Physics IV [2704] Midterm 2
Wednesday April 4th, 2018

Directions: This exam is closed book. You are allowed one 8.5”x11” sheet with equations etc., which should be turned in with your test. Read all the questions carefully and answer every part of each question. Please show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect. Please use a calculator only to check arithmetic – all steps of calculations should be explicitly shown. Unless otherwise instructed, you can express your answers in terms of fundamental constants like $k, h, \hbar, c, \varepsilon_0$ rather than calculating numerical values. If the question asks for an explanation, please write at least a full sentence explaining your reasoning. Please ask if you have any questions, including clarification about any of the instructions, during the exam.

<u>This test is designed to be gender and race neutral.</u>

Good luck!

A few useful numbers:

Speed of light $c = 3\text{x}10^8$ m/s
Planck’s constant $h = 6.6\text{x}10^{-34}$ J s = $4\text{x}10^{-15}$ eV s
Compton wavelength $h/(m_e c) = 2.4\text{x}10^{-12}$ m
Photon energy = $h\nu = hc/\lambda \sim 1240$ eV-nm /(λ in nm)
Bohr energies $E_n = -me^4/(32\pi^2\varepsilon_0^2\hbar^2) * Z^2/n^2 = -13.6\text{eV} * Z^2/n^2$
Bohr radius $a_0 = 4\pi\varepsilon_0\hbar^2/(me^2) = 0.0529$ nm

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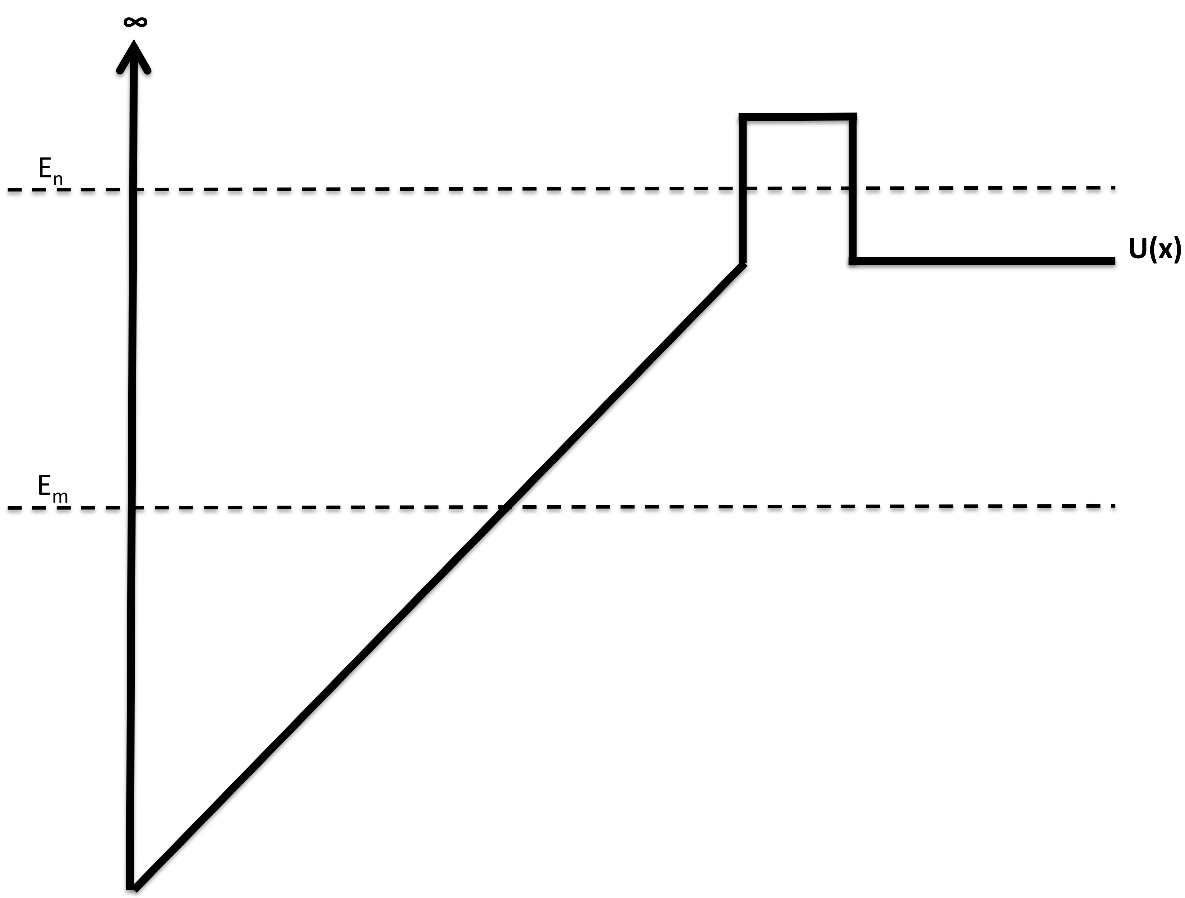
Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else’s test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name_________________________________

Print Your Name_________________________________

Question 1 (20 points): Sketch the real component of two wave functions that might occur in the asymmetric potential well shown below, for electrons at the energy levels E_n and E_m. Make sure your wave functions include at least three full wavelengths, and are qualitatively accurate in how their wavelength and amplitude vary across the drawing. *Don't worry about the relative number of wavelengths in the two functions – just use enough to make the variation clear in both cases. If you're not sure of your artistic skills, feel free to annotate the drawing to clarify the variation in wavelength, amplitude, etc.*



Question 2 (30 points): In a one dimensional world, we could write the potential energy for an electron bound to a proton as $U(x) = -a/x$, for $x > 0$.

2a (25 points): Show that $\psi(x) = A\, x\, e^{-bx}$ is a solution of the time-independent 1-d Schrödinger equation for this potential, and solve for the corresponding values of b and E in terms of a.

2b (5 points): Write down an equation (you do not need to solve it) that would allow you to determine the value of the normalization constant A.

Question 3 (15 points): Consider a boron atom ($Z = 5$), with four of its electrons removed, so that it is "hydrogenic" and the Bohr model applies. Calculate the longest wavelength at which this ion can absorb a photon, assuming the ion starts in its ground state. *Don't compute a numerical value - leave your answer as a symbolic expression in terms of fundamental constants.*

Question 4 (10 points): What physical quantities do the four quantum numbers n, l, m_l, m_s provide information about for the electron in a hydrogen atom?

Question 5 (15 points): The ground state radial wave function for the electron in the hydrogen atom is $R_{10}(r) = \frac{2}{{a_0}^{3/2}} e^{-r/a_0}$. Calculate the average radius $\langle r \rangle = \int_0^\infty rP(r)dr$ for the electron in this orbital (where *P(r)* is the radial probability density). *You will need to use the integral* $\int_0^\infty r^n e^{-br} dr = n!/(b^{n+1})$.

Question 6 (10 points): For the *n=3, l = 2, m$_l$ = 1* hydrogen wave function, the angular portion of the electron wave function is $\Theta(\theta)\Phi(\phi) = A sin\theta cos\theta e^{i\phi}$, where *A* is a normalization constant. Sketch the two portions of the angular probability density $|\Theta(\theta)|^2$and $|\Phi(\phi)|^2$ separately over the appropriate ranges of these two angular coordinates. At what approximate range(s) of angles would the electron have the highest probability of being found?