

Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Recap from Last Lecture

- Light is an electromagnetic wave
- But, the photoelectric effect shows that light comes in bits of energy of a specific size
 - The energy of a bit of light is $E = h * \text{frequency}$
- WTF???
- If you're a bit confused, that means you're paying attention!

Concept Check

- Which properties of light cannot be explained by treating it as a **wave**?
 - A. Reflection and the photoelectric effect
 - B. The photoelectric effect
 - C. Refraction and the photoelectric effect
 - D. Reflection and refraction

Concept Check

- Which properties of light cannot be explained by treating it as a **wave**?
 - A. Reflection and the photoelectric effect
 - B. The photoelectric effect
 - C. Refraction and the photoelectric effect
 - D. Reflection and refraction

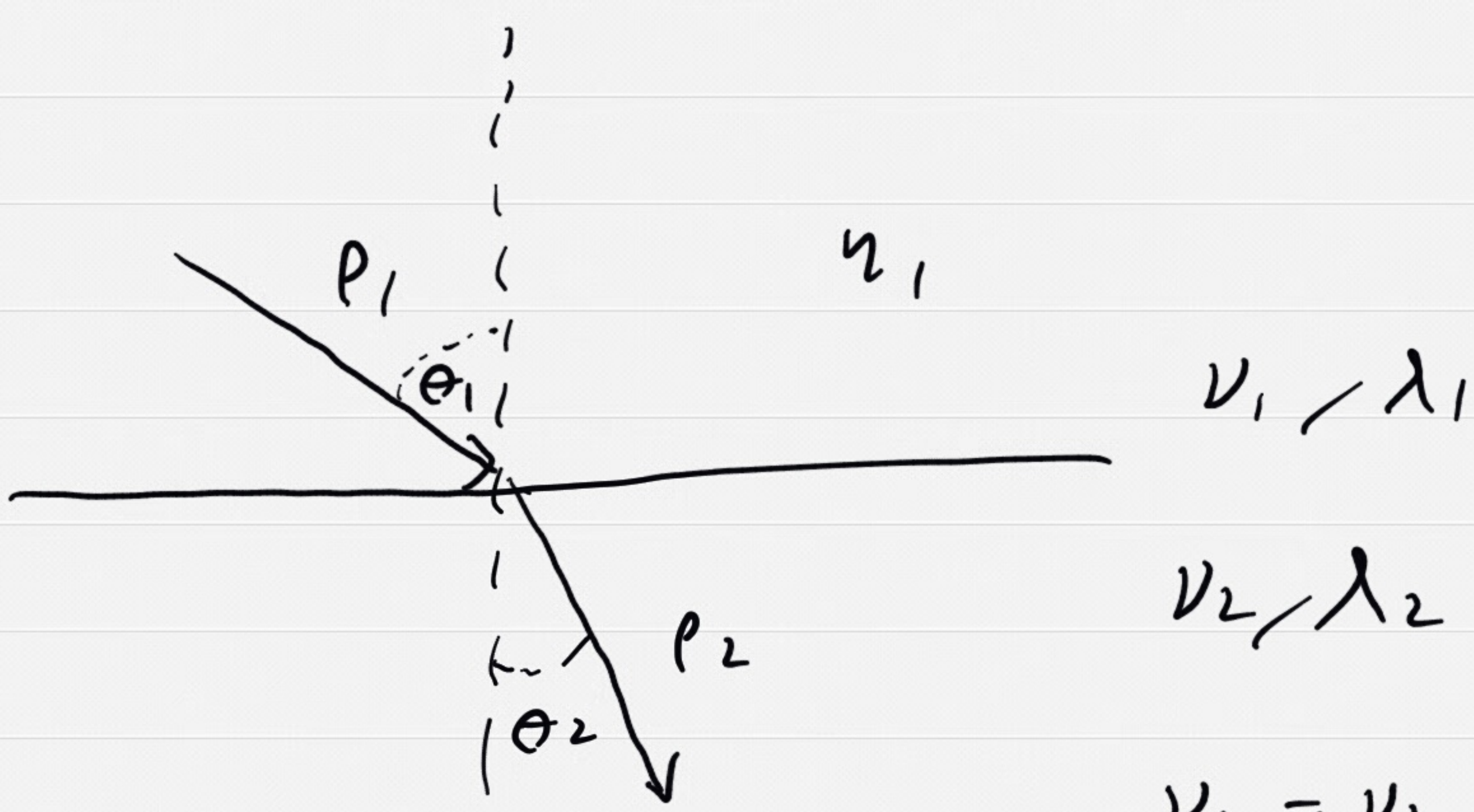
Concept Check

- Which properties of light cannot be explained by treating it as a **particle**?
- A. Reflection, refraction, interference
- B. Reflection, diffraction, interference
- C. Diffraction, refraction, interference
- D. Diffraction, interference

Concept Check

- Which properties of light cannot be explained by treating it as a **particle**?
 - A. Reflection, refraction, interference
 - B. Reflection, diffraction, interference
 - C. Diffraction, refraction, interference
 - D. Diffraction, interference

Photon Refraction



$$\begin{aligned} \nu_1 &= \nu_2 & \Rightarrow & E_1 = E_2 \\ \lambda_2 &< \lambda_1 & & \Rightarrow p_2 > p_1 \\ \nu_2 &< \nu_1 & & \end{aligned}$$

- p_x conserved

$$p_1 \sin \theta_1 = p_2 \sin \theta_2$$

- Energy conserved (since $\nu = \text{const.}$)

$$E_1 = E_2$$

- In vacuum $p = E/c = h/\lambda$


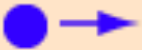

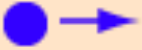












- In refractive medium $p = nE/c = h/\lambda$

$$\Rightarrow \frac{n_1 E_1 \sin \theta_1}{c} = \frac{n_2 E_2 \sin \theta_2}{c}$$

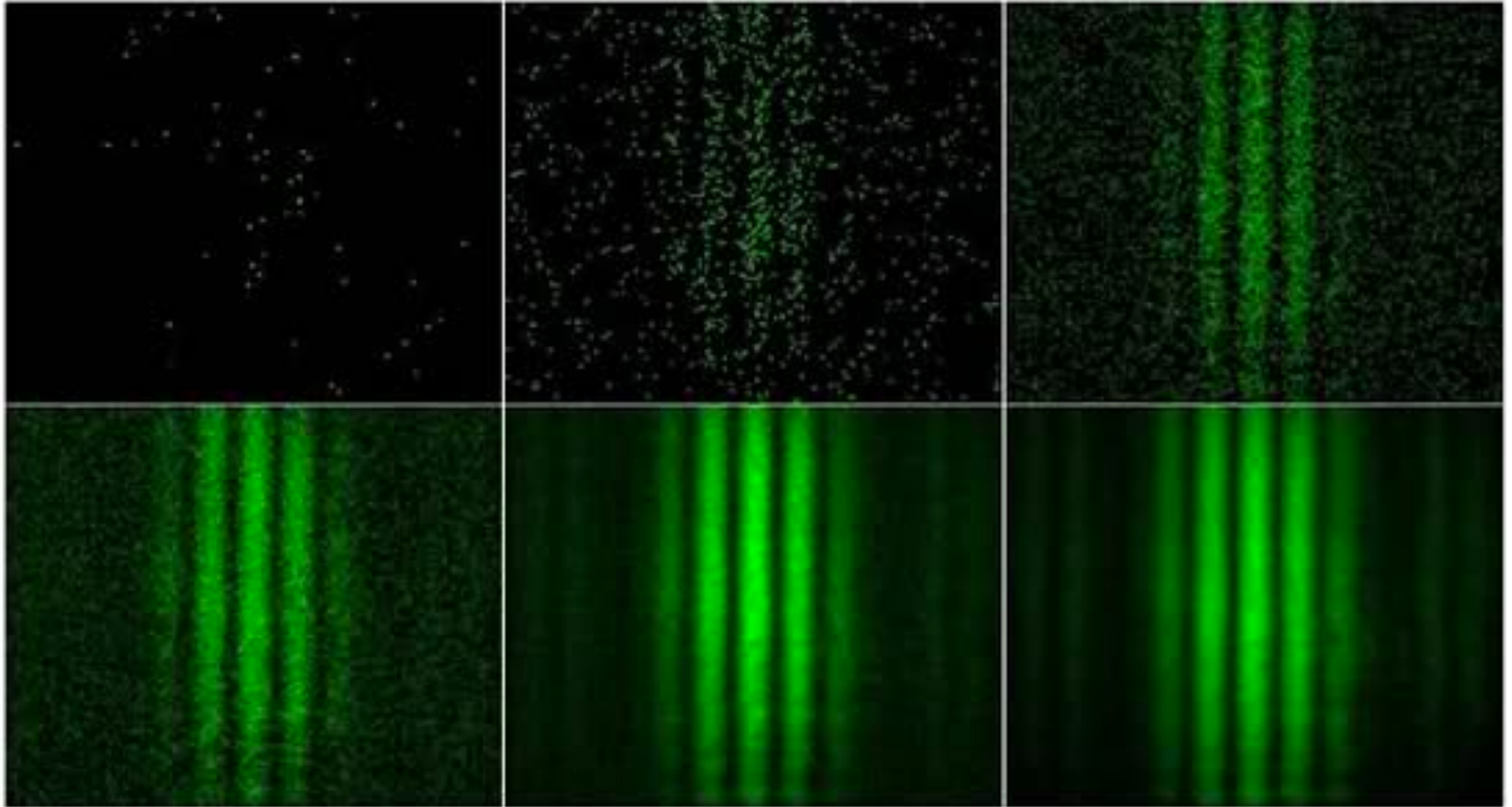
$$\Rightarrow n_1 \sin \theta_1 = n_2 \sin \theta_2$$

(Snell's Law)

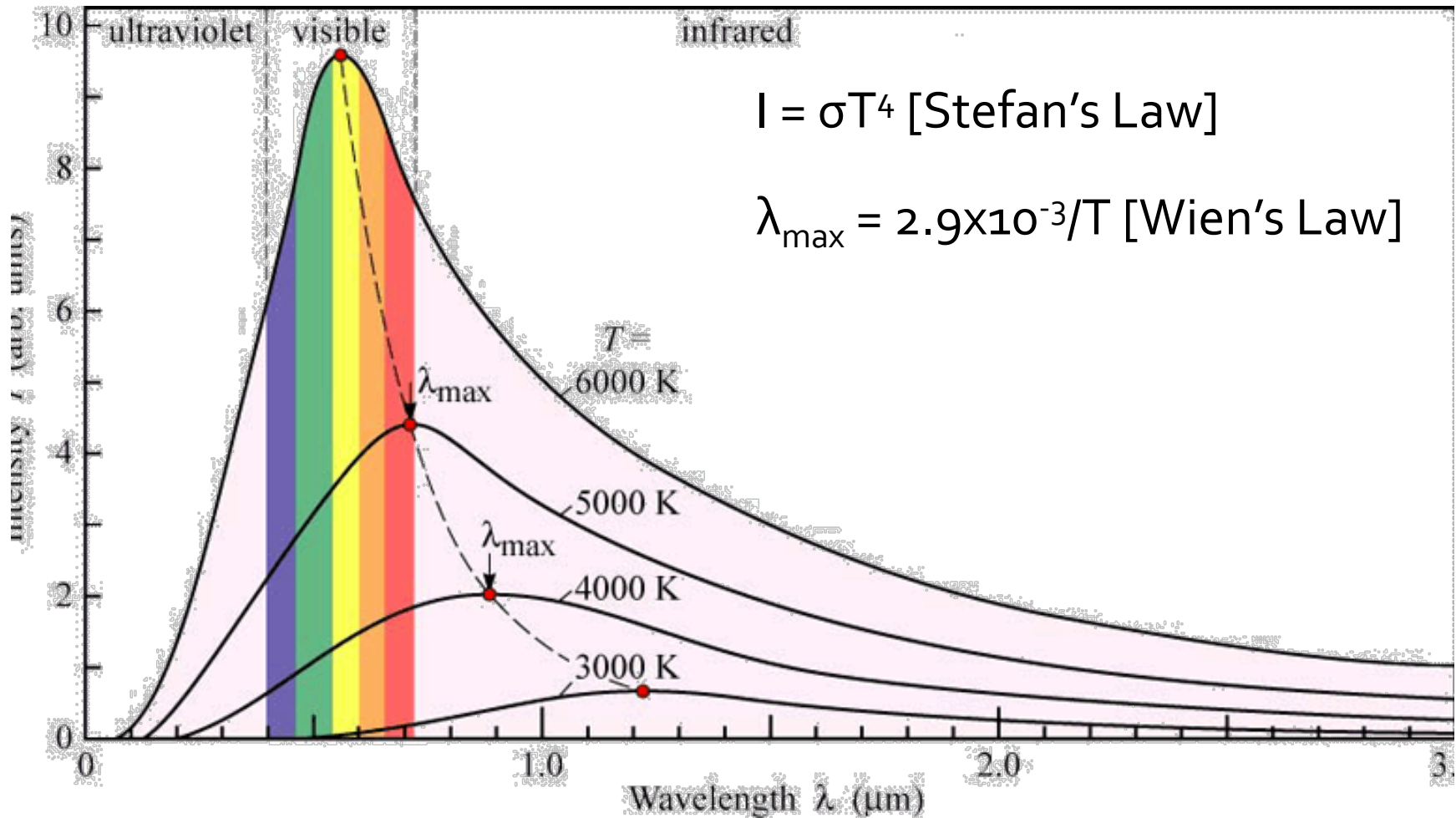
Wave-Particle Duality

1	Phenomenon	Can be explained in terms of waves.	Can be explained in terms of particles.
2	Reflection	 ✓	 ✓
3	Refraction	 ✓	 ✓
4	Interference	 ✓	 
5	Diffraction	 ✓	 
6	Polarization	 ✓	 
7	Photoelectric effect	 	 ✓

Light is Both a Particle and a Wave

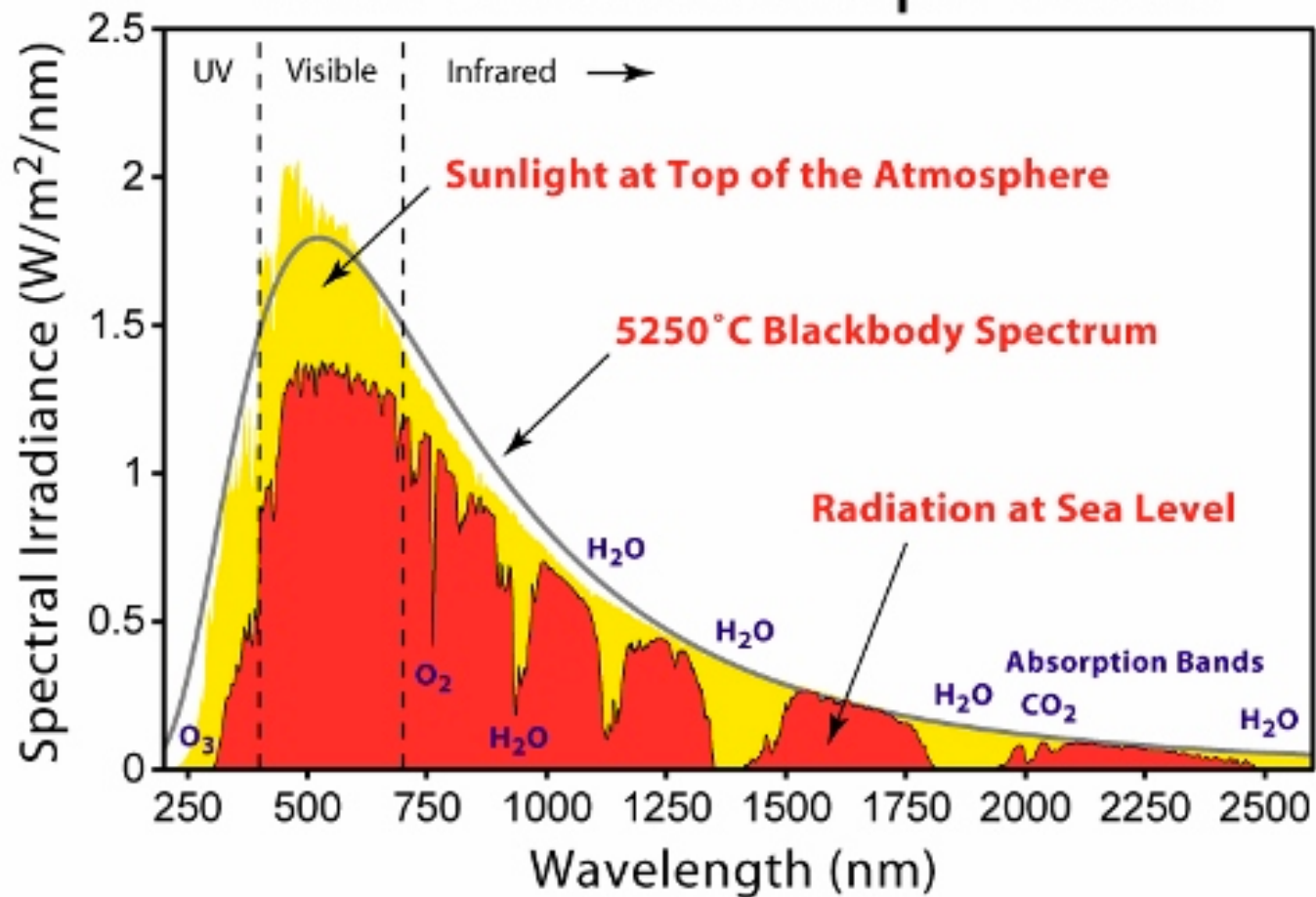


Black Body Radiation



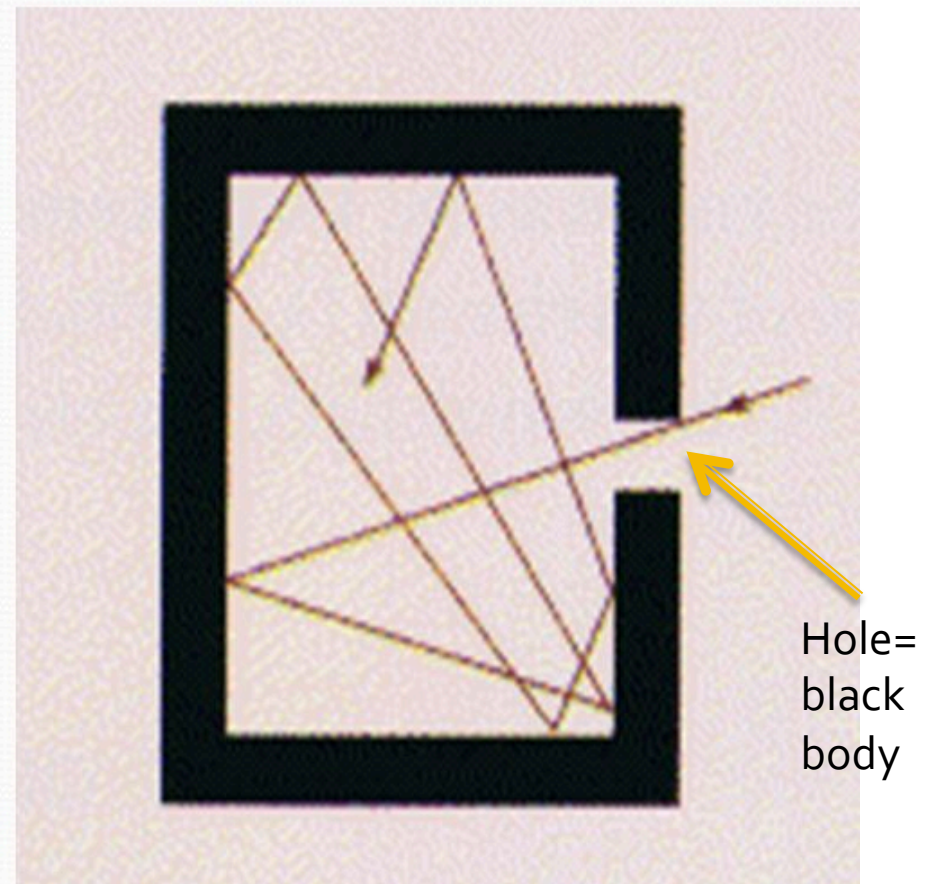
Solar Spectrum

Solar Radiation Spectrum

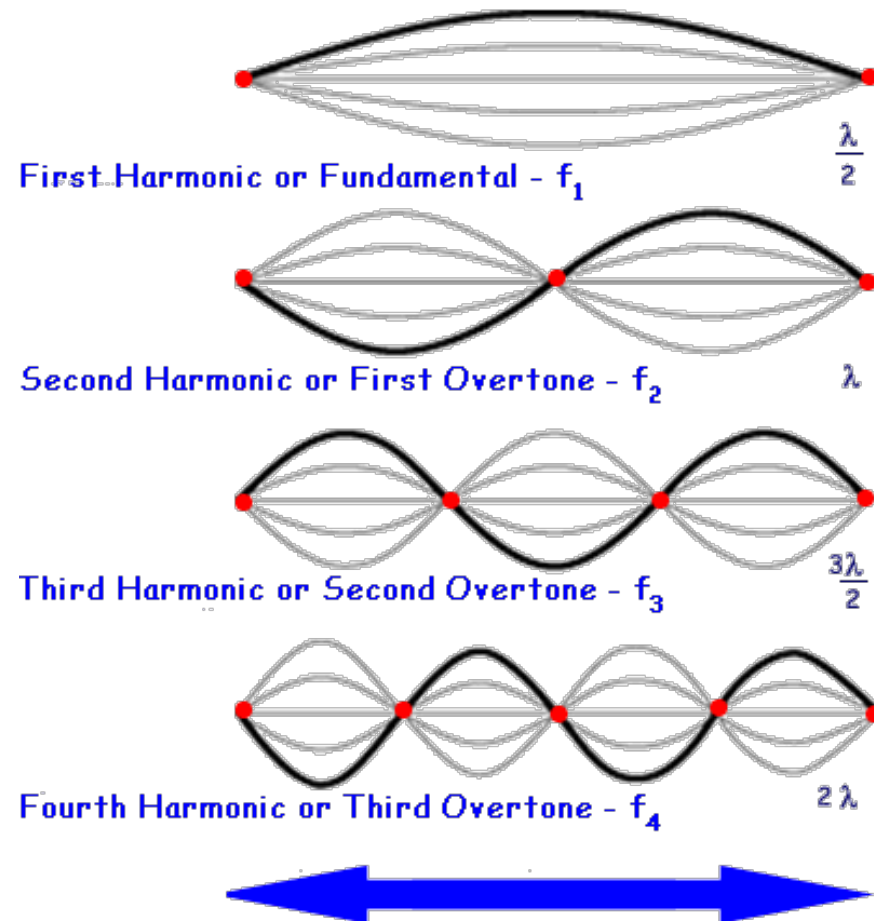


What is a Black Body?

- A black body is an ideal body which absorbs all the incident radiation within itself.
- The black body is an ideal absorber of incident radiation.
- The black body is an ideal radiator



Standing Waves



Rayleigh - Jeans Law

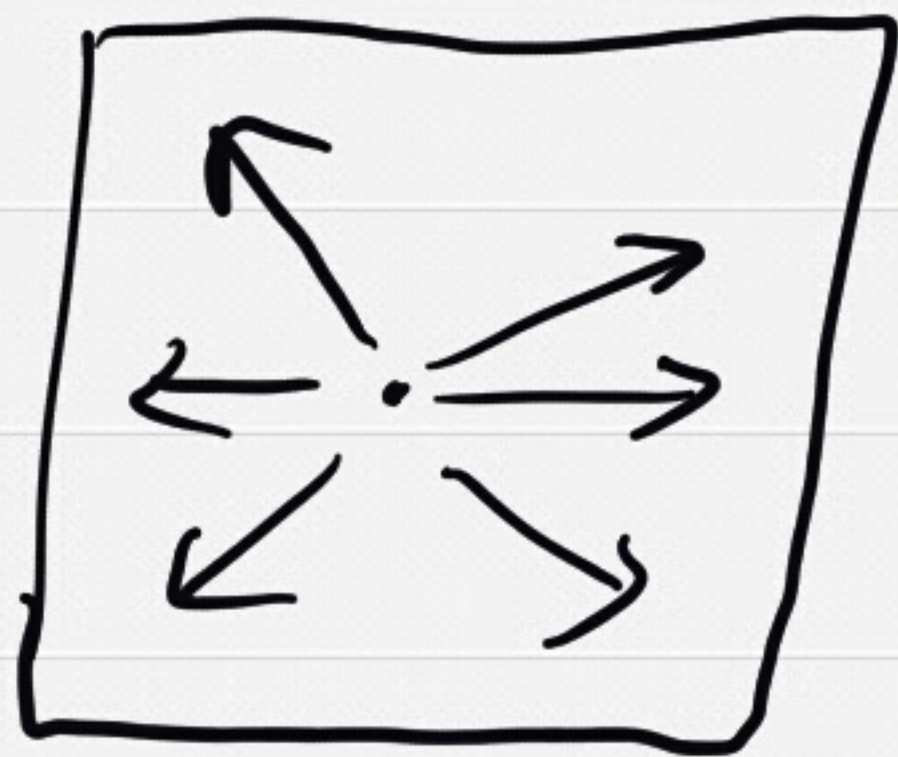
$$u(\lambda) = \frac{\text{energy}}{\text{volume}} \quad \text{at wavelength } \lambda$$

Total energy in box:

$$E = \int u(\lambda) d\lambda \cdot \text{Volume}$$

$$I(\lambda) = \frac{\text{energy}}{\text{area} \cdot \text{time}} = \text{intensity}$$

$$= \frac{\text{energy}}{\text{volume}} \cdot \frac{\text{distance}}{\text{time}}$$



$$\langle v_x \rangle = c/2$$

half to left,
half to right

$$\Rightarrow I(\lambda) = c/4 \cdot u(\lambda)$$

State w/o proof:

Number of modes of oscillation

$$N(\lambda) = \frac{8\pi \cdot \text{volume}}{\lambda^4} \quad \text{in 3-d box}$$

Assuming equipartition

- Each mode of oscillation has $\langle E \rangle = kT$

$$\begin{aligned}\Rightarrow u(\lambda) &= \frac{N(\lambda)}{\text{Volume}} \cdot kT \\ &= \frac{8\pi kT}{\lambda^4}\end{aligned}$$

$$I(\lambda) = c/4 u(\lambda)$$

$$= \boxed{\frac{2\pi c kT}{\lambda^4}}$$

"Rayleigh - Jeans Law"

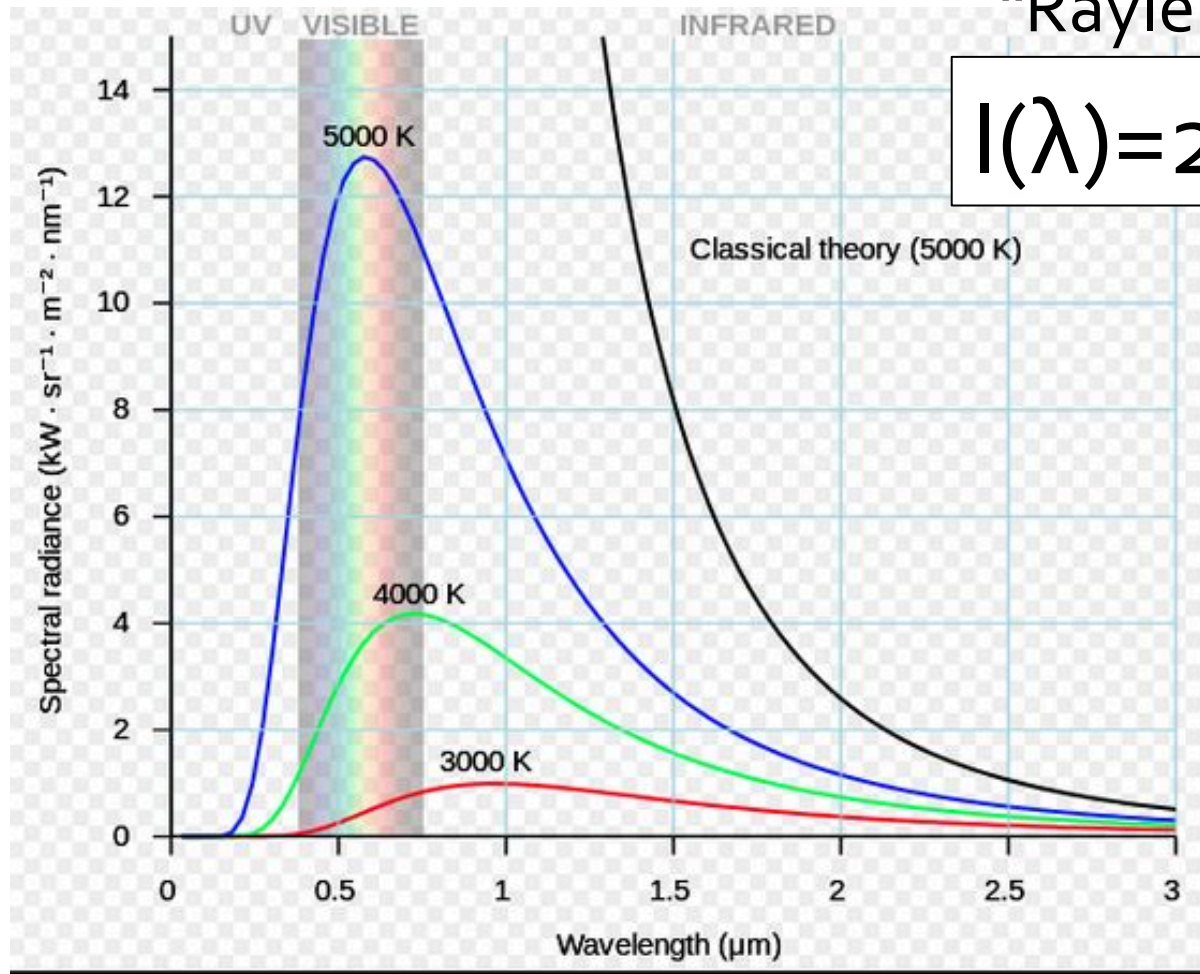
diverges for $\lambda \rightarrow 0$

= "ultraviolet catastrophe"

- Note total intensity

$$I = \int_0^{\infty} I(\lambda) d\lambda \text{ is infinite!!}$$

Ultraviolet Catastrophe



“Rayleigh-Jeans Law”

$$I(\lambda) = 2\pi c / \lambda^4 * kT$$

Maxwell-Boltzmann Statistics

With increasing energy E , it is progressively less likely that any given particle will attain that energy, so more particles will be found with lower energies. It is assumed that an unlimited number of particles can occupy any energy state.

The probability that a particle will have energy E

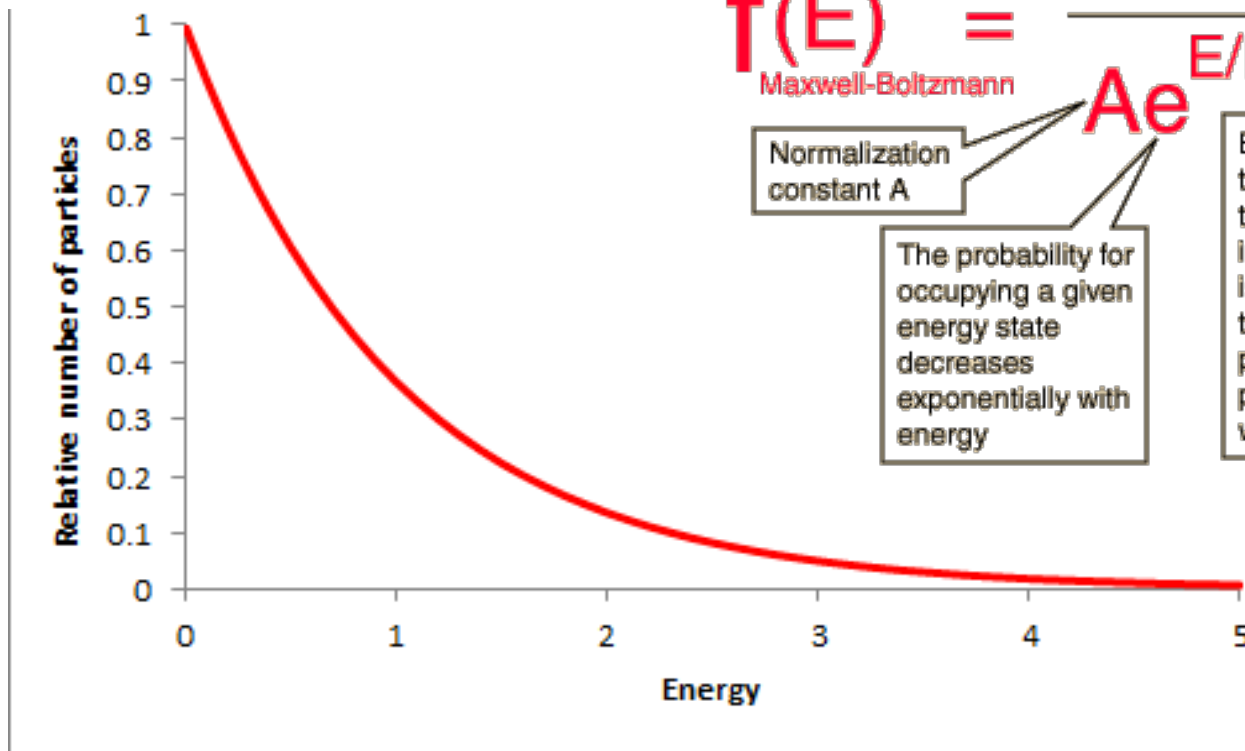
$$f(E) = \frac{1}{Ae^{E/kT}} = Be^{-E/kT}$$

Maxwell-Boltzmann

Normalization constant A

The probability for occupying a given energy state decreases exponentially with energy

Boltzmann's constant k times the absolute temperature T . The implication of this term is that for a higher temperature, it is more probable that a given particle can be found with energy E .



Concept Check

- The integral over all energies of $Be^{-E/kT}$ should equal N (the total number of oscillators).

What is B ?

- A. NkT
- B. N/kT
- C. kT/N
- D. $1/(NkT)$

Note: The integral of $\exp(-x/a)$ from $E=0$ to $E=\infty$ is a

Concept Check

- The integral over all energies of $Be^{-E/kT}$ should equal N (the total number of oscillators).
What is B ?

- A. NkT
- B. N/kT
- C. kT/N
- D. $1/(NkT)$

Note: The integral of $\exp(-x/a)$ from $E=0$ to $E=\infty$ is a

Maxwell - Boltzmann (continuous)

$$\int_0^{\infty} \beta e^{-E/kT} dE$$
$$= \beta \cdot (-kT) \cdot e^{-E/kT} \Big|_0^{\infty}$$
$$= \beta \cdot kT = N$$

$$\Rightarrow \beta = N/kT$$

$$\Rightarrow N(E) = \frac{N}{kT} e^{-E/kT}$$

= number of oscillators w/
energy E

$$\langle E \rangle = \frac{1}{N} \int_0^{\infty} N(E) \cdot E dE$$
$$= \frac{1}{N} \int_0^{\infty} \frac{N}{kT} e^{-E/kT} \cdot E dE$$

Trick $\int_0^{\infty} E e^{-aE} dE = \int_0^{\infty} \left(-\frac{d}{da} e^{-aE} \right) dE$
 $= -\frac{d}{da} \int_0^{\infty} e^{-aE} dE = -\frac{d}{da} \left(-\frac{1}{a} e^{-aE} \Big|_0^{\infty} \right)$

$$= -\frac{d}{da} \left(\frac{1}{a} \right) = \frac{1}{a^2}$$

$$\Rightarrow \langle E \rangle = \frac{1}{kT} \cdot \frac{1}{(1/kT)^2}$$
$$= \boxed{kT}$$