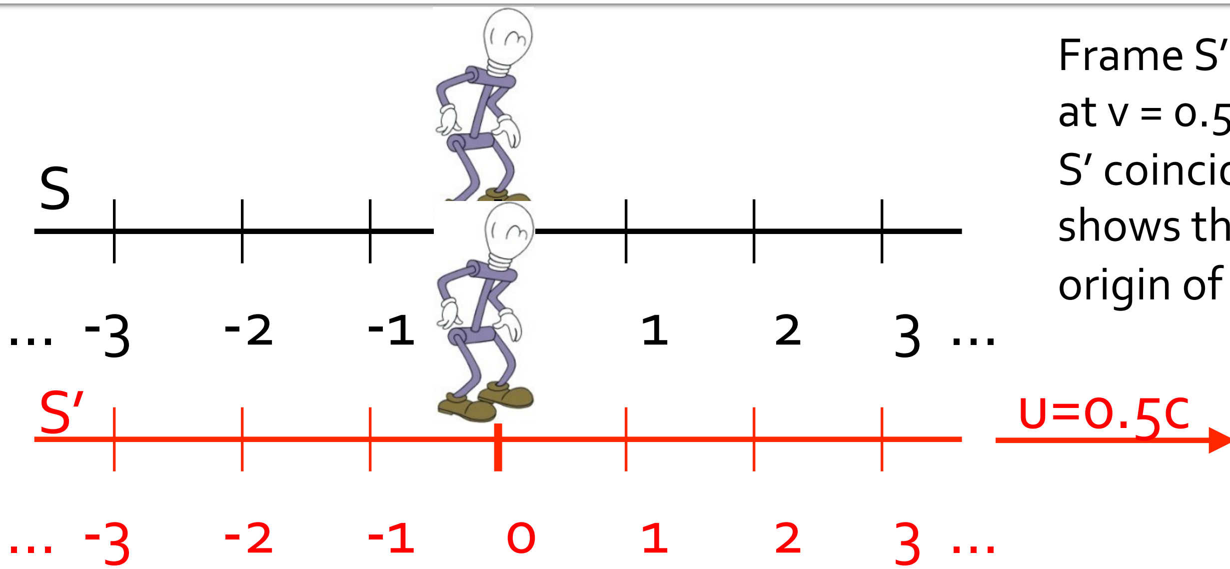
Concept Check: Worldlines



Frame S′ is moving to the right at v = 0.5c. The origins of S and S′ coincide at t=t′=0. Which shows the world line of the origin of S′ as viewed in S?



Concept Check: Worldlines



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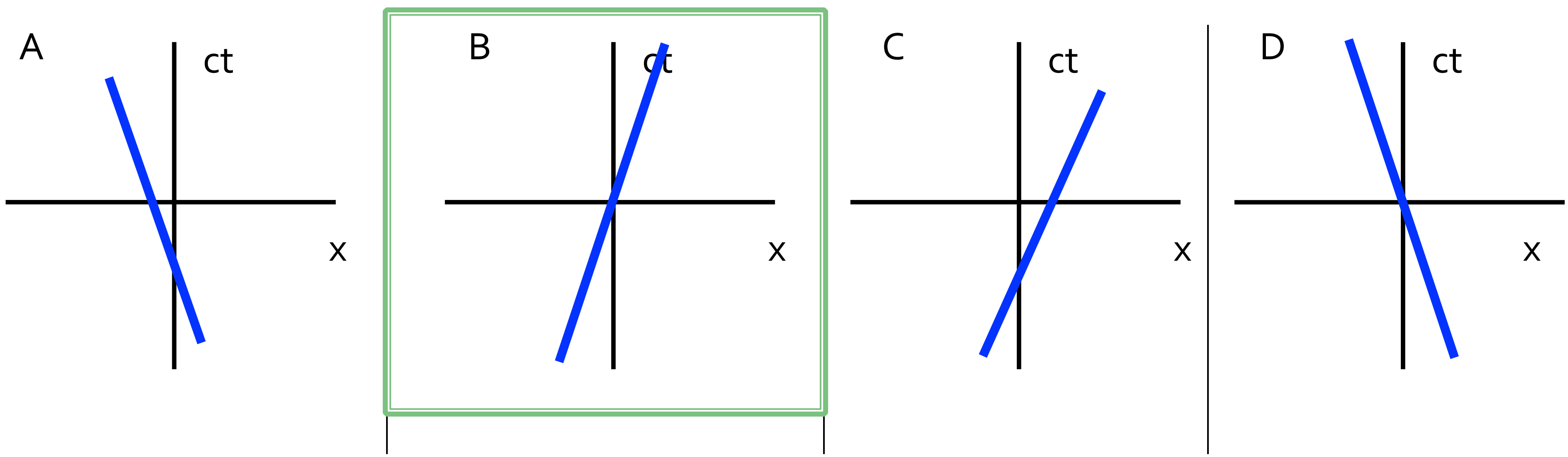


- Lorentz transformation for t', x' axes

$$x' = \gamma(x - ut)$$
$$t' = \gamma(t - \tfrac{u}{c^2}x)$$

$x' = 0$ on t' axis

$$\Rightarrow x = ut$$
$$= \tfrac{u}{c} \cdot ct$$

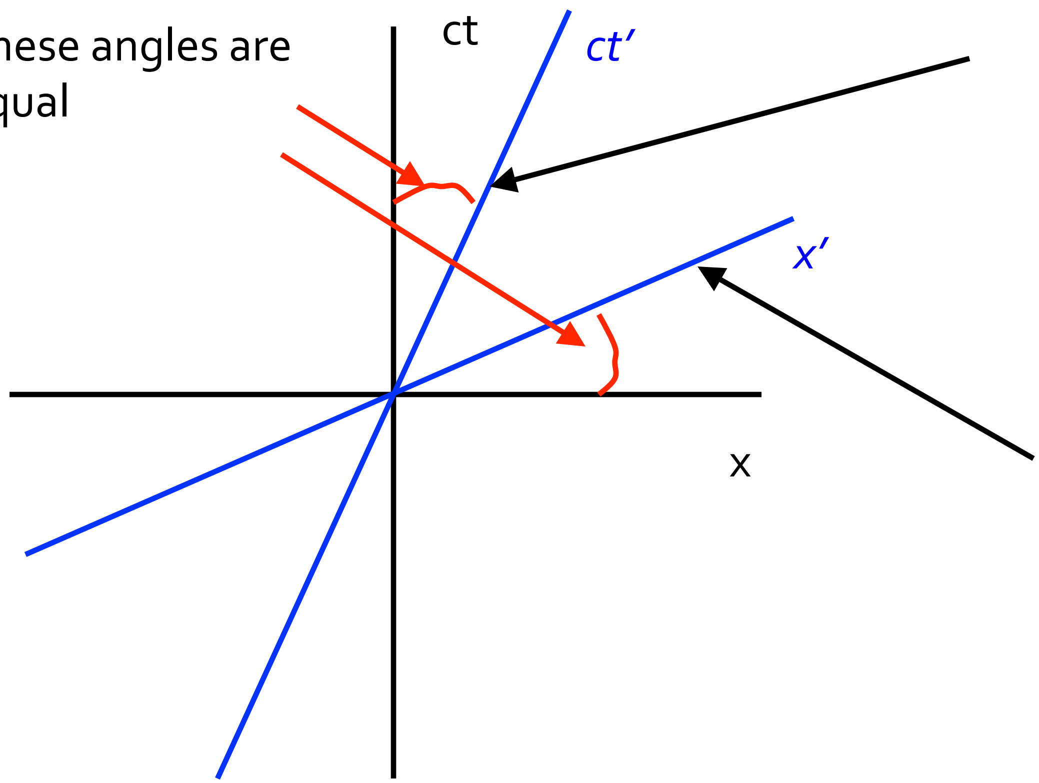
$t' = 0$ on x' axis

$$\Rightarrow t - \tfrac{u}{c^2}x = 0$$
$$\Rightarrow t = \tfrac{u}{c^2}x$$
$$ct = \tfrac{u}{c}x$$

- same equation but inverse slope

Frame S′ as viewed from S

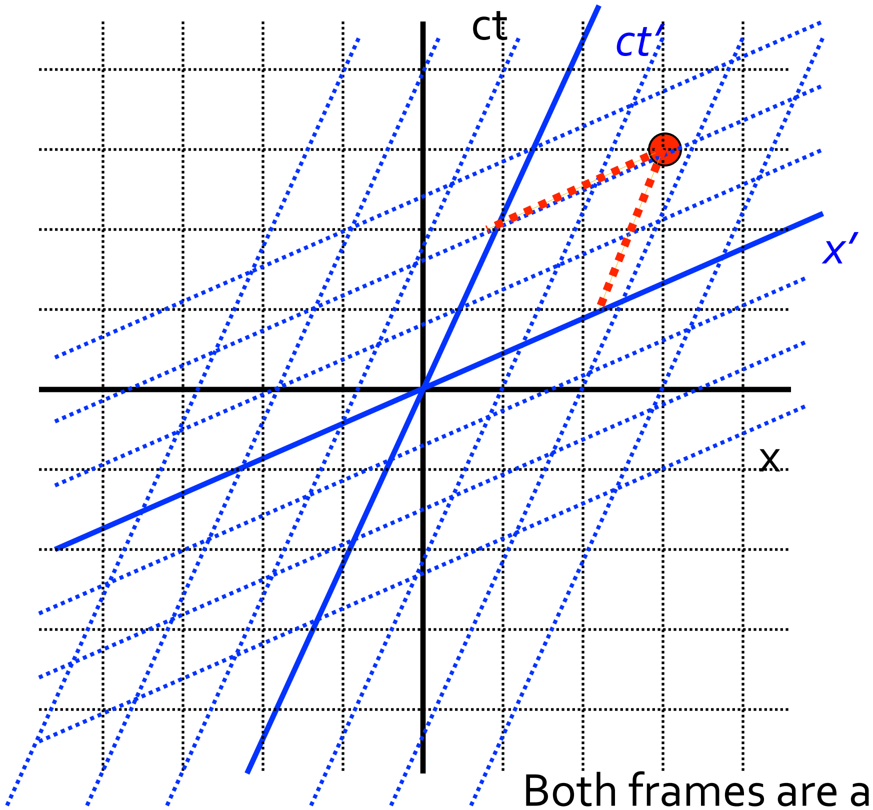
These angles are equal



This is the time axis of the frame S′

This is the space axis of the frame S′

Frame S′ as viewed from S



In S: (x=3,ct=3)

In S′: (x′=1.8,ct′=2)

Both frames are adequate for describing events – but will give different spacetime coordinates for these events, in general.

Interval Transformations

If S’ is moving with speed v in the positive x direction relative to S, then the coordinates of the same event in the two frames are related by:

Galilean transformation
(classical)

$$\Delta x' = \Delta x - u\Delta t$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

$$\Delta t' = \Delta t$$

Lorentz transformation
(relativistic)

$$\Delta x' = \gamma(\Delta x - u\Delta t)$$

$$\Delta y' = \Delta y$$

$$\Delta z' = \Delta z$$

$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}\Delta x)$$

Spacetime Interval

$$\Delta s^2 = (c\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$\Delta s'^2 = (c\gamma(\Delta t - u/c^2\,\Delta x))^2$$
$$- (\gamma(\Delta x - u\Delta t))^2$$
$$- \Delta y^2 - \Delta z^2$$

$$= (c\gamma\Delta t)^2 + (\gamma u/c\,\Delta x)^2 - 2\gamma^2 u\Delta t\Delta x$$
$$- (\gamma\Delta x)^2 - (\gamma u\Delta t)^2 + 2\gamma^2 u\Delta x\Delta t$$
$$- \Delta y^2 - \Delta z^2$$

$$= \Delta t^2(c^2\gamma^2 - \gamma^2 u^2)$$
$$- \Delta x^2(\gamma^2 - \gamma^2 u^2/c^2)$$
$$- \Delta y^2 - \Delta z^2$$

$$= c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$
$$= \Delta s^2$$

Spacetime interval

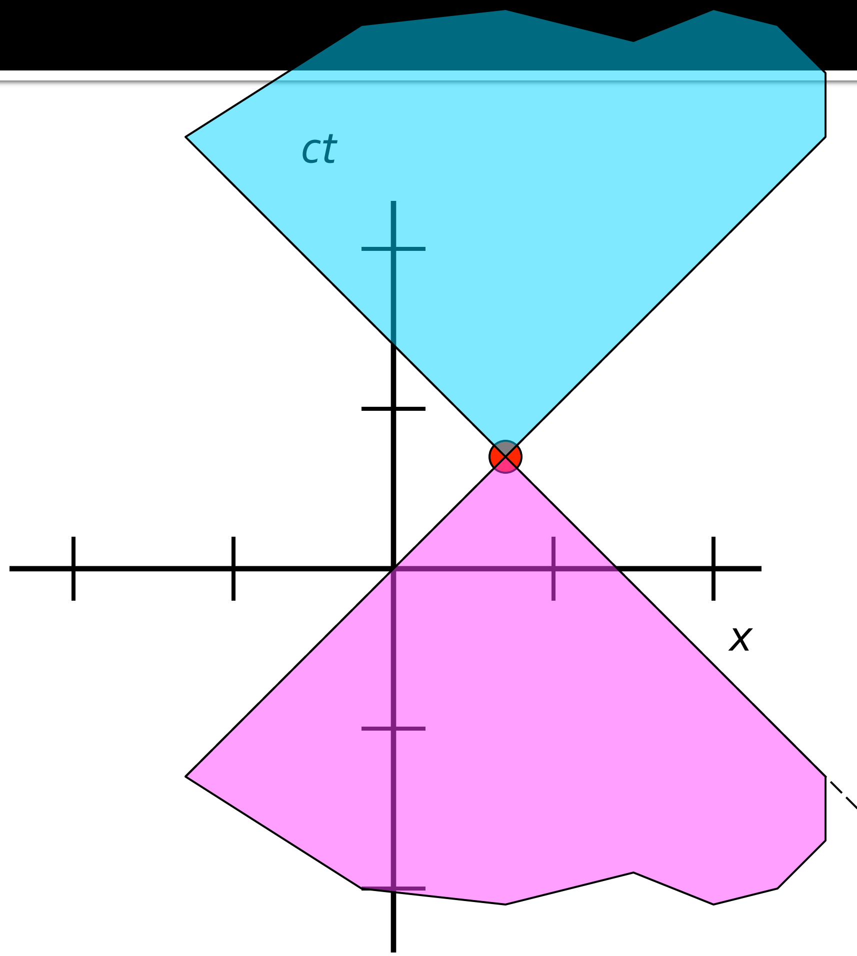
Say we have two events: (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2). **Define the spacetime interval** (sort of the "distance") **between two events** as:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

Spacetime interval

The spacetime interval has the same value in all reference frames! I.e. Δs^2 is "invariant" under Lorentz transformations.

Spacetime

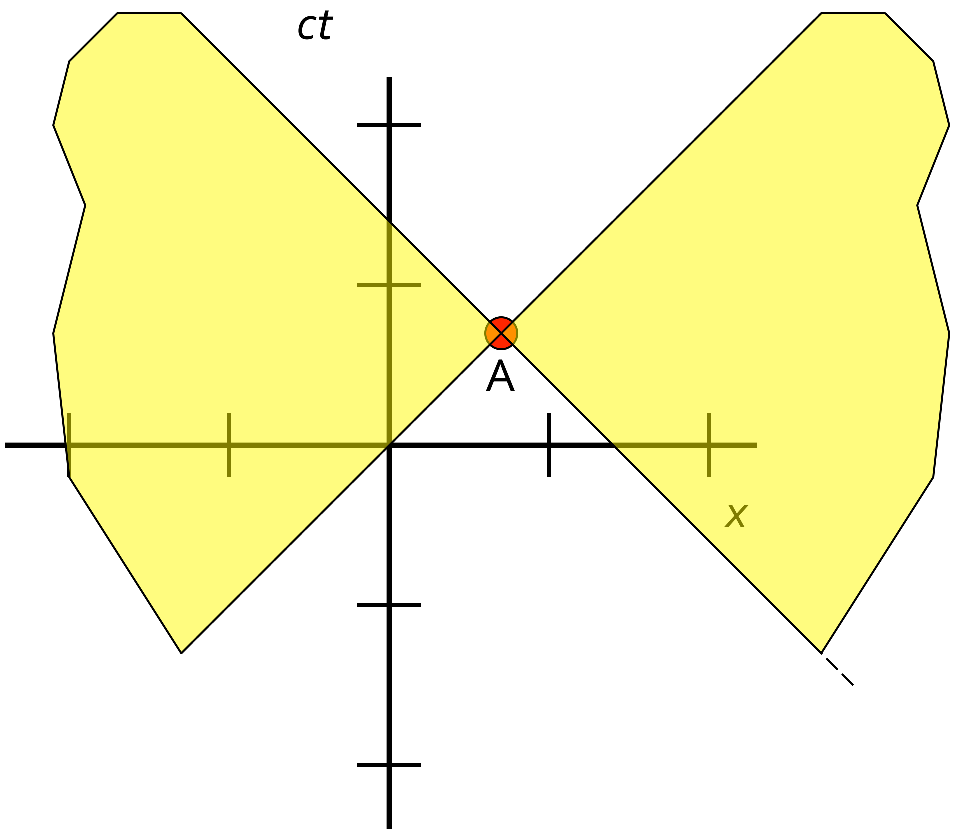


Here is an event in spacetime.

The blue area is the *future* on this event.

The pink is its *past*.

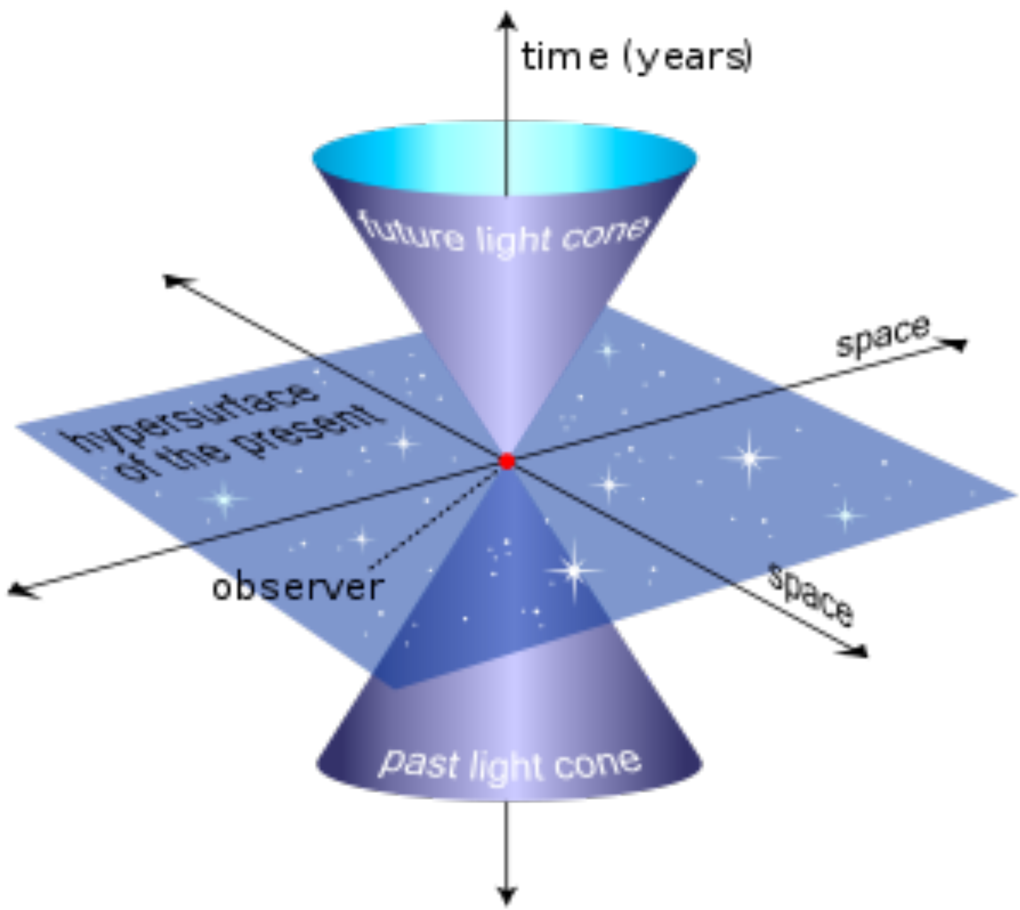
Spacetime



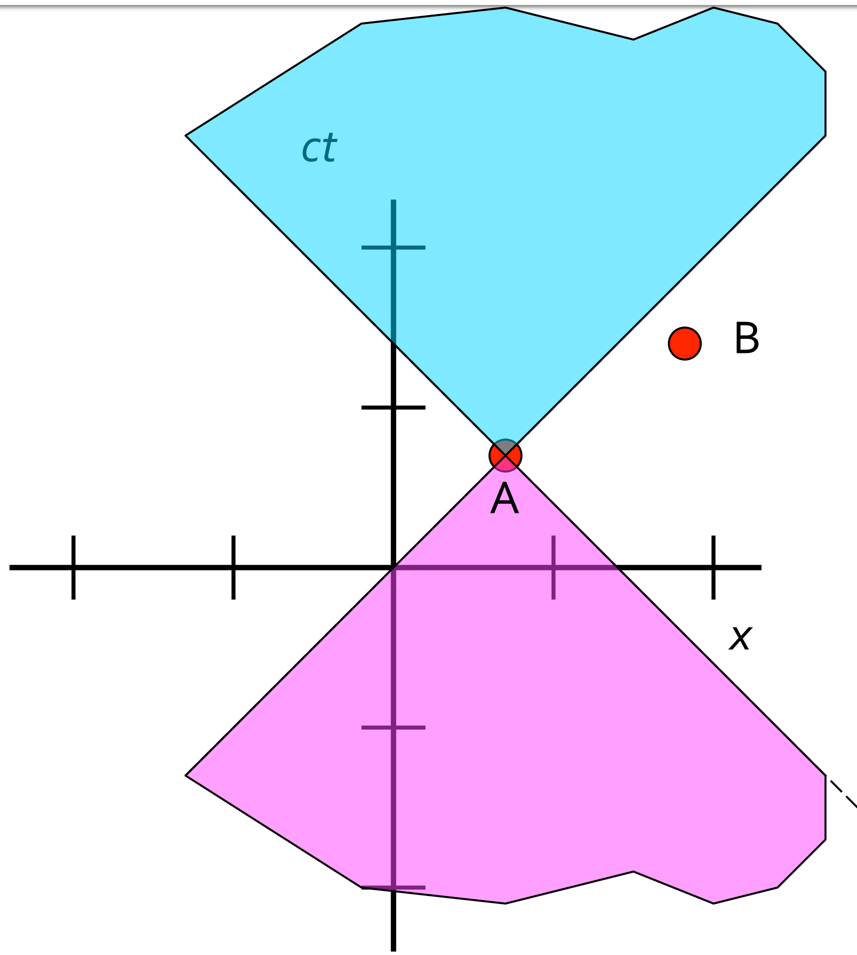
Here is an event in spacetime.

The yellow area is the "*elsewhere*" of the event. No physical signal can travel from the event to its elsewhere!

Spacetime in more than 1-d



Concept Check: Spacetime

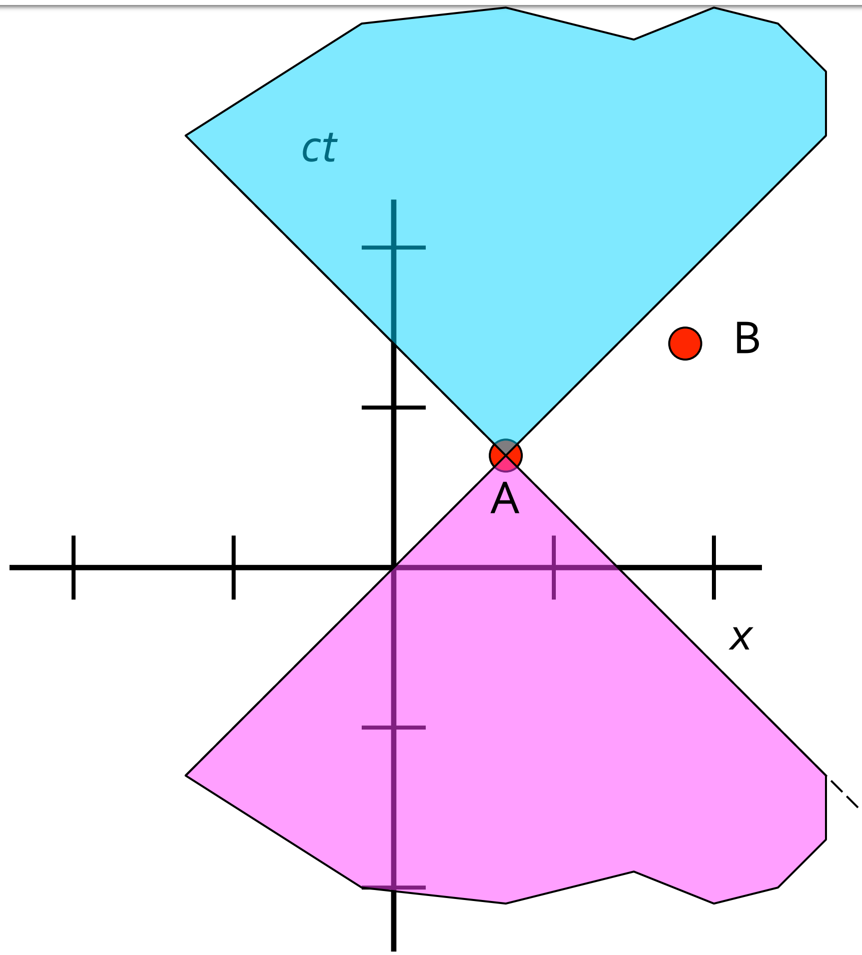


Now we have two events A and B as shown on the left.

The space-time interval $(\Delta s)^2$ of these two events is:

A) Positive
B) Negative
C) Zero

Concept Check: Spacetime

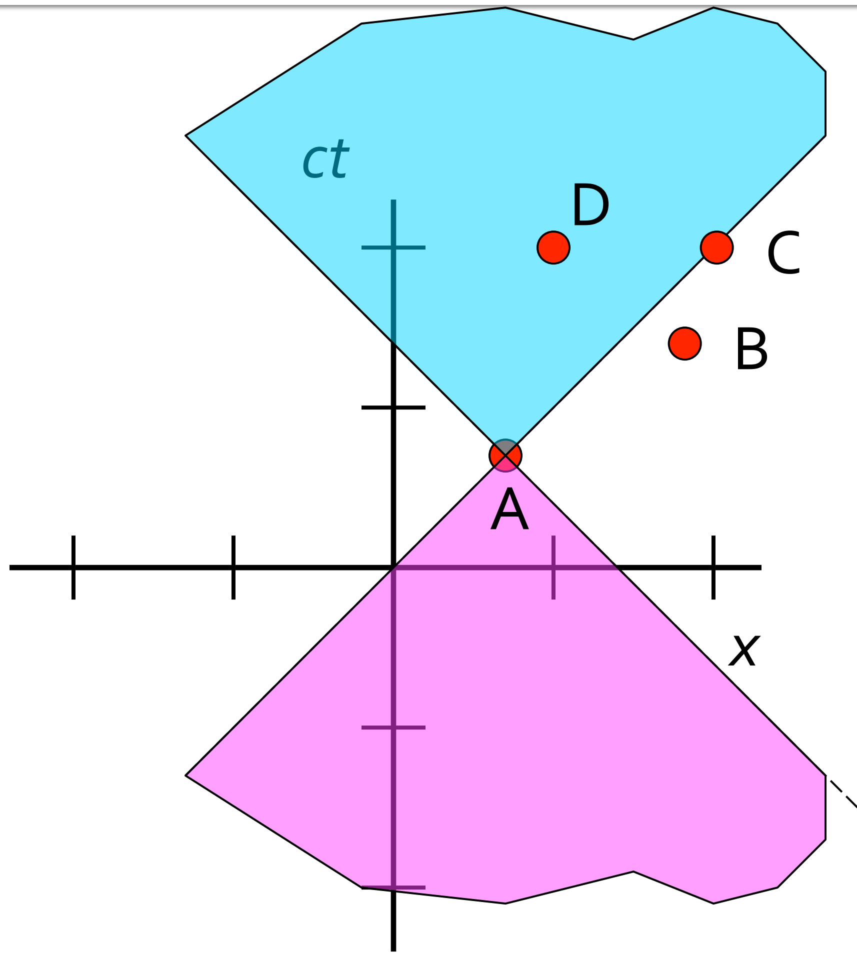


Now we have two events A and B as shown on the left.

The space-time interval $(\Delta s)^2$ of these two events is:

A) Positive
B) Negative
C) Zero

Spacetime



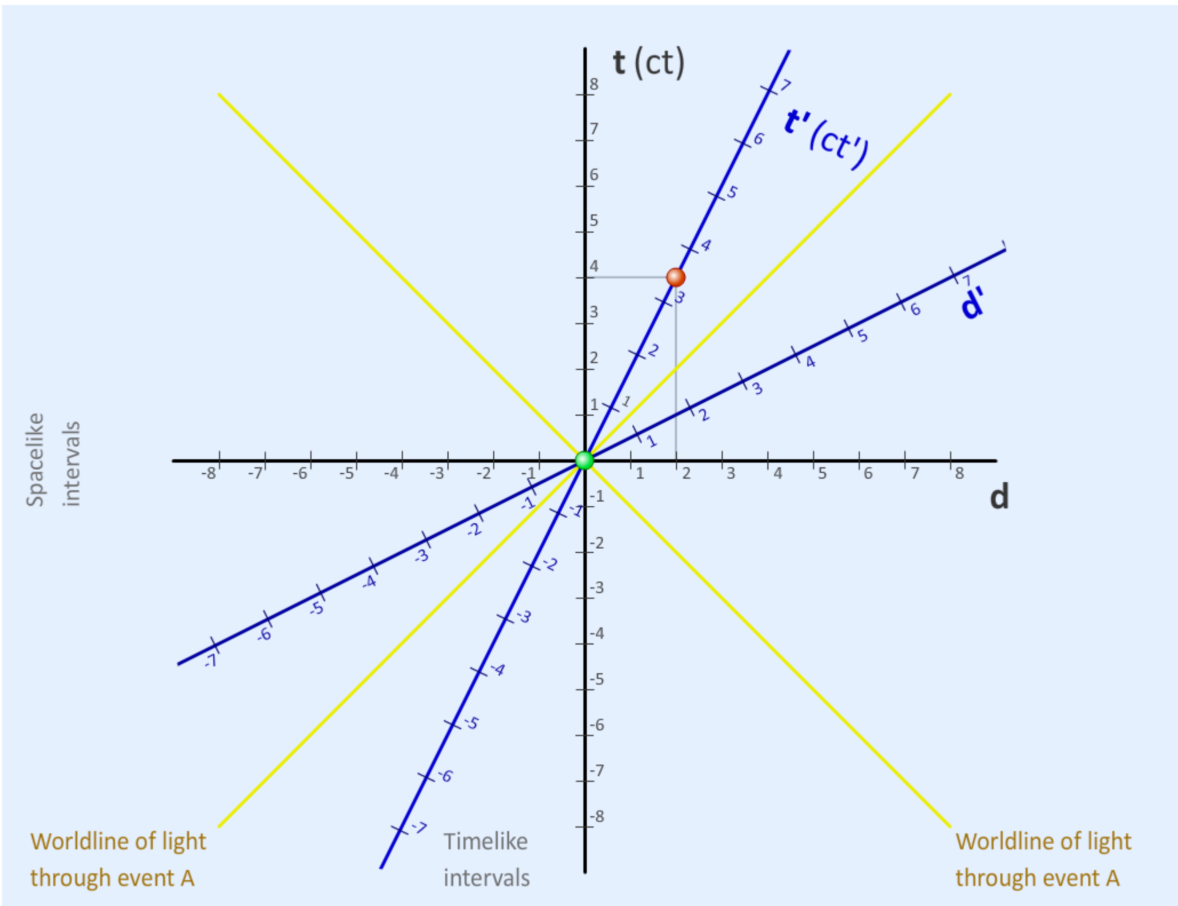
$(\Delta s)^2 > 0$: **Time-like events**
(A ➔ D)

$(\Delta s)^2 < 0$: **Space-like events**
(A ➔ B)

$(\Delta s)^2 = 0$: **Light-like events**
(A ➔ C)

Spacetime Intervals

http://www.trell.org/div/minkowski.html



Relative velocity:

0.5 c

Event A:

t = t' = 0

d = d' = 0

Event B:

t = 4

d = 2

t' = 3.4641

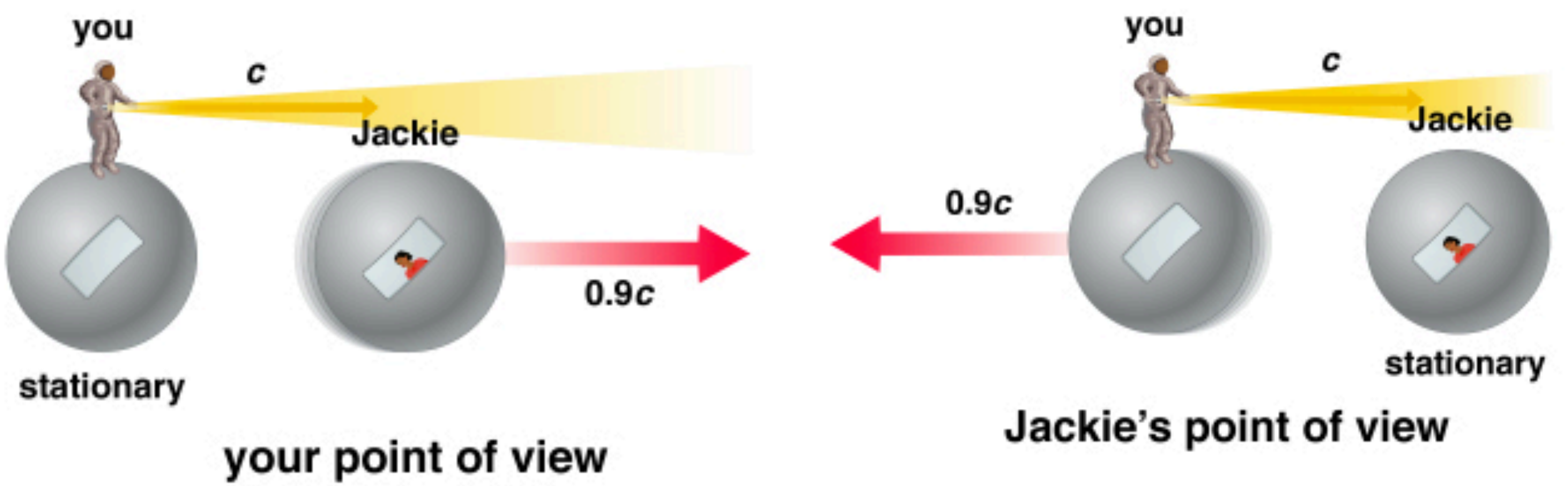
d' = 0

Invariant interval:

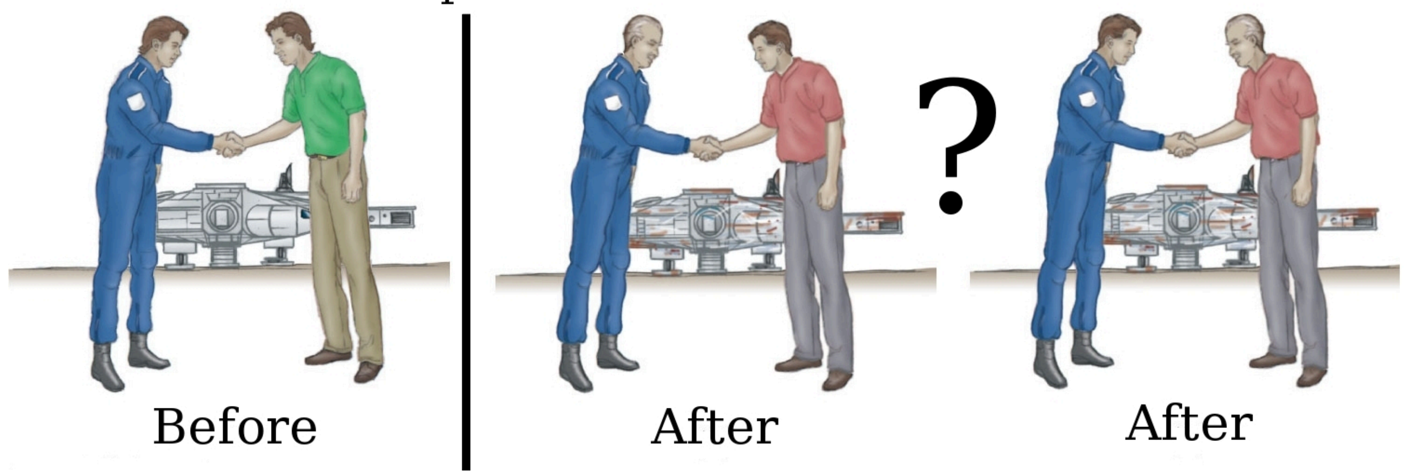
$i^2 = (ct)^2 - d^2 =$ 12

$i'^2 = (ct')^2 - d'^2 =$ 12

Twin Paradox



Twin paradox



Twin Paradox (Not a Paradox)