Physics IV [2704] Sample Final Exam

Directions: This exam is closed book. You are allowed both sides of an 8.5”x11” sheet with equations etc. Read all questions carefully and answer every part of each question. Please show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect. Please use a calculator only to check arithmetic – all steps of calculations should be explicitly shown. Unless otherwise instructed, you can express your answers in terms of fundamental constants like *k, h, ħ, c,* ε_0 rather than calculating numerical values. If the question asks for an explanation, please write at least a full sentence explaining your reasoning. Please ask if you have any questions, including clarification on instructions, during the exam.

This test is designed to be gender and race neutral.

Good luck!

A few useful numbers:

$v/c = 0.5$	$\gamma = 1.15$	|	$v/c = 0.8$	$\gamma = 5/3$
$v/c = 0.6$	$\gamma = 1.25$	|	$v/c = (\sqrt{3})/2 = 0.866$	$\gamma = 2$
$v/c = 1/\sqrt{2}$	$\gamma = \sqrt{2}$	|	$v/c = 0.98$	$\gamma = 5$
$v/c = 0.75$	$\gamma = 1.51$	|	$v/c = 0.999$	$\gamma = 22$

Speed of light $c = 3\text{x}10^8$ m/s

Planck’s constant $h = 6.6\text{x}10^{-34}$ J s $= 4\text{x}10^{-15}$ eV s $\quad \hbar = h/(2\pi)$

Compton wavelength $h/(m_e c) = 2.4\text{x}10^{-12}$ m

Photon energy $= h\nu = hc/\lambda \sim 1240$ eV-nm /(λ in nm)

Bohr energies $E_n = -m_e e^4/(32\pi^2\varepsilon_0^2\hbar^2) *Z^2/n^2 = -13.6\text{eV} * Z^2/n^2$

Bohr radius $a_0 = 4\pi\varepsilon_0\hbar^2/(m_e e^2) = 0.0529$ nm

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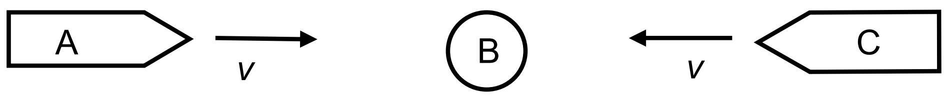
Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else’s test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name____

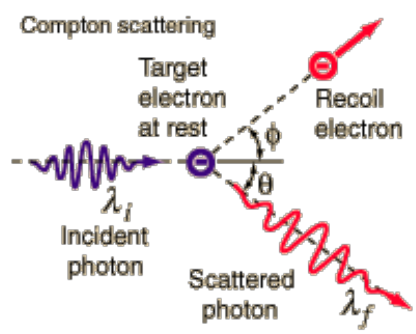
Print Your Name____

Question 1 (10 points): Rockets **A** and **C** move toward each other with identical speeds v = 0.5c in opposite directions relative to **B**, who is at rest in this frame of reference. A stick of length L_0 carried by **A** has a length of $L_0/1.15$ according to **B**. According to **C**, what is the length of the stick carried by **A**?



Relative motion according to B

Question 2 (10 points): In Compton scattering, an incident X-ray photon scatters from an electron initially at rest. Derive (do not solve) the three equations for conservation of relativistic energy and linear momentum in terms of the initial and final photon wavelengths λ_i and λ_f, the photon and electron scattering angles θ and ϕ, and the rest mass m_e and the final momentum p_e and energy E_e of the electron.



Question 3 (10 points): Light of wavelength 310 nm shines on a metallic surface with a work function of 1.2 eV.

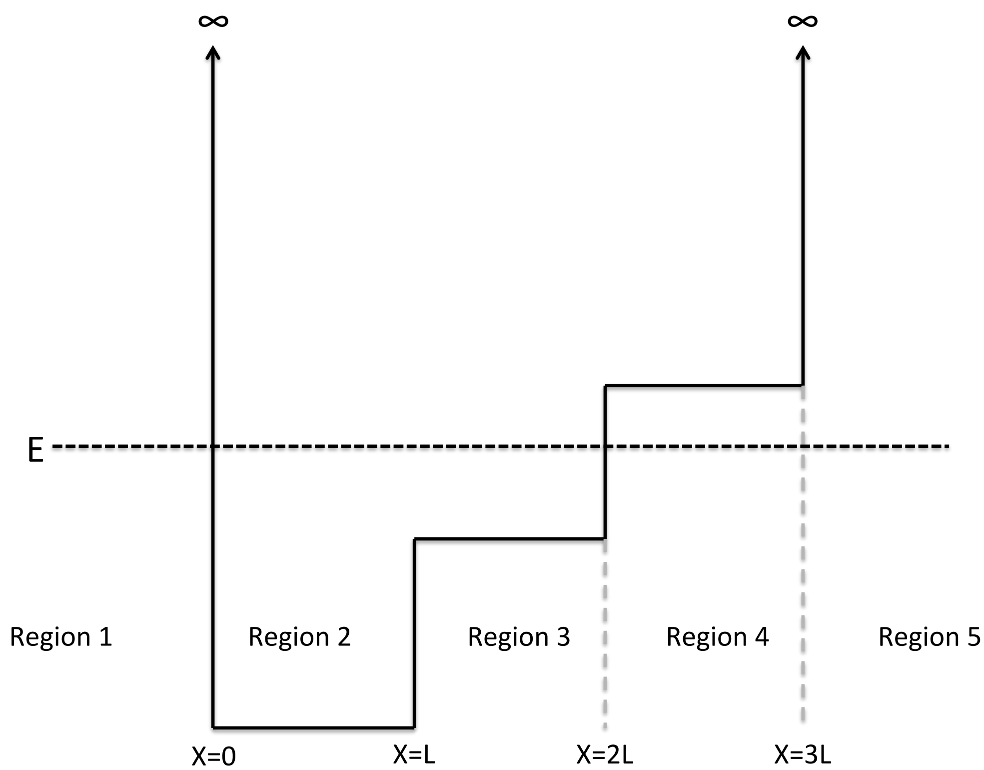
A (5 points). What is the maximum energy (in eV) of photoelectrons emitted from the surface?

B (5 points). What is the de Broglie wavelength (in nm) of an electron with the maximum energy from part A? You will need the mass and charge of the electron to solve this problem: $q = 1.6\text{x}10^{-19}$ C and $m_e = 9.1\text{x}10^{-31}$ kg. An answer correct to one order of magnitude is okay if you don't have a calculator. Note that at these energies, electrons are non-relativistic.

Question 4 (12 points): A particle of mass m moves in a one-dimensional region with a Coulomb-like potential energy $U(x) = -C/x$, where C is a positive constant. Show that $\psi(x) = Axe^{-ax}$ (with $a = mC/\hbar^2$) is a solution to the one-dimensional time-independent Schrödinger equation for this potential energy. What is the total energy E corresponding to this wave function?

Question 5 (12 points). An electron is trapped in an infinite potential well with an asymmetric potential structure in the well. The electron's energy is higher than the potential energy in regions 2 and 3, but lower in region 4 (and of course 1 and 5).

A (6 points). Sketch the real part of a possible wave function at the energy level shown, for a wave function with at least three wavelengths across the well, being careful to show qualitatively correct variations in amplitude and wavelength across the well.



B (3 points). In which one of the five regions will the electron have the maximum probability of being found? Why?

C (3 points). In which one or more of the five regions will the electron have zero probability of penetrating? Why?

Question 6 (12 points): The angular part of the hydrogen atom wavefunction for an electron with quantum numbers ($\boldsymbol{n=3, l = 2, m_l = 1}$) is proportional to $\sin\theta \cos\theta \, e^{i\phi}$.

A (4 points). Sketch the angular probability density for this electron wavefunction, as two separate line graphs of $|\psi|^2$ vs. θ and $|\psi|^2$ vs. ϕ.

B (4 points). How much energy (in eV) would be required to excite an electron from the hydrogen ground state to this state?

C (4 points). Compared to an electron with ($\boldsymbol{n=3, l = 0, m_l = 0}$), does the electron above have a higher or lower probability of being found very close to the nucleus? Why?

Question 7 (6 points): Helium has an atomic number of 2, while Lithium has an atomic number of 3. In neutral form, one of them is much easier (takes less energy) to ionize (remove one electron). Which one is it? Describe why in terms of the electron orbitals.

Question 8 (8 points): A K_α X-ray is produced when an electron is knocked out of the $n=1$ orbital and replaced by an electron that transitions from the $n=2$ orbital to fill the gap in the $n=1$ orbital. A K_α X-ray is observed with the energy 9180 eV. What is the atomic number Z of the element it was produced from?

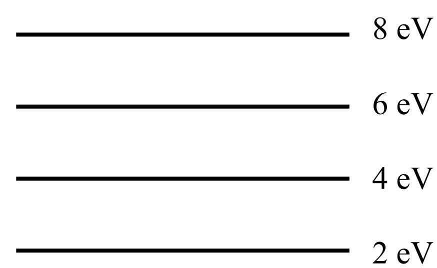
Question 9 (10 points): In an unknown molecule, the energy spacing between the ground state vibrational level and the first excited vibrational level is 1.5 eV. The energy spacing between the ground state rotational level and the first excited rotational level is 0.1 eV.

A (3 points). What is the energy spacing between the first excited vibrational level and the second excited vibrational level?

B (3 points). What is the energy spacing between the first excited rotational level and the second excited rotational level?

C (4 points). Assuming there are only the three vibrational and three rotational levels mentioned above, what are the only four possible energies of photons that could be absorbed by the molecule in question (starting in any of the nine possible states)?

Question 10 (10 points): Consider a collection of six particles at a temperature of T = 0 K, which can only occupy the four energy levels shown.



A (5 points). What is the average energy of these six particles if they are bosons?

B (5 points). What is the average energy of these six particles if they are fermions?